DIFFERENTIATION 1

DIFFERENTIATION FROM FIRST PRINCIPLES:

Examples :

Differentiate the following functions from first principles.

(a) $y = x^2$

DIFFERENTIATION RULES:

For any integer n $\text{if } y = x^n, \text{then } \frac{dy}{dx} = nx^{n-1}$.

In words we can say ***"multiply by the power and reduce the power by 1"***

General rules:

If $y = ku,$ where k is a constant and u is a function of x, then $\frac{dy}{dx} = k\frac{du}{dx}.$

If $y = u + v,$ where u and v are functions of x, then $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}.$

GRADIENT OF A CURVE AT A POINT:

1. The gradient of a curve $y = x^2 + ax + b$ at the point (2,6) is 7. Find the values of a and b.

   (3,-4)

2. The gradient of the curve $y = ax^2 + (b/x^2)$ at the point $(1/2, 5/2)$ is -6. Find the values of a and b. $(2, 1/2)$.
3. Find the gradient of the curve $y = x^2 + 7x - 2$ at the point (2,16) (11)
4. Find the points on the curve $y = x^3 + 3x^2 - 6x - 10$ where the gradient is 3. $(-3,8), (1,-12)$

TANGENTS AND NORMALS:

1. Find the equation of tangent and normal to the curve $y = x^2 - 4x + 1$ at the point (-2,13).

   $(y + 8x + 3 = 0, 8y = x + 106)$

2. Find the equation of tangent and normal to the curve $y = x^3 - 2x - 3$ at (0,-3).

   $(y + 2x + 3 = 0, 2y = x - 6)$

3. Find the coordinates of the point on the curve $y = x^2 - 6x + 3$ where the tangent is parallel to the line $y = 2x + 3.$ $(4, -5)$

4. Find the equation of tangent to the curve $y = x^3 - 9x^2 + 20x - 8$ at the point (1,4). At what points on the curve is the tangent parallel to the line $y + 4x - 3 = 0$?

   $y = 5x - 1, (2,4)$ and $(4, -8)$

5. Find the coordinates of the point on the curve $y = x - x^2$ where the tangent is parallel to the line $2y + x - 3 = 0.$

   $(3/4, 3/16)$

6.

7. Find the coordinates of the point on the curve $y = 3x^2 - 9x + 10$ where the normal is parallel to the line $3y - x + 4 = 0.$ $(1,4)$

8. T is a tangent to the curve $y = x^2 + 6x - 4$ at (1,3) and N is the normal to the curve $y = x^2 - 6x + 18$ at (4,10). Find the coordinates of the point of intersection of T and N.

   (2,11)

9. The tangent to the curve $y = ax^2 + bx + 2$ at $\left(1, \frac{1}{2}\right)$ is parallel to the normal to the curve $y = x^2 + 6x + 10$ at $(-2, 2)$. Find the values of a and b. $(1, -5/2)$

SECOND DERIVATIVE OF A FUNCTION:

STATIONARY AND TURNING POINTS OF A CURVE:

1. The curve $y = x^3 + ax^2 + b$ has a minimum point at (4,-11). Find the coordinates of the maximum point on the curve. (0,21)
2. Given that the curve $y = x^3 + px^2 + qx + r$ passes through the point (1,1) and has turning points where $x = -1$ and $x = 3$, find the values of p, q and r. (-3,-9,12)
3. Find the coordinates of any stationary points on the curve $y = x^3 - 2x^2 - 4x$ and distinguish between these points. max $\left(-\frac{2}{3}, \frac{40}{27}\right)$ min $(2, -8)$
4. Determine the turning points and their nature of those points on the curve $y = 3x^3 - 6x^2 - 12x + 20$ min(2,-4), max $\left(-\frac{2}{3}, 24.4\right)$

CURVE SKETCHING:

1. Sketch the graph of $y = 2 + x - x^2$.
2. Sketch the curve of $y = x^3 - 6x^2 + 9x.$
3. Sketch the curve $y = (x+1)(x-3)^3$

