

S.2 Mathematics | ALGEBRA

Algebraic expressions and Equations

Algebraic expressions are expressions that involve letters or symbols. Numbers can be represented by letters or symbols and what is formed in algebraic expressions. E.g. $a+3$, $2n$, $3ab$, $2a+5$ etc.

These expressions are treated like it were a number. We can add, subtract, multiply and divide like we do for numbers.

Algebraic equations

We can use algebraic expressions to form algebraic equations. e.g.

$$a + 3 = 5$$

$$2n = 12$$

$$3x + 5 = 17$$

$$2a + b = 36$$

Example 1:

Form an algebraic equations for each of the statements below.

- John is twice as old as his son James.
- My father gave me half his books. I had 10 books already but now I have 35 note books.
- I have 5 more pens than pencils.
- If had 5 more cows than I have now, I should have 9.

Solutions

a) Let John's age be y and James' age be x

$$y = 2x$$

b) Let the number of books my father had be n , he gave $\frac{n}{2}$

$$10 + \frac{n}{2} = 35$$

c) Let the number of pencils I have be a and pens be b

$$a + 5 = b \quad \text{or} \quad b - a = 5 \quad \text{or} \quad b - 5 = a$$

d) Let the number of cows I have now be n

$$n + 5 = 9$$

Example 2:

Simplify the following algebraic expressions

- a) $2a + 3 + 5a$
- b) $13 - b + 6b$
- c) $3(1 + a) - 4a$

Solutions

a) $2a + 3 + 5a$

Arrange the terms containing a together and collect like terms.

$$2a + 5a + 3$$
$$7a + 3$$

b) $13 - b + 6b$

$$13 + 6b - b$$
$$13 + 5b$$

c) $3(1 + a) - 4a$

$$3 \times 1 + 3 \times a - 4a$$
$$3 + 3a - 4a$$
$$3 - a$$

Example 3:

Simplify these expressions

a) $2p - 5 + 7p$

b) $2(x + 9) - 3x + 4$

c) $2xy - 1 + 4xy$

d) $xyz + yzx + zxy + xzy - 3xyz$

e) $ab + 2bc - 3ac + 1$

Solutions

a) $2p - 5 + 7p$

$$= 2p + 7p - 5$$
$$= 9p - 5$$

b) $2(x + 9) - 3x + 4$

$$2x + 18 - 3x + 4$$

$$2x - 3x + 18 + 4$$

$$-x + 22 \quad \text{or} \quad 22 - x$$

$$\begin{aligned} \text{c) } & 2xy - 1 + 4xy \\ & 2xy + 4xy - 1 \\ & 6xy - 1 \end{aligned}$$

$$\begin{aligned} \text{d) } & xyz + xyz + xyz + xyz - 3xyz \\ & = 4xyz - 3xyz \\ & = xyz \end{aligned}$$

$$\text{e) } ab + 2abc - 3ac + 1$$

The terms in part (e) do not have exactly the same letters; therefore, they are not like terms and cannot be simplified.

Example 4:

(Substitution)

a) Given that, $a = 2b - 3c$, find the value of a when

i) $b = 12, c = 5$

ii) $b = 9, c = 1$

iii) $b = 8, c = 6$

b) $m = n^2 + \frac{1}{4}p$. Find the value of m when

i) $n = 4, p = 8$

ii) $n = 6, p = 4$

iii) $n = 20, p = 400$

Solutions

a) $a = 2b - 3c$

i) when $b = 12, c = 5$

$$a = 2(12) - 3(5)$$

$$= 24 - 15$$

$$= 9$$

ii) When $b = 9, c = 1$

$$a = 2(9) - 3(1)$$

$$= 18 - 3$$

$$= 15$$

iii) When $b = 8, c = 6$

$$a = 2(8) - 3(6)$$

$$= 16 - 18$$

$$= -2$$

$$b) m = n^2 + \frac{1}{4}p$$

i) when $n = 4$, $p = 8$

$$\begin{aligned} m &= (4)^2 + \frac{1}{4}(8) \\ &= 16 + 2 \\ &= 18 \end{aligned}$$

ii) When $n = 6$, $p = 4$

$$\begin{aligned} m &= (6)^2 + \frac{1}{4}(4) \\ &= 36 + 1 \\ &= 37 \end{aligned}$$

iii) When $n = 20$, $p = 400$

$$\begin{aligned} m &= 20^2 - \frac{1}{4}(400) \\ &= 400 - 100 \\ &= 300 \end{aligned}$$

EXERCISE

1) Form an algebraic expression for each of the following quantities.

