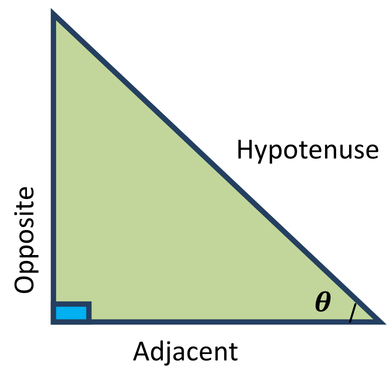
TRIGONOMETRY REVISION

The Sine, Cosine and Tangent of angles



$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

$$\sin\theta = \frac{Adjacent}{Hypotenuse}$$

$$\tan\theta = \frac{Opposite}{Adjacent}$$

Recall:

Hypotenuse is the longest side.

Opposite is the side where the angle is facing.

Adjacent is the side next to the angle and with which the hypotenuse makes the angle.

Note:

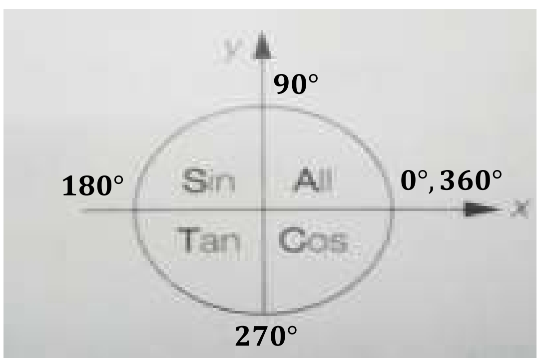
If one ratio is given, we can find the others (refer to the examples below).

Example:1

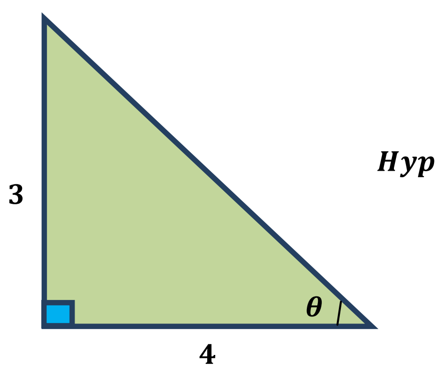
Given that θ is an acute angle and that $\tan\theta = \frac{3}{4}$, find the value of $\sin\theta$ and $\cos\theta$

Solution:

If θ is acute ($0° \leq \theta \leq 90°$) it lies within the first quadrant, where all ratios are positive.



Then,



$$\text{Tan}\,\theta = \frac{Opposite}{Adjacent} = \frac{3}{4}$$

$Hyp^2 = 4^2 + 3^2$

$= 16 + 9$

$= 25$

Hyp $= \sqrt{25}$

$= 5$ units

$$\therefore \ \sin\theta = \frac{Opposite}{Hypotenuse} = \frac{3}{5}$$

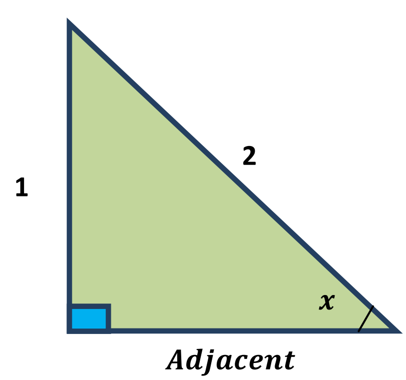
$$\cos\theta = \frac{Adjacent}{Hypotenuse} = \frac{4}{5}$$

<u>Example:2</u>

Given that $\sin x = 0.5$, find $\cos x$ and $\tan x$.

Note that $0.5 = \frac{1}{2}$

$$\sin x = \frac{Opposite}{Hypotenuse} = \frac{1}{2}$$



$(\text{Adjacent})^2 = (\text{Hypotenuse})^2 - (\text{Opposite})^2$

$= 2^2 - 1^2$

$= 4 - 1$

$= 3$

Adjacent = $\sqrt{3}$

$$\cos x = \frac{Adjacent}{Hypotenuse} = \frac{\sqrt{3}}{2}$$

$\tan x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ (With rational denominator)

Since $\sin x$ is positive, x may be in the 1st or 2nd quadrant.

