Motion under gravity

In absence of any resistance, all bodies regardless of their mass fall with same acceleration near the earth's surface.

Acceleration due to gravity is the rate of change of velocity for freely falling body and it is symbolized as g. $g = 9.81ms^2$ and it replaces "a" in equation of motion

For a body falling under gravity, g is positive and g is negative for the object moving upwards.

Example 1

A ball is thrown vertically upwards with initial speed $20ms^{-1}$. After reaching the maximum height and on the way down it strikes a bird 10m above the ground.

(a) Calculate the highest point reached

$u = 20ms^{-1}$, $g = -9.8ms^{-1}$, $v = 0$

from $v^2 = u^2 + 2as$

$0 = 20^2 + 2 \times -9.8 \times s$

The highest distance, s = 20.4m

(b) Calculate the speed at which it strike the bird

$u = 0$, $s = (20.4 - 10) = 10.4m$, $g = 9.8ms^{-2}$

from $v^2 = u^2 + 2as$

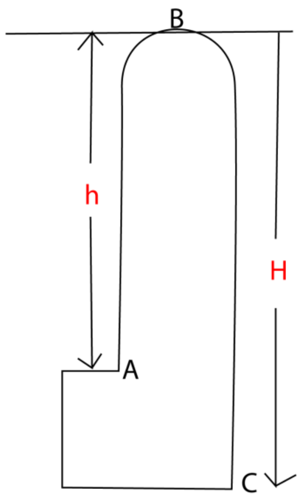
$v2 = 0^2 + 2 \times 9.8 \times 10.4$

$v = 14.3ms^{-1}$

Example 2

A stone is thrown vertically upwards with a speed of $10ms^{-1}$ from a building. If it takes 2.5 seconds to reach the ground, find the height of the building.

Solution



Time taken to move from A to B

v = u -at

0 = 10 – 9.81t; t = 1.02s

Height, h

h = ut - ½ gt^2

= 10 x 1.02 – ½ x 9.81 x $(1.02)^2$

= 5.1m

Time taken from B to C = 2.5 – 1.02 =1.48s

Distance, H, u = 0, t = 1.48, g -9.81ms^{-2}

H = 0 x 1.48 + ½ x 9.81 x 1.48^2

= 10.7m

Height of the building = H – h = 10.7-5.1 = 5.6m

Exercise

1 A ball is thrown straight upwards with a speed ams^{-1} from a point h m above the ground. Show that time taken to reach the ground is t = $\frac{u}{9}\left[1+\left(1+\frac{2gh}{u^2}\right)^{\frac{1}{2}}\right]$

2. A motorist travelling at a constant speed of 50 kmh^1 passes a motorcyclist just starting off in the same direction. If the motorcyclists maintains a constant acceleration of 2.8ms^{-2} calculate; (i) Time taken by motorcyclist to catch up with the motorist. (9.9s)

(ii) The speed at which the motorcyclist overtakes the motorist.(27.72ms^{-1})

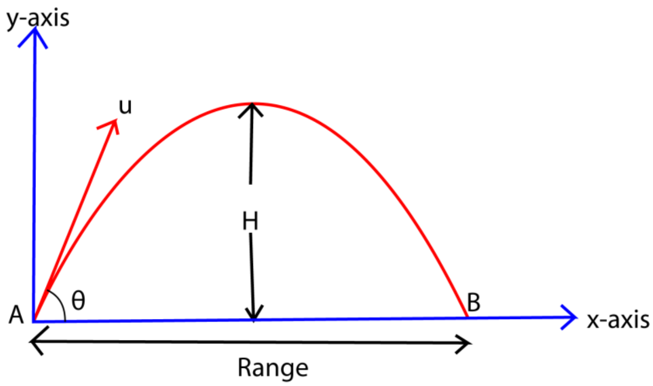
(iii) The distance travelled by the motorcyclists before overtaking.(137.2m)

3. A stone is thrown vertically upwards from a point at a height, h, above the ground level and initial velocity 20ms^{-1}. If the stone, hits the ground, 5s later; find h [Answer 22.625m

Projectile

A projectile is anything which is given an initial velocity and left to move on its own in the presence of a constant force field, e.g. gravitation force field. In this case, air resistance is negligible.