4. The curve $y = x^4 + ax^2 + bx + c$ passes through the point (-1,16) and at that point $\frac{d^2y}{dx^2} = -\frac{dy}{dx} = 16.$ Find the values of a,b,c and sketch the curve. (2,-8,5)
5. Sketch the curve $y = 5x^4 - x^5$.
6. Show that the curve $y = x^3 - 3x^2 + 6x - 4$ has one point of inflexion and find the gradient of the curve at this point. Sketch the curve. (1,0) , 3.
7. Find the coordinates of any stationary points on the curve $y = x^4 + 2x^3$ and distinguish between them. Hence sketch the curve. min $(-\frac{3}{2}, \frac{-27}{16})$ inflexion $(0,0)$.
8. Find the coordinates of any stationary points on the curve $y = 5x^6 - 12x^5$ and distinguish between them. Hence sketch the curve. min(2,-64), inflexion(0,0)
9. Find the coordinates and nature of any turning points on the curve $y = x^3 + 3x^2 - 9x + 6$. Hence sketch the curve. min(1,1), max(-3,33).

COMPOSITE FUNCTIONS:

1. Find $\frac{dy}{dx}$ if $y = (2x+1)^5$ $\qquad 10(2x+1)^5$
2. Find $\frac{dy}{dx}$ if $y = \sqrt{\left(1 - \frac{1}{x^2}\right)^3}$ $\qquad \frac{3}{x^3}\sqrt{\left(1 - \frac{1}{x^2}\right)}$
3. Differentiate $\frac{1}{1+x^3}$ $\qquad -\frac{3x^2}{(1+x^3)^2}$

4. Find $\frac{dy}{dx}$ if $y=\left\{1+\left(x^2-1\right)^3\right\}^{1/3}$ $\qquad \frac{2x(x^2-1)^2}{\left\{1+\left(x^2-1\right)^3\right\}^{2/3}}$

5. Find $\frac{dy}{dx}$ if $y=\left(3x^2-2\right)^4$ $\qquad 24x\left(3x^2-2\right)^3$

6. Find $\frac{dy}{dx}$ if $y=\frac{1}{\sqrt{\left(x^2-2\right)}}$ $\qquad -\frac{x}{\left(x^2-2\right)^{3/2}}$

7. Find $\frac{dy}{dx}$ if (a) $y=\left(4x^3-7x\right)^6$ (b) $y=\sqrt{\left(5x-2x^2\right)}$ (c) $y=\frac{1}{3x^3-4x}$

   (a) $6(4x^3-7x)^5(12x^2-7)$ (b) $\frac{5-4x}{2\sqrt{\left(5x-2x^2\right)}}$ (c) $\frac{4-9x^2}{\left(3x^3-4x\right)^2}$

8. Find the equation of the tangent and normal to the curve $y=\frac{5}{x^2-3}$ at the point (2,5).

   $\left(y+20x=45, 20y=x+98\right)$

9. Find the gradient of the curve $y=\left(2x^2-1\right)^3$ at the point where $x=-1$. Hence find the equation of the tangent to the curve at this point. $\left(-12, 12x+y+11=0\right)$

10. Find the turning points on the curve $y=x(5-x)^4$ and determine their nature. Sketch the curve.

    (1,256) max , (5,0) min.

11. Find any turning point on the curve $4/\left(x^2-2x+5\right)$ and show that the x-axis is an asymptote to the curve. Hence sketch the curve. $(1,1)$ max.