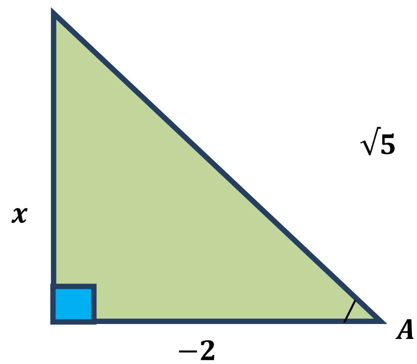
If it is in the second quadrant, then

$\cos x = \frac{-\sqrt{3}}{2}$ and $\tan x = \frac{-\sqrt{3}}{3}$

<u>Example:3</u>

Given that $\cos A = \frac{-2}{\sqrt{5}}$ $and\ 0° < A < 180°$

Find $\cos A = \frac{Adjacent}{Hypotenuse}$



$(\text{Hypotenuse})^2 = (\text{Adjacent})^2 + (\text{Opposite})^2$

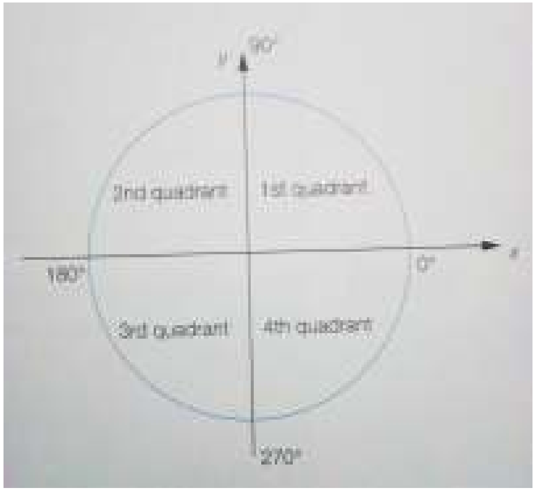
$(5)^2 = (-2)^2 + (x)^2$

$x^2 = 5 - 4$

$= 1$

$x = \sqrt{1}$

$= 1$



Since $\cos A$ is negative and A is less than 180°, then A is in the second quadrant where sines are positive and tangents are negative.

$$\sin A = \frac{Opposite}{Hypotenuse} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\tan A = \frac{Opposite}{Adjacent} = \frac{-1}{2}$$

<u>Exercise:</u>

1) Find the value of A in degrees, if;

a)$\sin A = \cos 30°$

b)$\cos A = \sin 70°$

c)$\cos A = \sin 2A$

Hint: Sin θ = Cos (90°- θ)

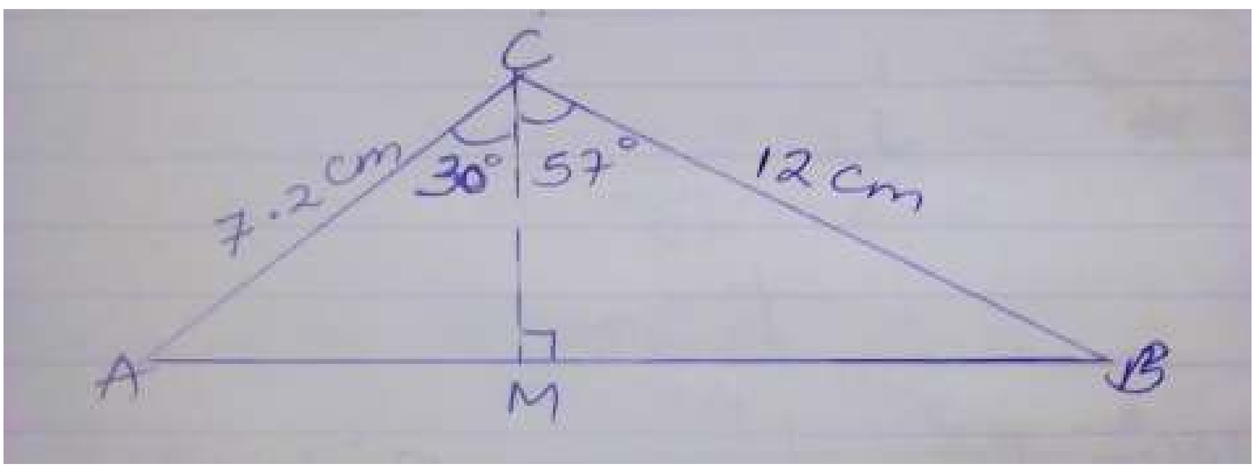
2) Given that A is acute angle and $\tan A = \frac{8}{15}$,find $\cos A$ and $\sin A$

3) Given that $\cos\theta = 0.6$ and $0° \leq \theta \leq 270°$, find sinθ and $\tan\theta$. Hence evaluate;$\sin\theta - \tan\theta$

