These are properties of material under the action of force

(a) Strength: this is the ability of a material to resist breaking when a force is applied. E.g. metals
(b) Stiffness: this is the ability of a material to resist change in shape or size when a force is applied to it e.g. glass, dry wood.
(c) Elasticity is the ability of a material to regain its original shape and size after its deforming force has been removed. E.g. rubber, nullified spring.
(d) Plasticity: This is the ability of a material to remain permanently deformed when a deforming force has been removed e.g. plasticine, wet clay
(e) Ductility: this is the ability of a material to be changed into various shapes without breaking or is the ability of a material to be deformed without breaking e.g. metals

    Ductile materials undergo both elastic and plastic deformation.

(f) Brittleness: this is the ability of a material to break easily or suddenly when a force is applied to it e.g. dry clay, glass, chalk. Brittle materials bend very little and break. Therefore they undergo only elastic deformation and their elastic region is very small.
(g) Hardness/toughness: this is the ability of a material to resist wearing e.g. rubber and metals

Hooke's law

This states that the extension produced in a wire is directly proportional to the force applied provided the elastic limit is not exceeded.

$F \propto e$

F = ke

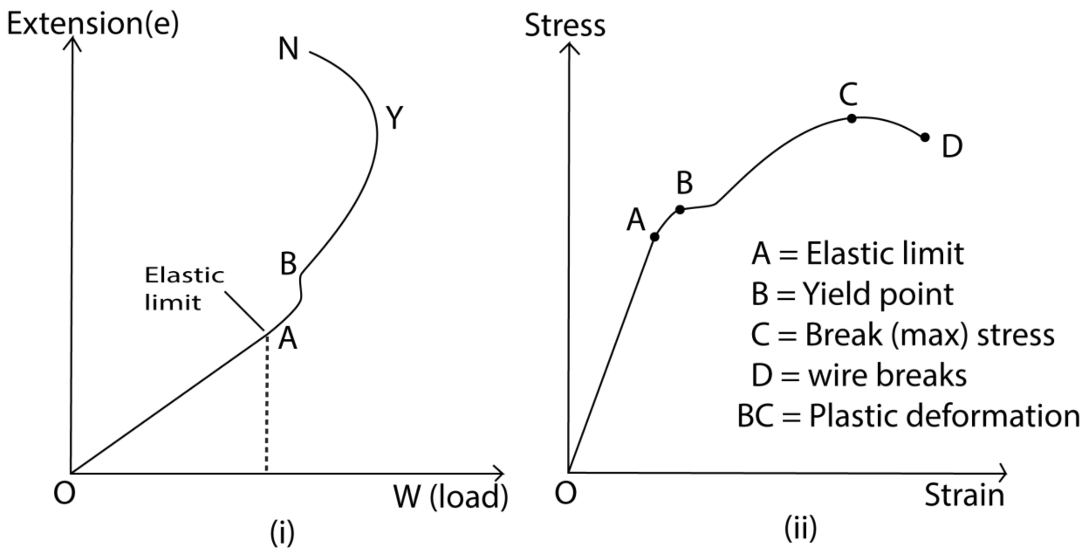
k= $\frac{F}{e}$ where k = force constant

Units of k

k = $\frac{MLT^{-2}}{L} = MT^{-2}$ or ks^{-1} or Nm^{-1}

Force constant, k, is the force required to change length of material by 1m.

Graph of force against extension



Extension versus Load

Description of section of stress- strain curve for ductile material above

Elastic limit:
This is the stress/load/point beyond which a material stops undergoing elastic deformation.
Yield point
Is the stress/load/point beyond which a material stops undergoing plastic deformation
NB. At yield point, there is a sudden increase in extension even though a small force is used.

Region OA
Extension is directly proportional to the force applied (stress is directly proportional to strain).
The material regains its original length and shape when a force/stress is removed. Extension produced is due to the molecules being displaced from their equilibrium position.

Region AB
Material regains its original shape when the force /stress are removed.

Stress is not proportional to strain or extension is not directly proportional to applied force therefore the material does not obey Hooke's law.

Region BC
the material does not fully regain its original shape and length when the force/stress is removed. Therefore Hooke's law is not obeyed.

The extension produced is due to the atoms of molecules being pulled apart breaking the bond between them. When the stretching forces are removed, these bonds are never recovered therefore, the material does not fully regain its original length.

Region CD

The material regains permanently stretched or deformed when the force/stress is removed. The bonds between the molecules are broken completely.