Consider a body projected with a speed u at an angle θ to the horizontal



θ = angle of project

A –point of projection

H- maximum height of projection

AB –range

The projection has both vertical and horizontal component which are independent of each other. the acceleration due to gravity for the vertical component is g while that of the horizontal component is zero, that is, the horizontal velocity is constant.

Terminology

(a) **Angle of projection** is the angle between the direction of the projection and the horizontal.
(b) **Trajectory** is the path followed by a projectile
(c) **Maximum height, H**, is the distance between the highest point reached and the horizontal plane through the point of projection.
(d) **Time of flight (T)** is the time taken by the projectile or particle to move from its initial position to the final position along its path.
(e) **Horizontal range** is the distance from the initial to the final position of projection.

Horizontal motion

Horizontal component of velocity is got by

$v_x = u_x + a_x t$.

Where v_x, is the velocity of a body at any time t, while u_x and a_x are the initial component of velocity and horizontal acceleration respectively.

But $u_x = u\cos\theta$, since $a_x = 0$

Hence $v_x = u\cos\theta$ -------------(1)

From the above equation the horizontal velocity is constant throughout motion.

The horizontal distance , x, travelled after time t is given by

$x = u_x t + ½ at^2$

But $a_x = 0$

$\therefore x = u_x t \cos\theta$……………………(2)

Vertical motion

$v_y = u_y + a_y t$ where v_y, is the vertical velocity of a body at any time, t, while u_y and a_y are initial velocity component of velocity and vertical acceleration respectively.

$u_y = u \sin\theta, a_y = -g$

$v_y = v \sin\theta - gt$………………………………………………………….......(3)

The vertical displacement, y, is obtained below

$y = u_y t + ½ a_y t^2$

But $u_y = u\sin\theta$, $a_y = -g$

Hence

$y = (u\sin\theta)\, t - ½gt^2$…………………………………………...…………(4)

Speed, V, at any time t is given by

$v = \left[\sqrt{v_x^2 + v_y^2}\right]$…………………………………………………………(5)

The angle, α, the body makes with the horizontal after t is given by

$\tan\alpha = \frac{v_y}{v_x} = \frac{u\sin\theta - gt}{u\cos\theta}$ ………………………………………………………… (6)

Maximum height, H

At maximum height, $v_y = 0$

$v_y^{2\,2} = u_y^2 + 2aH$

$0 = (u\sin\theta)^2 - 2ghH$

$H = \frac{u^2 \sin^2\theta}{2g}$ ………………………………… … … … … … … … … … … … … … ……..(7)

2 2

Time to reach the maximum heights

Using v = u + at

$0 = u_y + a_y t$

$0 = u\sin\theta - gt$

$t = \frac{u\sin\theta}{g}$ ……………………………………………………………………(8

Time of flight, T

The time taken by the projectile to move from the point of projection to a point on the plane through the point of projection where the projection lies i.e. time taken to move from A to B.

At B, y = 0

y = utsinθ $-\frac{gt}{2}$

0 = 2atsinθ – gt^2

0 = t(2usinθ –gt)

Either t =0 or T= $\frac{2u\sin\theta}{g}$ ………………………………………………..….(9)

Note: time of flight is twice the time taken to reach the maximum height

<u>Ranges , R:</u>

It is the distance between the point of projection and a point on the plane through the point of projection where the projectile lands i.e. horizontal distance AB.

x= ut cosθ

When x= R, t = T = $\frac{2u\sin\theta}{g}$

$\therefore R = u.\frac{2u\sin\theta}{g}\cos\theta = \frac{2u^2\sin\theta\cos\theta}{s}$ ………………………….….(10)

But 2sinθcosθ = sin2θ

$R = \frac{u^2\sin 2\theta}{g}$

For maximum range (R_{max})