12. Find the coordinates of any stationary points on the curve $y=(3x-1)^4$ and state the nature of any such points. min$\left(\frac{1}{3},0\right)$

PRODUCT AND QUOTIENTS:

1. Find $\frac{dy}{dx}$ if $y=(x^2+4)(x^5+7)$ $(7x^6+20x^4+14x)$
2. Differentiate the following with respect to x:
   (a) $y=x(x+3)^4$ (b) $y=(2x+1)^3(x-1)^4$
   (a) $(x+3)^3(5x+3)$ (b) $2(2x+1)^2(x-1)^3(7x-1)$
3. Differentiate the following with respect to x:
   (a) $y=x^3(3-x^4)^5$ (b) $y=x^2\sqrt{(x+3)}$
   (a) $x^2(3-x^4)(9-23x^4)$ (b) $\frac{x(5x+12)}{2\sqrt{(x+3)}}$
4. Find $\frac{dy}{dx}$ if $y=(x^2-1)^3(3x+1)^4$ $6(x^2-1)^2(3x+1)^3(5x^2+x-2)$
5. Find $\frac{dy}{dx}$ if $y=(x+4)\sqrt{(x^2-1)}$ $(2x^2+4x-1)/\sqrt{(x^2-1)}$
6. Find $\frac{dy}{dx}$ if $y=\frac{x}{x+1}$ $\frac{1}{(x+1)^2}$
7. Differentiate the following with respect to x:
   (a) $y=\frac{1-x}{x^2+8}$ (b) $y=\frac{4x+3}{\sqrt{(2x-1)}}$
   (a) $\frac{(x-4)(x+2)}{(x^2+8)^2}$ (b) $\frac{4x-7}{(2x-1)^{3/2}}$
8. Differentiate the following with respect to x: (a) $y=\frac{2x+3}{1-5x}$ (b) $y=\frac{(3x+1)^4}{(5x-2)^3}$
   (a) $\frac{17}{(1-5x)^2}$ (b) $\frac{3(3x+1)^3}{(5x-2)^4}(5x-13)$
9. Find $\frac{dy}{dx}$ if $y=\sqrt{\frac{x+1}{x^2+1}}$ $\frac{1-2x-x^2}{2(x+1)^{1/2}(x^2+1)^{3/2}}$
10. If $y=\sqrt{\frac{6x}{x+2}}$, find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x=4$.
11. Find the maximum and minimum values of the function $y=\frac{4x}{x^2+4}$. (-2,-1)min, (2,1)max.

IMPLICIT FUNCTIONS:

1. Given that $(x+1)^2+(y-2)^2=1$, find $\frac{dy}{dx}$. $-\frac{x+1}{y-2}$
2. Find the gradient of the curve $x^3+y^3=4y^2$ at the point (2,2). (3)
3. Find $\frac{dy}{dx}$ if $y^3-xy^2-x^3=1$. $\frac{3x^2+y^2}{3y^2-2xy}$
4. If $y^3+x^3=3x+7$, show that $y^2\frac{d^2y}{dx^2}+2y\left(\frac{dy}{dx}\right)^2+2x=0$.
5. Find the equation of tangent to the curve $x^2+2xy-2y^2+x=2$ at the point (-4,1).
6. Find the equation of the tangent to the curve $(x+1)y^2=x-2$ at the point (-2,2).

   $(4y=3x+14)$
7. If $x^2-y^2=1$, prove that $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=1$.
8. If $y^2-2xy=2x$, prove that $(x-y)\frac{d^2y}{dx^2}+2\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^2=0$.
9. Given that $(x+y)=(x-y)^2$; prove that $1-\frac{dy}{dx}=\frac{2}{2x-2y+1}$. Hence or otherwise, prove that $\frac{d^2y}{dx^2}=\left(1-\frac{dy}{dx}\right)^3$.
10. Prove that if $y=\sqrt{(3x^2+2)}$, then $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=3$.
11. Find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at the point (1,3) on the curve $3x^2+y^2=4y$. $(-3,-12)$
12. If $x^2+y^2=2y$, prove that $\frac{d^2y}{dx^2}=\frac{1}{(1-y)^3}$.
13. Find $\frac{dy}{dx}$ in terms of x and y: (a) $y^3+6x=x^2$ (b) $3y^2+2y+xy=x^3$

$(a)\frac{2x-y}{3y^2}$ (b) $\frac{3x^2-y}{x+6y+2}$

14. Find the equation of tangent and normal to the curve $3x^2-xy-2y^2+12=0$ at the point (2,3).

$(14y=9x+24, 9y+14x=55)$.