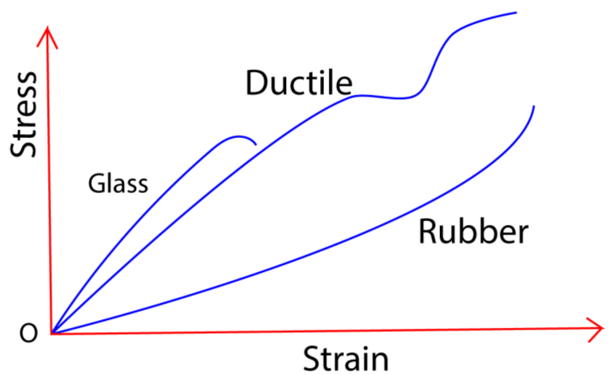
Region DE
The wire breaks in this region with any further increase in force or stress.

At breaking point the wire thins out, become hot. i.e. heat is given out.

Elastic deformation
During elastic deformation, molecular separation increases. This increases elastic potential energy when the force/stress is removed; the molecules regain their equilibrium position. Initial elastic potential energy is regained and stability is restored.

Plastic deformation
During plastic deformation, molecular separation increases leading to gain in elastic potential energy. Some of this energy is then lost in form of heat. When the force/stress is removed, the lost heat is never regained and therefore stability is never restored.

Stress-strain curve for non-ductile material



Glass: has he smallest elastic region and no plastic deformation regions. Glass is brittle due to small cracks on its surface. Any concentration of tensile stress/force on any of these cracks makes the glass break.

Rubber
Stretches easily without breaking and has a greatest range of elasticity. It it does not undergo plastic deformation

Unstretched rubber consist of coiled molecules when a tensile force is applied, they uncoil, become straight and hard. Any further increase in tensile force makes the rubber to break.

Work – hardening
When a metal is repeatedly deformed, it becomes brittle and its resistance to plastic deformation increases. This is called work- hardening of a metal.

Stress

This is the force acting on an area of 1m^2 of a material or it is force per unit area

$$Stress = \frac{Force}{Area}$$

$$[s] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

Units = kgm^{-1}s^{-2}, Nm^{-2} or Pascal (Pa)

Strain

This is the change in length per unit original length or is the change in length per 1m of original length

Strain = $\frac{Extension}{original\ length} = \frac{e}{L}$ (no units)

Young's modulus of elasticity

Young's modulus, E, is the ratio of tensile stress to tensile strain

E = $\frac{Tensile\ stress}{Tensile\ strain}$

[Y] = $\frac{MLT^{-2}}{L^2} = ML^{-1}T^{-1}$

Units = kgm^{-1}s^{-2}, Nm^{-2} or Pascal (Pa)

Or

E = $\frac{Tensile\ stress}{Tensile\ strain} = \frac{F}{A} \div \frac{e}{L} = \frac{FL}{eA}$

Then

F = $\frac{EeA}{L}$

Example 1

A mass of 2kg attached to the end of a wire 2m and diameter 0.64mm causes an extension of 0. 6mm. Find the Young's modulus.

F = 2 x 9.81 = 19.62N, A = $2\pi r^2 = \pi\ x\ (0.32\ x\ 10^{-3})^2 =$ 3.22 x 10^{-7}m^2

$Stress = \frac{19.62}{3.22x\ 10^{-5}}$ = 6.1 x 10^{7}Nm^{-2}

And

$Strain = \frac{e}{L} = \frac{0.6\ x\ 10^{-3}}{2}$ = 3 x 10^{-4}

E = $\frac{Tensile\ stress}{Tensile\ strain} = \frac{6.1\ x\ 10^7}{3\ x\ 10^{-4}}$ = 2.03 x 10^{11} Nm^{-2}

Alternatively

$$E = \frac{Tensile\ stress}{Tensile\ strain} = \frac{F}{A} \div \frac{e}{L} = \frac{FL}{eA}$$

$$E = \frac{2\ x\ 9.81\ x\ 2}{\pi\ 0.32\ x\ 10^{-6}\ x\ 0.3\ x\ 0.3\ x\ 10^{-3}} = 2.03 \text{ x } 10^{11} Nm^{-2}$$

It should be notes that Young's modulus, E, is calculated from ratio stress/ strain with the elastic limit of the material.

Example 2

Find the maximum load in kg in which may be placed on a steel wire of diameter 0.10cm if the permitted strain must not exceed 0.001 and Young's modulus for steel is 2.0 x $10^{11}Nm^{-2.}$

$$E = \frac{Tensile\ stress}{Tensile\ strain}$$

Maximum stress = maximum strain x Young's modulus

= 0.001 x 2 x 10^{11} = 2 x $10^{8}Nm^{-2}$

Area of cross-section in m^2 = $\frac{\pi d^2}{4} = \frac{\pi\ x\ (0.1\ x\ 10^{-2})^2}{4}$ = 7.85 x $10^{-7}m^2$

Stress = $\frac{Force}{Area}$

Force = stress x area = 2 x 10^8 x 7.85 x 10^{-7} = 157N

Mass = $\frac{F}{g} = \frac{157}{9.81} = 16kg$

Force in bar due to contraction or expansion

When a bar is heated, and then prevented from contracting as it cools, a considerable force is exerted at the end of the bar.