- I have n bananas, my sister has three more than I do. How many bananas does she have?
- I weigh y kg. I am 7kg lighter than my mother. How many kg does my mother weigh?
- Mariam's rope is b cm long and Milly's is 8cm longer. How long is Milly's rope in terms of Mariam's?
- I am f years old. My brother is twice as old as I am. How old is my brother?

2) Form an algebraic equation for each of these statements.

- Yesterday I picked m bags of oranges. Today, I picked n bags. Altogether I picked 25 bags.
- I have h hens. If I had six more, I should have 18 hens.
- There were ten trees in the field. A ranger cut down t of them. How many are left?
- My sister has a total of 3m beads. She has 12 white ones and n blue ones.

3. A pen costs b shillings and a pencil costs 900 shillings less than a pen. **Write an expression for the total cost of a pen and a pencil.**

4. Simplify the following by grouping the positives and negatives.

- a) $11a + 5a - 2a + a$
- b) $12d - 5d - 3d + 4d$
- c) $17p - 8p - 2p$
- d) $4b + 2b - 9b + 8b$

5. Simplify the following by collecting like term

- a) $15 - 2x + 10x$
- b) $4y - 2x + 5x - 3y$
- c) $20x - 4y - y - 3x$
- d) $9a + 10b - 5a - 4b$

Algebraic products

If an expression contains two or more terms, we may write it in brackets. E.g. $4 + 3$ may be written as $(4+3)$ or $a + b + c$ may be written as $(a + b + c)$. If multiply this number by a number say 2, it multiplies each term inside the bracket.

$$\begin{aligned} \text{i.e. } 2(4 + 3) &= 2 \times 4 + 2 \times 3 \\ &= 8 + 6 \\ &= 14 \end{aligned}$$

But $2(4 + 3)$ is also equal to

$$\begin{aligned} 2 \times 7 \\ = 14 \end{aligned}$$

Similarly $2(a + b + c)$

Example 1:

- Multiply out
- i) $5(2a + 4b)$
 - ii) $3(2w + 3x - 4y)$
 - iii) $(2m - 5n) 2l + (3l + 4m) 2n$

Solution

$$\begin{aligned} \text{i) } &= 5(2a + 4b) \\ &= 5 \times 2a + 5 \times 4b \\ &= 10a + 20b \end{aligned}$$

$$\begin{aligned} \text{ii) } &3x(2w + 3x - 4y) \\ &= 3x \times 2w + 3x \times 3x - 3x \times 4y \\ &= 6xw + 9x^2 - 12xy \end{aligned}$$

$$\text{iii) } (2m - 5n) 2l + (3l + 4m) 2n$$

$$= 4ml - 10ln + 6ln + 8mn$$
$$= 4ml - 4ln + 8mn$$

Note that $5(2a - 4b) = (2a - 4b)5$

The number outside the brackets is called a factor.

Example 2:

Evaluate each expression in two ways

- a) $3(6 + 3)$ d) $(2 + 5) \times 8$
b) $4(7 + 4)$ e) $(6 + 5) \times 7$
c) $6(9 - 7)$ f) $(11 - 6) \times 6$

Solution

a) $3(6 + 3) = 3 \times 6 + 3 \times 3$
 $= 18 + 9$
 $= 27$

OR $3(6 + 3) = 3 \times 9$
 $= 27$

b) $4(7 + 4) = 4 \times 7 + 4 \times 4$
 $= 28 + 16$
 $= 44$

OR $4(7 + 4) = 4 \times 11$
 $= 44$

c) $6(9 - 7) = 6 \times 9 - 6 \times 7$
 $= 54 - 42$
 $= 12$

OR $6(9 - 7) = 6 \times 2$
 $= 12$

d) $(2 + 5) \times 8 = 2 \times 8 + 5 \times 8$
 $= 16 + 40$
 $= 56$

OR $(2 + 5) \times 8 = 7 \times 8$
 $= 56$

e) $(6 + 5) \times 7 = 6 \times 7 + 5 \times 7$
 $= 42 + 35$
 $= 77$

$$\begin{aligned}\text{OR } (6 + 5) \times 7 &= 11 \times 7 \\ &= 77\end{aligned}$$