APPLICATIONS OF MAXIMUM AND MINIMUM

1. A carton of volume Vm^3 is made from a piece of cardboard as shown below. If the area of cardboard used is $6m^3$, find expressions for h and V in terms of x and the value of x which produces a box of maximum volume. $(x=1, V=1)$.
2. 1000m of fencing is to be used to make a rectangular enclosure. Find the greatest possible area and corresponding dimensions. $(l=w=250m, A=6500m^2)$
3. A company that manufactures dog food wishes to pack the food in closed cylindrical tins. What should be the dimensions of each tin if each is to have a volume of $128\pi cm^3$ and the minimum possible area.

   $(r=4cm, h=8cm)$.
4. A lump of modeling clay of volume $72cm^3$ is moulded into the shape of a cuboid with edges of length x cm,2x cm and y cm. Find the minimum surface area of this cuboid. ($108\ cm^2$)

5. A closed hollow right circular cone has internal height a and the internal radius of its base is also a. A solid circular cylinder of height h just fits inside the cone with the axis of the cylinder lying along the axis of the cone. Show that the volume of the cylinder is $V = \pi h(a-h)^2$. If a is fixed , but h may vary, find h in terms of a when V is a maximum. $\left(\frac{1}{3}a\right)$
6. A right circular cylinder of height 2h is contained in a sphere of radius R, the circular edges of the cylinder touching the sphere. The volumes of the cylinder and the sphere are denoted by V and W respectively. Express V in terms of R and h. By finding the maximum value of V, as h varies, show that $V/W \le 1/\sqrt{3}$.

   $V = 2\pi h\left(R^2 - h^2\right)$
7. A right circular cylinder is of radius r cm and height pr cm. The total surface area of the cylinder is S cm^2 and its volume V cm^3. Find an expression for V in terms of p and S. If the value of S is fixed, find the value of p for which V is a maximum. $\sqrt{\left(\frac{S^3p^2}{8\pi(1+p)^3}\right)}$;2
8. A piece of wire 80cm in length is cut into three parts , two of which are bent into equal circles and the third into a square. Find the radius of the circles if the sum of the enclosed areas is a minimum.

   $(20/(\pi+2))$

9. A right pyramid having a square base is inscribed in a sphere of radius R, all vertices of the pyramid lying on the sphere. The height of the pyramid is x; show that the four vertices forming the base of the pyramid lie on a circle of radius r, where $r^2 = 2Rx - x^2$. Hence or otherwise, show that the volume V, of the pyramid is given by the formula $V = \frac{2}{3}x^2(2R - x)$. If R is fixed but x may vary, find the greatest possible value of V. $\frac{64}{81}R^3$.

10. A cylinder of volume V is to be cut from a solid sphere of radius R. Prove that the maximum value of V is $\frac{4\pi R^3}{3\sqrt{3}}$.

11. Find the height of a right circular cylinder of greatest volume which can be cut from a sphere of radius a. $\frac{2a}{\sqrt{3}}$

12. A hemispherical bowl of radius a cm is initially full of water. The water runs out of a small hole at the bottom of the bowl at a constant rate which is such that it would empty the bowl in 24s.Given that , when the depth of the water is $\frac{1}{3}\pi x^2(3a - x)cm^3$; prove that the depth is decreasing at a rate of $a^3/36x(2a - x)\text{cms}^{-1}$. Find after what time the depth of water is $\frac{1}{2}acm,$ and the rate at which the water level is then decreasing.

$\left(16.5s, a/27\ \text{cms}^{-1}\right)$

13. A square of side xcm is cut from each of the corners of a rectangular piece of cardboard 15cm by 24cm. The cardboard is then folded to form an open box of depth xcm. Show that the volume of the box is $(4x^3 - 78x^2 + 360x)cm^3$. Find the value of x for which the volume is a maximum.