For a bar which is L m having Young's modulus, E, a cross-sectional area, A, a linear expansivity of magnitude, α, and decrease in temperature, t^0 C; the decrease length e if were free to contract will be = αLt

$$E = \frac{Tensile\ stress}{Tensile\ strain}$$

$$= \frac{F}{A} \div \frac{e}{L}$$

$$= \frac{FL}{eA}$$

$$F = \frac{EA\alpha Lt}{L}$$

$$= EA\alpha t$$

Example 3

A steel rod of cross section area 2.0cm^2 is heated to 100^{0}C, and then prevented from contracting when it is cooled to 10^{0}C. the linear expansivity of steel = 12 x 10^{-6}K^{-1} and Young's modulus, E = 2.0 x 10^{11}Nm^{-2}. Find the force exerted.

Solution

A = 2cm^2 = 2 x 10^{-4} m^2, t = 100 – 10 = 90^{0}C

$F = EA\alpha t$

= 2 x 10^{11} x 2 x 10^{-4} x 12 x 10^{-6} x 90

= 43200N

Energy stored in a stretching wire

When a wire is stretched by an amount, e, by applying a force, F without exceeding elastic limit. The average force = $\frac{(0+F)}{2} = \frac{1}{2}F$

Work done/ work stored in a wire = force x distance = $\frac{1}{2}F.e$

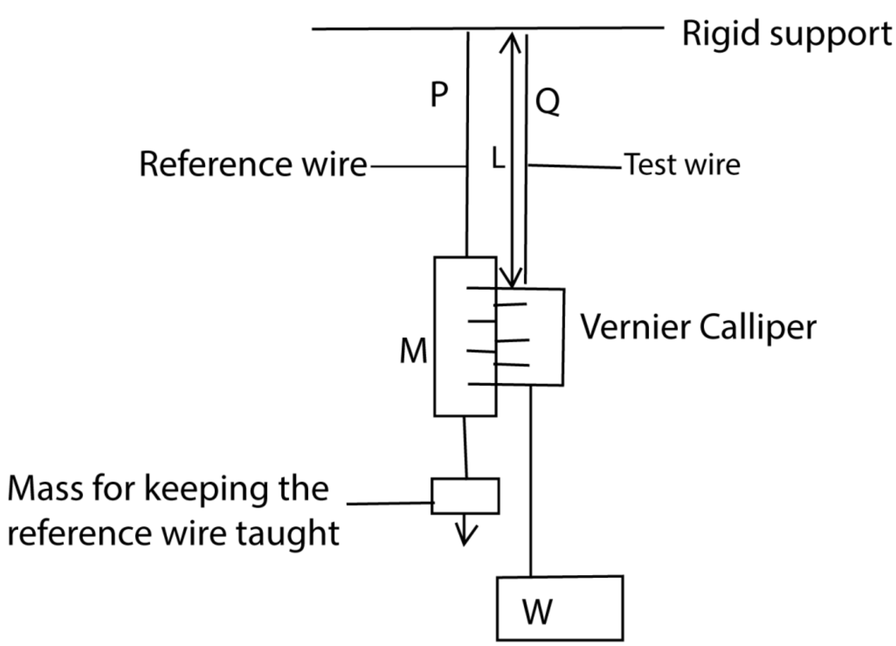
Work done per unit volume of a wire

The volume of the wire = AL

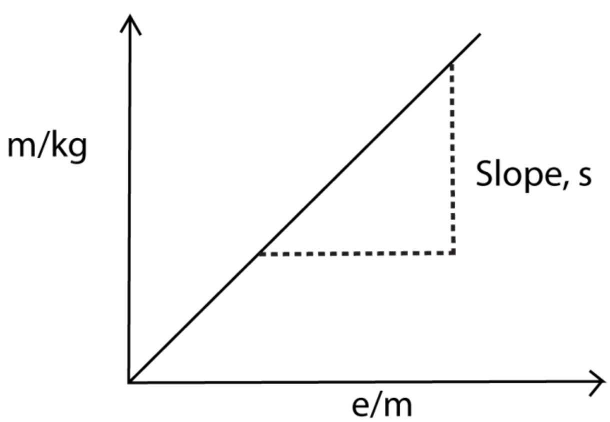
Energy per unit volume = $\frac{1}{2}F.e \div AL = \frac{1}{2}\ x\ \frac{F}{A}\ x\ \frac{e}{L}$