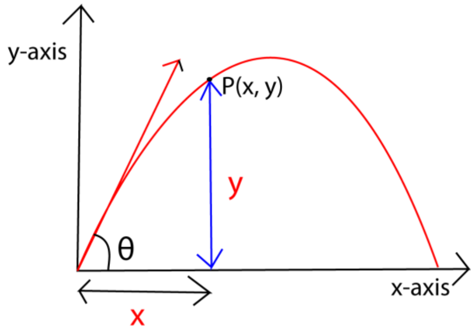
At R_{max}, θ = 45^0

Sin2θ= sin 90

$R_{max} = \frac{u^2}{g}$

Equation of trajectory

Consider a body project with a speed, u, from the ground and angle θ from horizontal.



Suppose the body passes through a point P(x, y) after time, t.

Consider vertical motion

x = (ucos θ)t

t = $\frac{x}{ucos\,\theta}$

Consider vertical motion

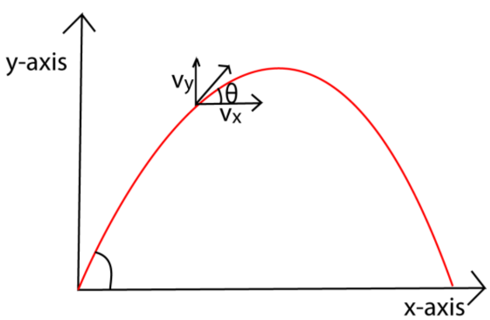
y = (usin θ)t – ½ gt^2

Substituting for t

y = (usin θ)$\frac{x}{ucos\,\theta}$ – ½ g$(\frac{x}{ucos\,\theta})^2$

= xtanθ - $\frac{gx^2}{2u^2}(1 + \tan^2\theta)$

Direction of motion



The direction of motion is determined by the direction of velocity of particles at any time, t. and its angle θ to which the velocity makes with the horizontal

$\tan\theta = \frac{v_y}{v_x}$

But v= u + at

v_y = usin θ -gt

$v_x = u\cos\theta$

$\theta = \tan^{-1}\left[\frac{u\sin\theta - gt}{u\cos\theta}\right]$

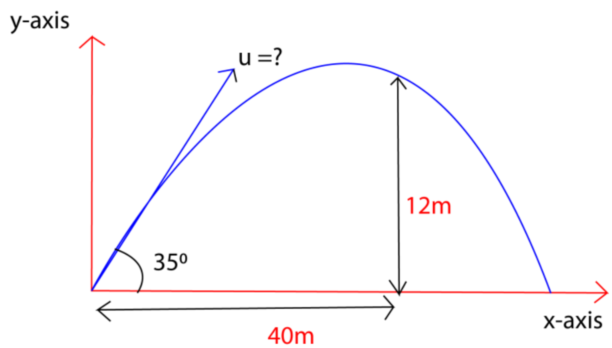
Magnitude of velocity, v= $\sqrt{[v_x^2 - v_y^2]}$

Example 3

A particle is projected at 35^0 to the horizontal and just clears a wall 12m high and 40m away from the point of projection. Find

(i) The speed of projection

(ii) Velocity of particle when it strikes the wall and time taken to reach the wall.