(3)

RATES OF CHANGE:

1. A piece of paper is burning round the circumference of a circular hole. After t seconds, the radius rcm of the hole is increasing at the rate $0.5cms^{-1}$. Find the rate at which the area A cm^2, of the hole is increasing when r = 5. $(5\pi cm^2 s^{-1})$
2. Water is being poured into a conical vessel at a rate of $10cm^3s^{-1}$. After t seconds, the volume Vcm^3, of water in the vessel is given by $V = \frac{1}{6}\pi x^3$; where x cm is the depth of water. Find, in terms of x, the rate at which water is rising. $(20/\pi x^2)cms^{-1}$
3. Water runs into a conical vessel fixed with its vertex downwards at the rate of $3\pi cm^3 s^{-1}$, filling the vessel to a depth of 15cm in a time of one minute. Find the rate at which the depth of water in the vessel is increasing when the water has been running for 7.5 seconds. $\left(\frac{1}{3} cms^{-1}\right)$

4. A large container in the shape of a right circular cone of height 10m and base radius 1m is catching the drips from a tap leaking at the rate of $0.1m^3$ per minute. Find the rate at which the surface area is increasing when the water is half way up the cone.

   $\left(0.04m^2 \min^{-1}\right)$

5. Suppose the volume of the cylinder disc is $54\pi m^3$, what will the dimensions of the cylinder if the surface area is minimum. $(r = 3m, h = 6m)$

6. If the radius of a sphere is increasing at $2cms^{-1}$, find the rate at which the volume of the sphere is increasing when the radius is 3cm. $\left(72\pi cm^3 s^{-1}\right)$

7. Water is pumped in an empty trough which is 200cm long , at the rate of $33000cms^{-1}$.The uniform cross-section of the trough is an isosceles trapezium with hypotenuse 50cm, short side 80cm and long side 140cm. Find the rate at which the depth of the water is increasing when the depth is 20cm.

   $\left(\frac{3}{2} cms^{-1}\right)$

8. When the radius of sphere is 21cm, the radius is increasing at a rate of $0.01cms^{-1}$, find the rate at which the surface area and volume are increasing at this point. $(\pi = 22/7)$

   $\left(5.28cm^2, 55.44cm^3\right)$

9. A hollow right circular cone has base radius 4m and vertical height 20m. It is held upside down with its

axis vertical. It contains water which is being added at a constant rate of $1.5m^3$ per minute and which leaks away through a small hole in vertex at a constant rate of $2m^3$ per minute. At what rate is the depth of water changing when the depth is 12m?

$\left(0.03m\min^{-1}\right)$

10. A container is in the shape of a cone of semi-vertical angle 30^0, with its vertex downwards . Liquid flows into the container at the rate of $\frac{\sqrt{3}}{4}\pi cm^3 s^{-1}$. At the instant when the radius of the circular surface of the liquid is 5cm,find the rate of increase in

(a) the radius of the circular surface of the liquid.

$\left(0.01cms^{-1}\right)$

(b) the area of the circular surface of the liquid.

$\left(0.1\pi cm^2 s^{-1}\right)$

SMALL CHANGES AND ERRORS:

1. In an experiment , the diameter xcm of a sphere is measured and volume Vcm^3, calculated using the formula $V = \frac{1}{6}\pi x^3$.If the diameter is found to be 10cm with a possible error of 0.1cm, estimate the possible error in the volume calculated from this reading.

