Linear momentum

Linear momentum is a product of the body's mass and its velocity.

The S1 unit of momentum is $kgms^{-1}$

When a force F is applied to a body, it changes the body's velocity from u to v, the size of the force and the time for which it acts on a body.

From F = ma

$$a = \frac{v-u}{t}$$

$$F = \frac{m(v-u)}{t}$$

$$Ft = m(v-u)$$

Impulse of force is the product of force and the duration of its action or impulse is the change in momentum of the body which is acted on by the force

Example 1

A body of mass 3kg initially moving with a velocity of $5ms^{-1}$ is acted on by a horizontal force of 15N for 2s. Find the impulse and final speed.

Solution

Impulse = Ft

= 15 x 2

= 30N

Impulse = change in momentum

30 = m(v-u)

30 = 3(v-5)

$v = 15ms^{-1}$

Example 2

A tennis ball has a mass of 0.07kg. it approaches a racket with a speed of $5ms^{-1}$ and bounces off and returns to the way it come with a speed of $4ms^{-1}$. The ball is in contact with the racket for 0.2 seconds. Calculate

(i) The impulse given to the ball

(ii) The average force exerted on the ball by the racket.

Solution

(i) Impulse = Ft = m(v-u)

= 0.07(-4-5)

= -0.63Ns

(ii) $F = m\left[\frac{v-u}{t}\right] = \frac{0.63}{0.2} = 3.15N$

Collisions and principles of conservation of linear momentum

When two or more bodies collide, the total momentum of the system is conserved provided there is no external force on the sysytem.

Consider a body of mass m_1 moving with a velocity u_1 to the right. Suppose the body makes a head on collision with a nother body of mass m_2 moving with velocity u_2 in the same direction

Let v_1 and v_2 be the velocities of the 2 bodies respectively after collision

Let F1 be the force exerted on m_2 by m_1 and F_2 the force exerted on m_1 by m_2 using Newton's 2nd law.

$F_1 = m_1\left(\frac{v_1 - u_1}{t}\right)$, $F_2 = m_2\left(\frac{v_2 - u_2}{t}\right)$, where t is the time of collision

Using Newton's third law

$F_1 = -F_2$

$m_1\left(\frac{v_1 - u_1}{t}\right), = -m_2\left(\frac{v_2 - u_2}{t}\right),$

$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$

Hence, total momentum before collision = total momentum after collision, in other words momentum is conserved.

When two bodies collide, there is a short period of contact during which each exerts a force on each other at that instant, the force which each exert on each other is equal and opposite.

Types of collision

Collisions can be categorized as inelastic collision, perfectly inelastic, elastic or perfectly elastic collisions.

Elastic or perfectly elastic collision	Inelastic collision	Perfectly inelastic collision
Kinetic energy is conserved	Kinetic energy is not conserved	Kinetic energy is not conserved
Linear momentum is conserved	Linear momentum is conserved	Linear momentum is conserved
Bodies separate after collision, e.g. collision of gas molecules	Bodies separate after collision e.g. a ball bouncing from a concrete floor	Bodies stick together and move with a common velocity. E.g. a trailer colliding with a saloon car.

Elastic collision

Momentum is conserved

$m_1v_1 + m_2v_2 = m_1u_1+m_2u_2$

$m_1(u_1 - v_1) = m_2(v_2 - u_2)$ ……………………………(i)

Kinetic energy is conserved

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$m_1(u_1^2 - v_1^2) = m_2(u_2^2 - v_2^2)$ ……………………….. (ii)

Equation (i) ÷ (ii)

$\frac{m_1(u_1-v_1)}{m_1(u_1^2-v_1^2)} = \frac{m_2(u_2-v_2)}{m_2(u_2^2-v_2^2)}$

$\frac{(u_1-v_1)}{(u_1-v_1)(u_1+v_1)} = \frac{(u_2-v_2)}{(u_2-v_2)(u_2+v_2)}$

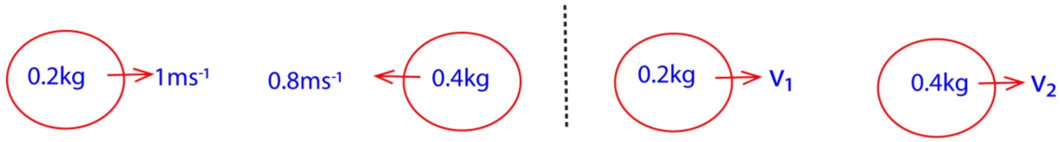
$\frac{1}{(u_1+v_1)} = \frac{1}{(u_2+v_2)}$

$(u_1 - u_2) = (v_2 - v_1)$

Example 3

A 200g block moves to the right at a speed of 100cms^{-1} and meets a 400g block moving to the left with a speed of 80cms^{-1}. Find the final velocity of each block if the collision is elastic.