$$\begin{aligned}\text{f) } (11 - 6) \times 6 &= 11 \times 6 - 6 \times 6 \\ &= 66 - 36 \\ &= 30\end{aligned}$$

$$\begin{aligned}\text{OR } (11 - 6) \times 6 &= 5 \times 6 \\ &= 30\end{aligned}$$

Example 3:

Multiply each term in the brackets by the factor.

- a) $4(a + b)$
- b) $2(3g + h)$
- c) $3(3n - 2)$
- d) $4(r - 2s + 3t)$
- e) $(5n - 3)p$
- f) $3a(b + 2c - 4d)$

Solution

$$\text{a) } 4(a + b) = 4 \times a + 4 \times b = 4a + 4b$$

$$\text{b) } 2(3g + h) = 2 \times 3g + 2 \times h = 6g + 2h$$

$$\text{c) } 3(3n - 2) = 3 \times 3n - 3 \times 2 = 9n - 6$$

$$\text{d) } 4(r - 2s + 3t) = 4 \times r - 4 \times 2s + 4 \times 3t = 4r - 8s + 12t$$

$$\text{e) } (5n - 3)p = 5n \times p - 3 \times p = 5np - 3p$$

$$\begin{aligned}\text{f) } 3a(b + 2c - 4d) &= 3a \times b + 3a \times 2c - 3a \times 4d \\ &= 3ab + 6ac - 12ad\end{aligned}$$

Example 4:

Expand and simplify each of the following expressions as much as possible

- a) $2(a + b) + 3(a - b)$
- b) $4(2c + d) + 5(d - c)$
- c) $5(3j - k) - 4(5k + m)$
- d) $2g(h - 2j) - 3g(h - 2j)$

Remember

*When there is a positive sign before the bracket, the signs of the terms inside the bracket remain the same when the bracket are removed.

*When there is negative sign before the bracket, the signs of the terms inside the brackets change when the brackets are removed.

Solutions

$$\begin{aligned} \text{a)} &= 2(a + b) + 3(a - b) \\ &= 2a + 2b + 3a - 3b \\ &= 2a + 3a + 2b - 3b \\ &= 5a - b \end{aligned}$$

$$\begin{aligned} \text{b)} &= 4(2c + d) + 5(d - c) \\ &= 8c + 4d + 5d - 5c \\ &= 8c - 5c + 4d + 5d \\ &= 3c + 9d \end{aligned}$$

$$\begin{aligned} \text{c)} &= 5(3j - k) - 4(5k + m) \\ &= 15j - 5k - 20k - 4m \\ &= 15j - 25k - 4m \end{aligned}$$

$$\begin{aligned} \text{d)} &= 2g(h - 2j) - 3g(h - 2j) \\ &= 2gh - 4gh - 3gh + 6gj \\ &= 2gj - gh \end{aligned}$$

EXERCISE

1) Remove the brackets and then simplify.

- a) $6x + (3 - x)$
- b) $(2p + 6q) - q$
- c) $(x - 3y) + (x + 3y)$
- d) $(12a + 7) + (4 - 5a)$

2) Remove the brackets from the following (or expand).

- a) $6(7 - h)$
- b) $7(2x + 9y)$
- c) $3x(2y - 7)$
- d) $(w - x)4x$

3) Simplify the expressions as much as possible.

- a) $2(3e + 2f) + 5(e - 2f)$
- b) $2(3p + q) + 3(p - 2q)$
- c) $3(4g - 3h) - 2(5g + 2h)$
- d) $2d(e - 3f) - d(e - 2f)$

Binomial Products

A binomial is an expression with two terms. E.g. $a + b$, $2x + 3$, $2 + a$, etc.

An expression with two or more terms may be written with brackets as $(a + b)$, $(2x + 3)$, $(2 + a)$.