   $\left(5\pi cm^3\right)$

2. Suppose the error in measuring the radius of a circle is 0.02,find the percentage error made in measuring the area of circle. (4%)

3. If $y = 2x^2 - 3x,$ find the approximate change in y when x increases from 6 to 6.02. (0.042)
4. In calculating the area of a circle, it is known that an error of $\pm 3\%$ could have been made in the measurement of the radius. Find the possible percentage error in the area. $(\pm 6\%)$
5. The time T seconds taken for one complete swing of a pendulum , length $lm,$ is given by $T = 2\pi\sqrt{\frac{l}{g}}$ where g is a constant. If a 1% error is made in measuring the length of a pendulum, estimate the percentage error in the value of T. (0.5%)
6. Find an approximate value for $\sqrt[3]{1003.}$

   (10.01)
7. Find an approximate value for $\sqrt{(16.08)}$

   (4.01)
8. Find an approximate value of $\sqrt{(101)}$

   (10.05)
9. Find the cube root of 64.08

   (4.001)

DIFFERENTIATION OF TRIGONOMETRIC FUNCTIONS:

1. Differentiate (a) $\sin^3 x$ (b) $\cos(4-3x)$ (c) $\sin x^0$

   $3\sin^2 x\cos x, 3\sin(4-3x), \frac{\pi}{180}\cos x^0$
2. Differentiate (a) $y = \sin^2 x$ (b) $\sin 3x \cos x$ (c) $\frac{\cos^3 5x}{6x}$ (d) $x^3 \tan 3x$

   (a) $\sin 2x$ (b) $3\cos 3x\cos x - \sin 3x \sin x$

(c) $\frac{-\cos^2 5x}{6x^2}(15x\sin 5x+\cos 5x)$ (d) $3x^2\left(\tan 3x + x\sec^2 x\right)$

3. Find $\frac{dy}{dx}$ if (a) $y=\sin 4x$ (b) $y=\cos\left(x^3-1\right)$

(a) $4\cos 4x$ (b) $-3x^2\sin\left(x^3-1\right)$

4. Find $\frac{dy}{dx}$ if (a) $y=\tan^5 x$ (b) $y=\sec^3 x$

$\left(5\tan^4 x\sec^2 x, 3\sec^3 x\tan x\right)$

5. Find $\frac{dy}{dx}$ if $y=\cot^4\left(3x^2+2x-1\right)$ $-8(3x+1)\cot^3\left(3x^2+2x-1\right)\cos ec^2\left(3x^2+2x-1\right)$

6. Find $\frac{dy}{dx}$ if (a) $y=\sin^2 x\cos 3x$ (b) $y=\frac{\sin^4 3x}{6x}$

(a) $\sin x(2\cos 3x\cos x-3\sin x\sin 3x)$

(b) $\frac{\sin^3 3x}{6x^2}(12\cos 3x-\sin 3x)$

7. Find $\frac{dy}{dx}$ if (a) $y=x^2\sin x$ (b) $y=x\sin x\cos x$

(a) $2x\sin x+x^2\cos x$ (b) $\sin x\cos x+x\left(\cos^2 x-\sin^2 x\right)$

8. Find $\frac{dy}{dx}$ when (a) $y=\frac{\sin x}{x}$ (b) $\frac{1+\sin x}{1-\sin x}$ (c) $\frac{\sin x-\cos x}{\sin x+\cos x}$

(a) $\frac{x\cos x-\sin x}{x^2}$ (b) $\frac{2\cos x}{(1-\sin x)^2}$ (c) $\frac{2}{(\sin x+\cos x)^2}$

9. Find $\frac{d}{dx}$ for (a) $\frac{x}{\tan x}$ (b) $\frac{\sin x}{1+\tan x}$ (c) $\frac{1+\sin^2 x}{1-\sin^2 x}$

(d) $\sin^3 x\sin 3x$

(a) $\cot x-x\cos ec^2 x$ (b) $\frac{\cos^3 x-\sin^3 x}{(\sin x+\cos x)^2}$ (c) $4\sec^2 x\tan x$

(d) $3\sin^2 x\sin 4x$

10. If $y=\sqrt{1+\sin x}$, show that $\frac{dy}{dx}=\frac{1}{2}\sqrt{(1-\sin x)}$.

11. If $y=\sqrt{\left(\frac{1+\sin x}{1-\sin x}\right)}$, show that $\frac{dy}{dx}=\frac{1}{1-\sin x}$

12. Find the maximum and minimum value(s) of the function $f(x)=2\sin x+\cos 2x$ for $0<x<\pi$. max $\left(\frac{\pi}{6},\frac{3}{2}\right),\left(\frac{5}{6}\pi,\frac{3}{2}\right)$ min $\left(\frac{\pi}{2},1\right)$