Energy stored per unit volume = $\frac{1}{2}\ x\ stres\ x\ strain$

Experiment to determine Young's Modulus for a metal wire



(i) Two thin, long wires of the same material and length P and Q are suspended from a rigid support.
(ii) P carries a scale M in mm and its straightened by attaching a weight at its end.
(iii) Q carries a vernier scale which is alongside scale M
(iv) Various loads are added to the test wire and corresponding extensions caused are read off from a vernier scale.
(v) The diameter (2r) of the wire is obtained by a micrometer screw gauge, and the cross section area of the wire A = $4\pi r^2$
(vi) A graph of mass (m) of the load against extension e is plotted



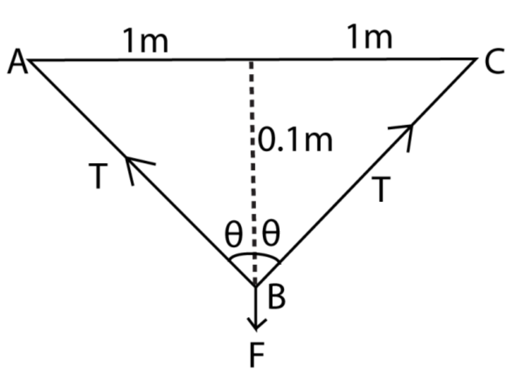
Young's modulus, Y = $\frac{gsL}{A}$

Precaution in the experiement above

1. After each reading, the load is removed to check that the wire returns to its original length, to ensure that elastic limit is not exceeded.
2. Long wires are used to achieve measurable expansion
3. Thin wires are used to produce high tensile stress
4. Identical wires are used to eliminate error of expansion or contraction due to changes in temperature.

Example 4

A metal wire of diameter 2.0 x 10^{-4}m and length 2m is fixed horizontally between two points 2m apart. Young's modulus for the wire is 2 x 10^{11}Nm^{-2}.
(i) What force should be applied at the midpoint of the wire to depress it by 0.1m
(ii) Find the work done



$\cos\theta = \frac{0.1}{AB}$ but $AB = \sqrt{(1^2 + 0.1^2)}$ = 1.005m

$\cos\theta = \frac{0.1}{1.005}$

Length ABC = 2AB = 2 x 1.005 = 2.01
Extension, e = 2.01 – 2 = 0.01m

T = $\frac{YAe}{l}$ and A = $A = \pi r^2 = \frac{\pi d^2}{4}$
Resolving vertically
F = 2Tcosθ
F = $\frac{2YAecos\,\theta}{L} = \frac{2Y\pi d^2 ecos\theta}{4L}$

F = $\frac{2\,x\,2\,x\,10^{11} x\,\pi\,x\,(2\,x\,10^{-4})^2\,x\,0.01\,x\,0.1}{1\,x\,4\,x\,1.005} = 12.5N$

(iii) Work done = $\frac{1}{2}Fe = \frac{1}{2}\,x\,12.5\,x\,0.01 = 0.0625J$

Example 5

A uniform bar of length 1.0m and diameter 2.0cm is fixed between two rigid supports at 25^0C. If the temperature of the rod is raised to 75^0C.

Find

(i) The force exerted on the supports.
(ii) The energy stored in the rod at 75^0C.

(Young's modulus for the metal = 2.0 x 10^{11}Pa,
coefficient of linear expansion = 1.0 x $10^{-5}K^{-1}$)

Solution

(i) F = YAαΔθ
= 2.0 x 10^{11} x $(\pi\,x\,0.01^2)$ x 1.0 x 10^{-5} (75 – 25)
= 31400N

(ii) Energy stored = $\frac{1}{2}Fe$ but e = αLΔθ
= $\frac{1}{2}\,x\,31400\,x\,1\,(75 - 25) = 7.85$J

Exercise

1. A uniform wire of upstretched length 2.49m is attached to two points A and B which are two meters apart and in the same horizontal line. When 5kg mass is attarched to the midpoint, c, of the wire, the equilibrium point of c is 0.75m between the line AB.
(Young's modulus for the wire =2.0 x 10^{11}Pa)
Find
(i) Strain and stress in the wire [strain = 0.00402, stress = 8.04 x 10^8Pa]
(ii) Energy stored in the wire [Ans. 2.02 x 10^3J]

2. A thin steel wire initially 1.5m long and diameter 0.5mm is suspended from a rigid support. Calculate
   (i) extension (Ans. 3.53×10^{-3}m)
   (ii) the energy stored in the wire, when a mass of 3kg is attached to the lower end. [5.19×10^{-2}J]
   (Young's modulus = 2.0×10^{11}Nm^{-2}]
3. Two thin wires, one of steel and the other of bronze each 1.5mg and the diameter 0.2cm are joined end to end to form a composite wire of 3m. What tension will produce a total extension of 0.064cm? (Young's modulus for steel = 2.0×10^{11}Pa, Young's modulus of bronze = 1.2×10^{11}Pa) {Ans. 1009N]