Example:
A quadrilateral with coordinates $\mathrm{A}(2,1), \mathrm{B}(1,3)$ and $\mathrm{C}(2,4)$,
$C(3,3)$ is reflected in the line $x=1$. Draw a diagram to show the object and the image and state the point which does not move under the transformation.

$B(1,3)$ does not move under the transformation (invariant)

## Example:

A square $A B C D$, whose vertices are $A(1,1), B(3,1), C(3,3), D(1$, 3 ) is reflected in the line
i. $X=1$, to give $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$
ii. $Y=2$, to give $A^{\prime \prime} B^{\prime \prime} C^{\prime \prime} D^{\prime \prime}$
iii. $Y=X$, to give $A^{\prime \prime \prime} B^{\prime \prime \prime} C^{\prime \prime \prime} D^{\prime \prime \prime}$

Find the co-ordinates of the vertices of those three images

## of square $A B C D$.



Example:
The image of triangle PQR, $P(-5,1), Q(-2,1)$ and $R(0,3)$ is triangle $P^{\prime} Q^{\prime} R^{\prime}$ whose vertices are $P^{\prime}(-1,3), Q^{\prime}(-1,0), R(1,2)$.

Find the equation of the reflection line.

## Solution:

*Join any point to its image say $P$ to $P^{\prime}$ and draw a perpendicular bisector of PP' $^{\prime}$. This perpendicular bisector is the reflection line.
*Identify any two points on this line and use them to find the equation of the line.

## Question 1:

Draw the quadrilateral PQRS which has vertices $\mathrm{P}(2,2), \mathrm{Q}(-3,2)$, $R(-2,6)$ and $R(2,5)$. Draw the image $P^{\prime} Q^{\prime} R^{\prime} S^{\prime}$ after reflection in the line $y=2$. Give the coordinates of $P^{\prime} Q^{\prime} R^{\prime}$ and $S^{\prime}$ and any invariant point.

## Question 2:

A triangle with coordinates $\mathrm{P}(4,0), \mathrm{Q}(5,5)$ and $\mathrm{R}(3,4)$ is given a translation equivalent to the vector $\binom{3}{-2}$. Find the coordinates of $P^{\prime}, Q^{\prime}$ and $S^{\prime}$, the images of $P, Q$ and $R$ respectively after this translation. $P^{\prime} Q^{\prime} R^{\prime}$ is reflected in the line $x=6$ to give $P^{\prime \prime} Q^{\prime \prime} R^{\prime \prime}$. Find the coordinates of $P^{\prime \prime} Q^{\prime \prime}$ and $R^{\prime \prime}$.

## ROTATION

A transformation in which an image is formed by turning the object is called rotation. Rotation is also aninsometry because the size and shape are maintained.

Rotation is fully specified by stating the;

- center of rotation,
- angle of rotation and
- direction of rotation

If the center of rotation is one of the points on the object, then that point is invariant.

The angle of rotation is positive if the rotation is anticlockwise and it is negative if the rotation is clockwise.

## Example:

Given the points, $A(2,3), B(-1,2)$, find the images if rotated about $(1,1)$ through $180^{\circ}$

Solution

- Mark the points and the center of rotation on the Cartesian plane.
- Join point A to the center of rotation and use your protractor to get an angles of $180^{\circ}$ anticlockwise.
- Repeat step two for point B.

Note: Even if it is a figure e.g. polygon, the same procedure is followed.

$A^{\prime}(0,-1), \quad B^{\prime}(3,0)$

## Example:

Triangles $A B C$ has vertices at $A(3,1), B(6,1)$, and $C(6,3)$. Find its image
$A^{\prime} B^{\prime} C^{\prime}$ after a rotation of $+90^{\circ}$ about $P(0,2)$.


## Example:

The object PQ has been rotated to give the image $P^{\prime} Q^{\prime}$. Find the center of rotation and angle of rotation.

\# The center of rotation is obtained from the intersection of the two mediators i.e. the mediator of $\mathrm{QQ}^{\prime}$ and $\mathrm{PP}^{\prime}$
\#The angle of rotation is given either by <POP' or <QOQ'

## Finding the angle and center of rotation

1. Join an object point i.e. A to its image $A^{\prime}$.
2. Draw a mediator ( perpendicular bisector) $m_{1}$ of $A A^{\prime}$
3. Join a second point $B$ to its image $B^{\prime}$
4. Draw a mediator $m_{2}$ of $B^{\prime}$
5. Mark the point of intersection (C) of the two mediators which is the center of rotation.
6. Join one point and its respective image to the center of rotation.
7. Measure the angle either $<A C A^{\prime}$ or $<B C B^{\prime}$..

## Note:

The angle of rotation, measured from the object to the image, is negative if it is a clockwise rotation and positive if it is an anticlockwise rotation

## Question 1:

The point $\mathrm{P}(2,3)$ and $\mathrm{Q}(-1,1)$ map onto $\mathrm{P}^{\prime}(0,-1)$, and
$Q^{\prime}(3,4)$ under a rotation. Use squared paper to find
a) The coordinates of the center of rotation.
b) The angle of rotation
c) The image of $(0,0)$ under this rotation.
d) The coordinates of $R$ if $R^{\prime}$ is the point $(-4,-3)$

## Question 2:

Square $A B C D$ where $A(2,1), B(1,1), C(1,2)$ and $D(2,2)$ is rotated through $90^{\circ}$ about $(3,3)$. Find the image of the square under this rotation.