13. If $y^2=\tan 2x+\sec 2x,\text{ show that }\frac{dy}{dx}=y\sec 2x$. State any assumption made.

14. If $y=\frac{\sin x}{x}$, prove that $x\frac{d^2y}{dx^2}+2\frac{dy}{dx}+xy=0.$

DIFFERENTIATION OF INVERSE TRIGONOMETRIC FUNCTIONS:

1. Find $\frac{dy}{dx}$ if $y=\sin^{-1}(3x-1)$. $\frac{3}{\sqrt{(6x-9x^2)}}$
2. Differentiate : (a) $\sin^{-1}x$ (b) $\cos^{-1}x$ (c) $\tan^{-1}x$
   (a) $\frac{1}{\sqrt{(1-x^2)}}$ (b) $-\frac{1}{\sqrt{(1-x^2)}}$ (c) $\frac{1}{1+x^2}$
3. Differentiate : (a) $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ (b) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

DIFFERENTIANTING EXPONENTIAL FUNCTIONS:

1. Find $\frac{dy}{dx}$ when $y=e^{3x^2}$ $6xe^{3x^2}$
2. Differentiate: (a) $e^{2x}\sin 3x$ (b) e^{x^2}
   (a) $e^{2x}(2\sin 3x+3co3x)$ (b) $2xe^{x^2}$
3. Differentiate e^{x^2+1} with respect to x. $2xe^{x^2+1}$
4. Sketch the curve $y=x^2e^{-x}$.
5. If $y=e^x\sin x,$ show that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0.$
6. If $e^xy=\sin x,$ show that $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$.

7. If $y\cos x = e^x$, show that $\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} - 2y = 0.$
8. If $\sin y = 2\sin x,$ show that $\cot y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 1 = 0.$
9. A particle moves in a straight line so that after t seconds its distance from a fixed point O is s metres, where $s = t^2e^{2-t}$. Find the distance of the particle from O when it first comes to rest and its acceleration at that point. $(4m, -2ms^{-2})$
10. Find the values of x for which the function $(x^2 - 2x - 1)e^{2x}$ has maximum or minimum values, distinguish between them.
    max $(x = -1)$, min $(x = 2)$.
11. If $y = xe^{-x}$, show that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0.$
12. If $y = e^x \sin x,$ show that $\frac{d^2y}{dx^2} = 2e^x \sin\left(x + \frac{\pi}{2}\right)$
13. If $y = e^{-2x}\cos 4x,$ prove that $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 20y = 0.$
14. Show that $e^{\ln x} = x$ and that $e^{-2\ln x} = x^{-2}$.

DIFFERENTIATION OF LOGARITHMIC FUNCTIONS:

1. Solve the equation $e^x - 2 - 3e^{-x} = 0.$ $(\ln 3)$
2. Differentiate (a) $\ln \sec x$ (b) $x^2 \ln x$ (c) $x(\ln x - 1)$
   (a) $-\ln \cos x$ (b) $x(1 + 2\ln x)$ (c) $\ln x$
3. Differentiate $\ln(x^2 + 1)$ $\frac{2x}{x^2 + 1}$.