Solution



$(v_2 - v_1) = -(-0.8 - 1)$

$(v_2 - v_1) = -1.8$ …………………… (i)

Using conservation of momentum

$m_1v_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(0.2 \times 1) + (-0.4 \times 0.8) = 0.2v_1 + 0.4v_2$

$-0.12 = 0.2v_1 + 0.4v_2$

$-0.6 = v_1 + 2v_2$ ……………………….(ii)

Eqn (i) and Eqn (ii)

$v_2 = -1.8 + v_1$

$-0.6 = v_1 + 2(-1.8 + v_1)$

$V_1 = 1ms^{-1}$

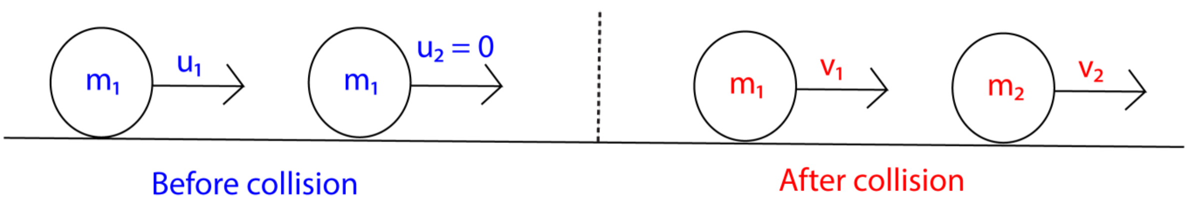
$V_2 = -0.8ms^{-1}$

Example 4

A particle of mass m_1, travelling with velocity u_1 makes a perfectly elastic collision with a stationary particle of mass m_2. After the collision, the first particle moves a velocity v_1 while the second particle moves in the same direction with velocity, v_2. Show that

$v_2 = \frac{2m_1u_1}{m_1 + m_2}$ and $v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)}$

Solution



From the principle of conservation of momentum

Momentum before collision = momentum after collision

$m_1u_1 + m_2(0) = m_1v_1 + m_2v_2$

$v_1 = \frac{m_1u_1 - m_2v_2}{m_1}$ ……………………………….(i)

From conservation of kinetic energy

Kinetic energy before collision = kinetic energy after collision

$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2 x\ 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$m_1u_1^2 = m_1v_1^2 + m_2v_2^2$ …………………………………. (ii)

Substituting equation (i) into equation (ii)

$$m_1u_1^2 = m_1\left[\frac{m_1u_1 - m_2v_2}{m_1}\right]^2 + m_2v_2^2$$

$$m_1^2u_1^2 = m_1^2u_1^2 - 2m_1u_1m_2v_2 + m_2^2v_2^2 + m_1m_2v_2^2$$

$$2m_1u_1m_2v_2 = m_2^2v_2^2 + m_1m_2v_2^2$$

Dividing through by m_2v_2

$$2m_1u_1 = v_2(m_1 + m_2)$$

$$v_2 = \frac{2m_1u_1}{(m_1 + m_2)}$$

From the principle of conservation of momentum

Momentum before collision = momentum after collision

$$m_1u_1 + m_2(0) = m_1v_1 + m_2v_2$$

$$m_2v_2 = m_1(u_1 - v_1)$$

$$v_2 = \frac{m_1((u_1 - v_1)}{m_2}$$

$$v_2^2 = \frac{m_1^2}{m_2^2}(u_1 - v_1)^2 \text{ ....(i)}$$

From conservation of kinetic energy

Kinetic energy before collision = kinetic energy after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2x\ 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2$$

$$v_2^2 = \frac{m_1}{m_2}(u_1^2 - v_1^2) \text{ .... (ii)}$$

Equation (i) and (ii)