(i) From $y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$

$12 = 40\tan 35 - \frac{9.81 \times 40 \times 40}{2u^2}(1 + \tan^2 35^0)$

$u = \sqrt{730.6085} = 27.03ms^{-1}$

(ii) From $t = \frac{x}{u\cos\theta} = \frac{40}{27.03\cos 35^0}$

t = 1.8s

Velocity, v

From v= $\sqrt{[v_x^2 - v_y^2]}$

$v_x = 27.03cos\ 35$ = 22.14ms^{-1}

v_y = 27.03sin 35^0 -9.81 x 1.8 = -2.658

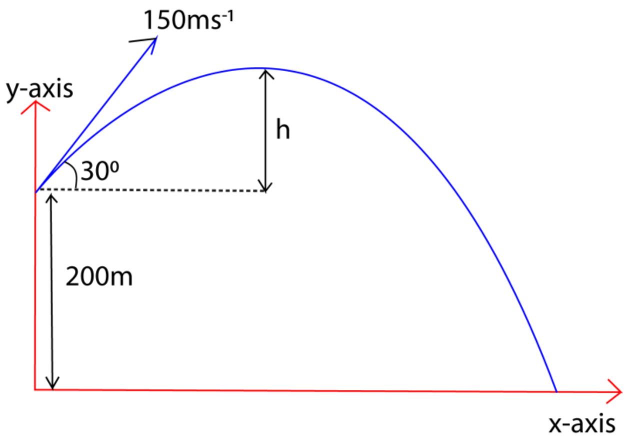
v = $\sqrt{[22.14^2 + (-2.658)^2]}$ = 22.3ms^{-1}

Example 4

A bullet is fired from a gun placed at a height of 200m with a velocity of 150ms^{-1} at an angle of 30^0. Find the

(i) Maximum height attained
(ii) Time taken for the bullet to hit the ground

Solution



(i) From h = $\frac{u^2 \sin^2 \theta}{2g}$

h = $\frac{150^2 \sin^2 30}{2\ x\ 9.81}$ = 286.7m

H = 200 + h

= 200 + 286.7 = 486.7m

(ii) Let time taken be t

From y = (usin θ)t – ½ gt^2

$-200 = (150\sin 30)t - \frac{1}{2} 9.81t^2$

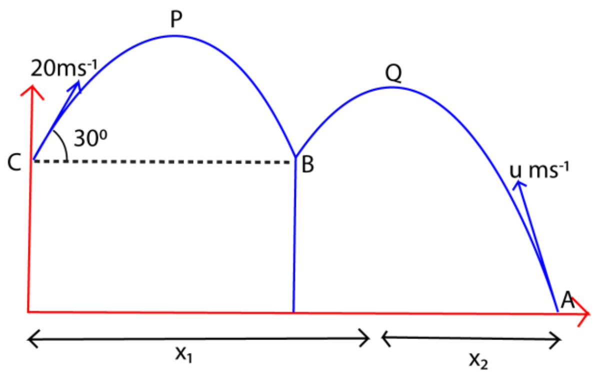
$t = 17.6s$

Example 5

An object P is projected upwards from a height 60m above the ground from a height 60m above the ground with a velocity 20m/s at 30^0 to the horizontal, at the same time an object Q is projected from the ground upwards towards P at 30^0 to the horizontal. P and Q collided at a height of 60m above the ground. Find

(i) The speed of projection of the object Q.

(ii) The horizontal distance between the point of projection



(a) Speed of Q

Time of flight A and B = time of flight from C to B

$$T = \frac{2u\sin\theta}{g} = \frac{2 \times 20 \times \sin 30}{9.81} = 2.04s$$

Speed u of Q

(b) From $y = (u\sin\theta)t - \frac{1}{2} gt^2$

$60 = u\sin30 \times 2.04 - \frac{1}{2} \times 9.81 \times (2.04)2$

$u = 78.84ms^{-1}$

(c) Range $x_1 = u\cos30t$

$= 20 \times \cos 30 \times 2.04$

$= 35.33m$

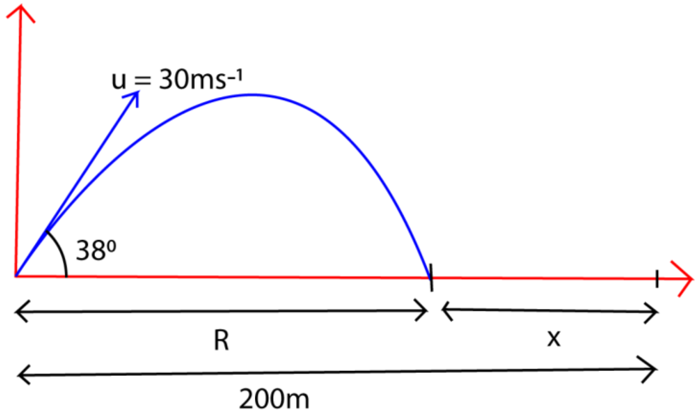
Distance $x_2 = 78.84 \times \cos 30 \times 2.04$