4. Differentiate (a) $\ln\sqrt{\dfrac{x^2-1}{x^2+1}}$ (b) $\ln\left(x+\sqrt{x^2+1}\right)$

(a) $\dfrac{2x}{x^4-1}$ (b) $\dfrac{1}{\sqrt{x^2+1}}$

5. Differentiate (a) $\ln\left(\dfrac{x}{\sqrt{x^2+1}}\right)$ (b) $\ln\left(x\sqrt{x+1}\right)$ (c) $\ln\left(\dfrac{x}{(x-1)^2}\right)$

(a) $\dfrac{1}{x(x^2+1)}$ (b) $\dfrac{3x+2}{2x(x+1)}$ (c) $\dfrac{1+x}{x(1-x)}$

6. Differentiate (a) $\ln(\sec x+\tan x)$ (b) $\ln\left(\dfrac{\sin x+\cos x}{\sin x-\cos x}\right)$

(a) $\sec x$ (b) $2\sec 2x$

7. Differentiate (a) $\ln\sqrt{\dfrac{1-x}{1+x}}$ (b) $\ln\left(x\sqrt{x^2-1}\right)$ (c) $\ln\dfrac{(x+1)^2}{\sqrt{x-1}}$

(a) $\dfrac{1}{x^2-1}$ (b) $\dfrac{2x^2-1}{x(x^2-1)}$ (c) $\dfrac{3x-5}{2\left(x^2-1\right)}$

8. Differentiate (a) $x\ln y$ (b) $y\ln x$ (c) $\dfrac{\ln x}{x^2}$ (d) $\dfrac{x}{\ln x}$

(a) $\ln y+\dfrac{x}{y}\dfrac{dy}{dx}$ (b) $\dfrac{y}{x}+\dfrac{dy}{dx}\ln x$ (c) $\dfrac{1-2\ln x}{x^3}$ (d) $\dfrac{\ln x-1}{(\ln x)^2}$

9. If $y=\ln\left(\dfrac{2x+1}{1-3x}\right)$, find $\dfrac{dy}{dx}$ $\qquad \dfrac{5}{(2x+1)(1-3x)}$

10. Differentiate 2^x with respect to x. $\qquad \left(2^x\ln 2\right)$

11. If $y=\dfrac{x}{\sqrt{\left(x^2-2\right)}}$, find $\dfrac{dy}{dx}$ $\qquad -\dfrac{2}{\sqrt{\left(x^2-2\right)^{3/2}}}$

12. Sketch the curve $y=\dfrac{1}{x}\ln x$.

13. If $y=\ln\left(x^2-5\right)$, show that $\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2=2e^{-y}$.

14. If $y=\sin 2x\ln(\tan x)$, show that $\dfrac{d^2y}{dx^2}+4y=4\cot 2x$.

15. If $\tan y = \ln x^2$, show that $x\frac{dy}{dx} = 2\cos^2 y$. Hence show that

$$x^2\frac{d^2y}{dx^2} + 2(1 + 2\sin 2y)\cos^2 y = 0$$

16. Show that if $y = e^{4x}\cos 3x$, then $\frac{d^2y}{dx^2}$ can be expressed in the form $25e^{4x}\cos(3x + \alpha)$. Give the value of $\tan\alpha$.

17. If $f(x) = e^{5x}\sin 12x$, show that $f'(x) = 13e^{5x}\sin(12x + \alpha)$ where $\tan\alpha = 12/5$.

18. If $y = e^{4x}\cos 3x$, prove that $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 25y = 0$.

19. If $y = e^{3x}\sin 4x$, show that $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0$.

20. If $y = x\tan^{-1}x$, show that $(1 + x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 2.$

21. Find $\frac{dy}{dx}$ if $y = \ln\left(\frac{3 + 4\cos x}{4 + 3\cos x}\right)$ $\left(\frac{-7\sin x}{(3 + 4\cos x)(4 + 3\cos x)}\right)$

22. Differentiate (a) $x^{\sin x}$ (b) $(\sin x)^x$ (c) $x^y = \sin x$

(a) $\left(\frac{1}{x}\sin x + \cos x\ln x\right)x^{\sin x}$ (b) $(\ln x + x\cot x)(\sin x)^x$

(c) $\ln x\frac{dy}{dx} + \frac{y}{x} = \cot x$

23. If $x = e^t$ and $y = \sin t$, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

24. Given that $y = \ln(1 + \sin x)$, prove that $\frac{d^2y}{dx^2} + e^{-y} = 0.$