DIFFERENTIATION 2

1. Different the following from first principles

   (a) $\cos x$ (b) $\sin x$ (c) $\sec x$

2. If $y^2 = \tan 2x + \sec 2x,$ show that when $y \neq 0, \frac{dy}{dx} = y \sec 2x.$

3. Differentiate the following

   (a) $\sin^2(2x-5)$ (b) $x^4 \tan 4x$ (c) $\frac{\sin x}{\sqrt{(\cos 2x)}}$ (d) $\frac{\sec x + \tan x}{\sec x - \tan x}$

   solution: (a) $2\sin(4x-10)$ (b) $4x^3(\tan 4x + x\sec^2 4x)$ (c) $\frac{\cos x}{(\cos 2x)^{3/2}}$

   (d) $\frac{2\sec x}{(\sec x - \tan x)^2}$

4. Differentiate the following by either quotient or product rule:

   (a) $\sin^2 x$ (b) $\frac{\cos^3 5x}{6x}$ (c) $\sin 3x \cos x$ (d) $x^3 \tan x$

   Solution: (a) $2\sin x \cos x$ (b) $-\frac{\cos^2 5x}{6x^2}(15x \sin 5x + \cos 5x)$

   (c) $3\cos 3x \cos x - \sin 3x \sin x$ (d) $3x^2(\tan 3x + x\sec^2 3x)$

Differentiation of inverse trigonometric functions

1. Differentiate the following

   (a) $\sin^{-1} x$ (b) $\cos^{-1} x$ (c) $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ (d) $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ (e) $\cos^{-1}\left(\frac{x}{5}\right)$

(f) $\cos^{-1}\left(\frac{2}{\sqrt{(2-x^2)}}\right)$ (g) $y=\sec^{-1}x$ (h) $x\sin^{-1}x$

Solution: (a) $\frac{1}{\sqrt{(1-x^2)}}$ (b) $\frac{-1}{\sqrt{(1-x^2)}}$ (c) $\frac{2}{1+x^2}$ (d) $\frac{2}{1+x^2}$ (g) $\frac{1}{x\sqrt{(x^2-1)}}$

2. If $y=\frac{\sin^{-1}x}{\sqrt{(1-x^2)}}$, show that $(1-x^2)\frac{dy}{dx}-xy=1.$

3. Show that $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$

Differentiation of exponential functions

1. Differentiate the following with respect to x

(a) $e^{3x}\sin 2x$ (b) xe^{-x} (c) e^{5x^2} (d) e^{4x} (e) $5e^{x^2+1}$
(f) $5x^2+3/e^{x^2}$ (g) $e^x\sin 2x$

2. Given that $y=-e^x\cos 2x,$ prove that $\frac{d^2y}{dx^2}=5e^x\sin(2x+\beta)$ where $\tan\beta=3/4.$

3. Given that $y=xe^{-x},$ show that $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$.

4. If $y=e^{-2x}\cos 4x,$ prove that $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+20y=0.$

5. If $y=e^{\tan^{-1}x}$, show that $(1+x^2)\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx}=0.$

6. If $y=\cos(\ln x)$; prove that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0.$

7. Sketch the curve $y=x^2e^{-x}$.

8. If $y=e^x\sin x,$ show that $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0.$

9. If $y=e^x\sin x$, show that $\frac{dy}{dx}=\sqrt{2}e^x\sin\left(x+\frac{\pi}{4}\right);\frac{d^2y}{dx^2}=2e^x\sin\left(x+\frac{\pi}{2}\right).$

Solution: (a) $e^{3x}(3\sin 2x+2\cos 2x)$ (b) $e^{-x}(1-x)$

Differentiation of logarithmic functions

1. Differentiate the following

(a) $\ln x$ (b) $\ln(1+x^2)$ (c) $\ln(5x^2-6)$ (d) $\ln(\sin 2x)$ (e) $\ln\left(\frac{x+2}{x+3}\right)$ (f) 2^x (g) $\ln\left(\frac{2x+1}{1-3x}\right)$ (h) $\ln\sqrt{\left(\frac{x^2-1}{x^2+1}\right)}$ (i)

2. Use natural logarithms to show that if $y=\frac{x^2}{\sqrt{(x-1)}}$,

$\frac{dy}{dx}=\frac{x(3x-4)}{2(x-1)^{3/2}}$.

3. If $y=\frac{x}{\sqrt{(x^2-2)}}$, show that $\frac{dy}{dx}=-\frac{2}{(x^2-2)^{3/2}}$.

4. If $y=e^x\ln x,$ show that $x\frac{d^2y}{dx^2}=(2x-1)\frac{dy}{dx}+(1-x)y.$

5. If $\tan y=\ln x^2,$ show that $x^2\frac{d^2y}{dx^2}+2(1+2\sin 2y)\cos^2 y=0.$

6.

Solutions: (a) $\frac{1}{x}$ (b) $\frac{2x}{x^2+1}$ (c) $\frac{10x}{5x^2-6}$ (d) $2\cot 2x$ (e) $\frac{1}{(x+2)(x+3)}$

(f) $2^x \ln 2$ (g) $\frac{5}{(2x+1)(1-3x)}$

APPLICATION OF NATURAL LOGARITHMS IN SIMPLIFYING DERIVATIVES:

Find the derivatives of the following with respect to x using natural logarithms.

(a) $y=\left(x-\frac{3}{x^2}\right)^2$ (b) $y=\sqrt{\left(\frac{1+x}{1-x}\right)}$ (c) $y=\sqrt{1+\sin x}$ (d) $y=\sqrt{\left(\frac{1+\sin x}{1-\sin x}\right)}$

Solution: (a) $2\left(x-\frac{3}{x^2}\right)\left(1+\frac{6}{x^3}\right)$ (b) $\frac{1}{(1+x)^{1/2}(1-x)^{3/2}}$ (c) $\frac{1}{2}\sqrt{(1-\sin x)}$

(d) $\frac{1}{1-\sin x}$.

Uneb questions: 2010 no. 5, no. 10(b), no.12(a)

2009 no. 3, no. 10(a) , no. 14(b) 2000. no.3

2008 no. 7, no. 11(a) 1999. no.5

2007 no. 8, no.16(a) 1998(November). no.5, no. 11

2006 no. 2 1998(march). no. 11

2005 no. 8, no. 13(a)

2004 no. 4, no.14

2003 no.6, no.8

2002 no.5, no.7

1993. no.6

1991. no.4(a)

1997. no.4

1996. no.7

1995. no. 9(a) , no.11

1994 no. 9

1992. no.12(b)