$= 139.29$

Distance between the point of projection = 139.29 + 35.33 = 174.62m

Example 6

Two foot ballers 120m apart standing facing each other, one kicks a ball from the ground such that the ball takes off at a velocity $30ms^{-1}$ at38^0 to the horizontal. Find the speed at which the second footballer must run towards the first footballer in order to trap the ball as it touches the ground if he starts running at the instant the ball is kicked.



Range, R =$\frac{u^2 sin2\theta}{g}$ = $\frac{30^2 \sin(2 \; x \; 38)}{9.81}$ 89.02m

Time T taken to cover distance, R,

T = $\frac{2usin\theta}{g}$ = $\frac{2usin\;\theta}{g}$ =$\frac{2 \; x \; 30x \sin 38}{9.81}$ =3.77s

Distance x to be covered by the second footballer = 120 – R = 120 – 89.02 = 30.98m

Speed = $\frac{Distance}{time} = \frac{30.98}{3.77} = 8.22ms^{-1}$

Exercise

1. A projectile is fired horizontally from the top of a cliff 250m high. The projectile landed 1.414 x 103m from the bottom of the cliff. Find the
   (i) Initial speed (Ans. $198.06ms^{-1}$)
   (ii) Velocity of the projectile just before it hits the ground. (Ans. $210.03ms^{-1}$)
2. (a) Define the term of flight and range as applied to the projectile motion.
   (b) A projectile is fired in air with a speed u ms-1 at an angle θ to the horizontal. Find the time of flight of the projectile.(T = $\frac{2usin\theta}{g}$)
3. (a) Define the term of flight and range as applied to the projectile motion.
   (b) A stone is projected at an angle 200 to the horizontal and just clears a wall which is 10m high and 30m from the point of projection. Find the
   (i) speed of projection [Ans. $73.75ms^{-1}$]
   (ii) Angle at which the stone makes with the horizontal at it clears the wall.[Ans. 16.84^0]
4. Prove that the time of flight T and the horizontal range, R, of a projectile are connected by equation, gT2 = 2Ttan α. Where α is the angle of projections.

5. A projectile is fired from ground level with a velocity of $500ms^{-1}$ at 30^0 to the horizontal. Find the horizontal range, the greatest height to which it rises and time taken to reach the greatest height. What is the least speed with which it could be projected in order to achieve the same horizontal range? [Ans. Range = 22069.96m, H = 3185.m, u_{min} =$465,3ms^{-1}$)
6. A body is thrown from the top of a tower 30.4m high with a velocity of $24ms^{-1}$ at an elevation of 300 above the horizontal. Find the horizontal distance from the roof of the tower of the point where it hits the ground. [Ans. 61.1m]
7. A body is projected at such an angle that the horizontal range is three times the greatest height. Given that the range is 400m, fins the necessary velocity of projection and angle of projection. [velocity = $64ms^{-1}$, angle = 53.13^0]
8. A projectile fire at an angle of 60^0 above the horizontal trikes a building 30 away at a point 5m above the point of projection. Find
   (i) The speed of projection. [time = 3.094s]
   (ii) Velocity of the projectile when it strikes the building.[u = $19.39ms^{-1}$]
9. An object P is projected upwards from a height of 60m above the ground with a velocity of $20ms^{-1}$ at 30^0 to horizontal. P and Q collide at a height 60m above the ground while they are both moving downward. Find
   (i) The speed of projection Q.[Ans. $78.84ms^{-1}$]
   (ii) The horizontal distance between the points of projection [174.62m]
   (iii) The kinetic energy of P before the collision with Q if the mass of P is 0.5kg [Answer200J]