$$\frac{m_1^2}{m_2^2}(u_1 - v_1)^2 = \frac{m_1}{m_2}(u_1^2 - v_1^2)$$

$$\frac{m_1}{m_2}(u_1 - v_1) = (u_1 + v_1)$$

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1$$

$$u_1(m_1 - m_2) = v_1(m_2 + m_1)$$

$$v_1 = \frac{u_1(m_1 - m_2)}{(m_2 + m_1)}$$

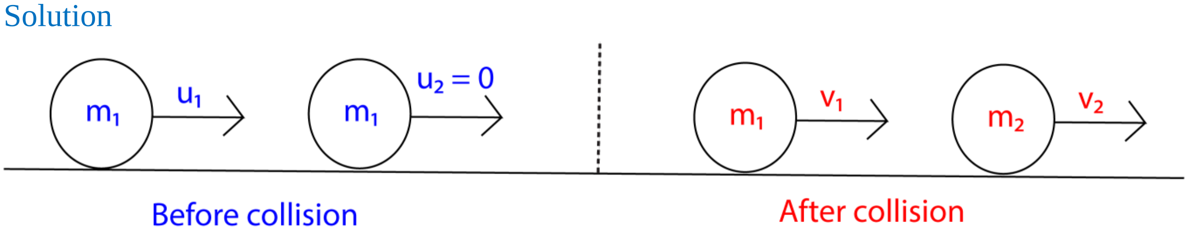
Example 5

A particle P of mass m_1, travelling with a speed u_1 makes a head on collision with a stationary particle Q of mass m_2. If the collision is elastic and speed of P and Q after impact are v_1 and v_2 respectively show that if b = $\frac{m_1}{m_2}$

(i) $\frac{u_1}{v_1} = \frac{b+1}{b-1}$

(ii) $\frac{v_2}{v_1} = \frac{2b}{b-1}$

Solution



From the principle of conservation of momentum

Momentum before collision = momentum after collision

$$m_1u_1 + m_2(0) = m_1v_1 + m_2v_2$$

$$v_2 = \frac{m_1(u_1 - v_2)}{m_2}$$

$$v_2 = b(u_1 - v_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

From conservation of kinetic energy

Kinetic energy before collision = kinetic energy after collision

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2x\ 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2$$

$$v_2^2 = \frac{m_1}{m_2}(u_1^2 - v_1^2)$$

$$v_2^2 = b(u_1^2 - v_1^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Squaring equation (i)

$$v_2^2 = b^2(u_1 - v_1)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(iii)$$

Equation (ii) and (iii)

$$b(u_1^2 - v_1^2) = b^2(u_1 - v_1)^2$$

$$(u_1 + v_1) = b((u_1 - v_1)$$

$$u_1(b-1) = v_1(b+1)$$

$$\frac{u_1}{v_1} = \frac{(b+1)}{(b-1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

(ii) Consider $\frac{u_1}{v_1} = \frac{(b+1)}{(b-1)}$

$$u_1 = v_1\frac{(b+1)}{(b-1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(v)$$

Substitution Eqn (v) into (i)

$$v_2 = b\left(v_1\frac{(b+1)}{(b-1)} - v_1\right)$$

Dividing through by v_1

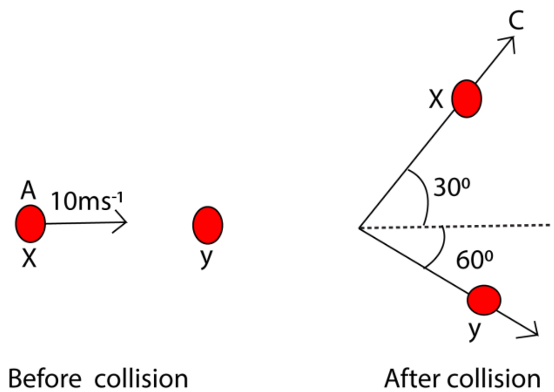
$$\frac{v_2}{v_1} = \frac{2b}{b-1}$$

Example 6

An object X of mass m moving with velocity $10ms^{-1}$ collides with a stationary object Y of equal mass. After collision, x, moves with speed u at an angle 30^0 to its initial direction, while Y moves with a speed of V at an angle 90^0 to the new direction of x.