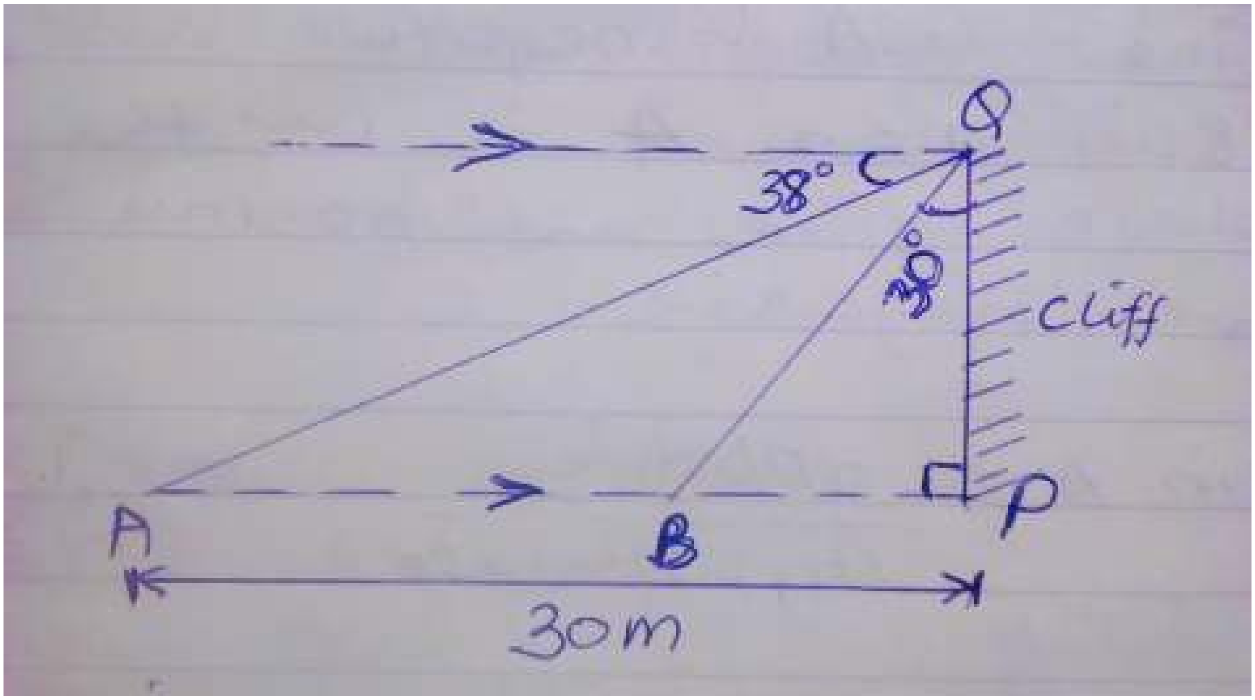
4) Given that $\tan \propto = \frac{8}{15}$, calculate without using tables or calculator, the value of;$\sin \propto + 4\cos \propto$.

5) Given that $\cos\theta = \frac{-5}{13}$ and $0° \leq \theta \leq 180°$, find without using tables or calculators, the value of;$5\tan\theta + 13\sin\theta$

6) In the triangle below, find the length AB



7) From the top of the cliff PQ, a man observes two boats A and B as shown in the diagram.



i) What is the angle of elevation of the top Q from boat A, and from boat B?
ii) What is the angle of depression from Q of the nearest boat B, and of further boat A?
iii) If boat A is 30m from the base of the cliff, Calculate the height of the cliff. Hence or otherwise, calculate how far boat B is from the base of the cliff.

8) a) Copy and complete the table of values for the function $y = 1 + 2\sin\theta$ given that $0° \leq \theta \leq 360°$

ϴ°	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
Sinϴ°	0.00	0.50	0.87	1.00				-0.50	-0.87	-1.00			0.00
2Sinϴ°	0.00	1.00	1.74	2.00				-1.00	-1.74	-2.00			0.00
+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
y	1.00	2.00	2.74	3.00				0.00	-0.74	-1.00			1.00

b) Using the table above, draw the graph of $y = 1 + \sin\theta$ and use it to find the value of θ for which $y = 1.5$

9) a) Plot the graph $y = 3\cos\theta$ for $0° \leq \theta \leq 180°$

b) Use the graph to solve the equations below.

i) $y = 2.5$

ii) $y = -1.5$

Stay Home

Stay Safe