(i) Calculate the speed u and v.

(ii) Determine whether the collision is inelastic or not.



Consider horizontal momentum

From the principle of conservation of conservation of momentum

m x 10 + m x 0 = m x ucos 30^0 + mvcos 60^0

$$10 = u\frac{\sqrt{3}}{2} + \frac{v}{2} \text{ ……………………………..(i)}$$

Consider vertical momentum

m x 0 + m x 0 = $\frac{u}{2} - v\frac{\sqrt{3}}{2}$

$$u = v\sqrt{3} \text{ …………………………………(ii)}$$

Putting Eqn (ii) into Eqn (i)

$20 = (v\sqrt{3}\ x\ \sqrt{3} + v$

$20 = 4v$

$v = 5ms^{-1}$

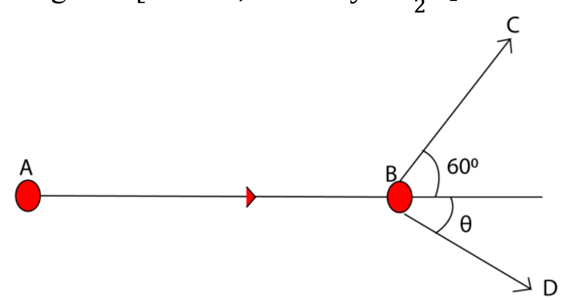
$u = 5\sqrt{3}\ ms^{-1}$

(iii) Kinetic energy before = $\frac{1}{2}\ x\ m\ x\ (10)^2$ = 50M J

Kinetic energy after collision =

Exercise

1. A bullet of mass 300g travelling at a speed of $8ms^{-1}$hits a body of mass 450g moving in the same direction as the bullet at 1.5ms-1. The bullet and the body move together after collision. Find the loss in kinetic energy. [Ans. 3.8025J]
2. A particle A of mass 4kg is incident with velocity V on a stationary helium nucleus B of mass 4kg. After collision, A moves in direction BC with velocity v/2 where BC makes an angle of 600 with the initial direction of AB and the helium nucleus moves along BD. Calculate the angle made in direction AB and the velocity of the helium along BD. [θ = 30^0, velocity = $\frac{v\sqrt{3}}{2}$]



3. (a) State Newton's laws of motion

(b) Use Newton's laws of motion to show that when two bodies collide, their momentum is conserved.

(c) Two balls P and Q travelling in the same direction line in opposite direction with speeds $6ms^{-1}$ and $15ms^{-1}$ inelastic collision. If the masses of P and Q are 8kg and 5kg respectively, find the

(i) final velocity [Ans. $2.08ms^{-1}$]

(ii) change in kinetic energy [Ans. 28.03J]

(d) (i) What is an impulse of force?

(ii) Explain why a long jumper should normally land on sand.

The force exerted on a long jumper on coming to rest is given by F = change in momentum over time taken. Since the change in momentum is constant, it Implies that if the time taken to when coming to rest is increased, then the force exerted on the knees of the jumper reduces. Sand increases the time taken for the jumper to stop reducing the damaging force to the knee.

4. (a)(i) State the law of conservation of linear momentum
   (ii) Use Newton's laws to derive the above.
   (b) Distinguish between elastic and inelastic collisions
   (c) An object X of mass m moving with velocity 10ms-1 collides with a stationary object Y of equal mass. After collision, x, moves with speed u at an angle 30^0 to its initial direction, while Y moves with a speed of V at an angle 90^0 to the new direction.
   (i) Calculate the speed u and v. [Ans. u = $5\sqrt{3}$ ms^{-1}, v =$5ms^{-1}$]
   (ii) Determine whether the collision is inelastic or not. [Ans. kinetic energy on both sides = (50M)J, since kinetic energy is conserved, the collision is elastic]
5. (a) (i) Define linear momentum
   (ii) State the law of conservation of linear momentum.
   (iii) Show that in (ii) above follows Newton's laws of motion.
   (iv) Explain why, when catching a fast moving ball, the hands are drawn back while the ball is being brought to rest.
   (b) A car of mass 100kg travelling at uniform velocity of 20ms-1 collides perfectly in elastically with a stationary car of mass 1500kg. Calculate the loss in kinetic energy of the car as a result of collision. [Ans. 168000J]