Physical quantities

Physical quantities are divided into two groups

(a) Fundamental quantities

These are physical quantities which cannot be expressed in form of other quantities using any mathematical equations. They include

Quantity	S.I unit	Symbol of S.I unit
Mass	kilogram	kg
Time	second	s
Length	metres	m
Temperature	Kelvin	K
Current	Ampere	A

(b) Dimensional/derived quantities

These are physical quantities which can be expressed in terms of fundamental quantities. For example velocity, work, volume, density

Dimensions of physical quantities

This is a way in which derived quantities can be expressed in form of fundamental quantities. i.e.

Mass- M

Length- L

Time –T

The square bracket, [ ] is used to show dimensions

For example

(i) [Area] = length x length or length x width

$L \times L = L^2$

(ii) Volume = length x width x height

$= L \times L \times L = L^3$

(iii) Density = $\left[\frac{mass}{volume}\right] = \frac{M}{L^3}$ or ML^{-3}

(iv) Velocity = $\left[\frac{lenght}{time}\right] = \frac{L}{T}$ = LT^{-1}

(v) Acceleration =$\frac{velocity}{time}$ = $\frac{LT^{-1}}{T}$ = LT^{-2}

(vi) Force = mass x acceleration

= M x LT^{-2} = MLT^{-2}

Application of dimensions

(i) To check the validity of equation (equations that are not dimensionally consistent are obviously wrong expression and should be discorded.

(ii) To derive equations: for correct equation, the units of the left hand side must be similar to the units of the right hand side.

All right equations must be dimensionally consistent but not all dimensionally consistent equations are correct.

Examples 1

(a) The centripetal force F on a body of Mass M moving at constant speed V round a circular path of radius, r, is given by F = $\frac{MV^2}{r}$.

Show that the equation is dimensionally consistent

Solution

LHS = F = Ma = M x LT^{-2} = MLT^{-2}

RHS = $\frac{MV^2}{r}$. = $\frac{M\,x\,(LT^{-1})^2}{L}$ = MLT^{-2}

Since [LHS] =[RHS], the equation is dimensionally consistent.

(b) Show that the second equation of motion is dimensionally correct

s = ut + ½ at^2

[LHS] = [s] = L

[RHS]= [u][t] + ½ [a]$[T]^2$

=LT^{-1} x T + ½ LT^{-2} xT^2

= L + ½ L = $\frac{3}{2}$ L

Since $\frac{3}{2}$ is a constant,

[LHS] =[RHS] showing that the equation is dimensionally consistent.

(c) The equation of a transverse wave of a rod of youngers modulus (E) and density, ρ, is given by v = $\sqrt{\frac{E}{\rho}}$. Show that it is dimensionally consistent.

Solution

[LHS] = [v] = LT^{-1}

[RHS]

E = ——

E = $\frac{stress}{strain}$ = $\left[\frac{MLT^{-2}}{L^2}\right]$ = $ML^{-1}T^{-2}$

$\sqrt{\frac{ML^{-1}T^{-2}}{ML^{-3}}} = \sqrt{(L^2T^{-2})}$ = LT^{-1}

[LHS] =[RHS] , the equation is dimensionally consistent

Derivation of equation

The method of dimensions can be used to derive equations.

Examples 2

The period (T) of a pendulum bob depends on the length of the pendulum (L), mass of the pendulum ball, M, and acceleration due to gravity, g. Determine an expression for the period of a simple pendulum T in terms of the quantities mentioned.

Solution

T ∝ MLg

T = $kM^xL^yg^z$ where k is dimensionless constant

[T] =$[M]^x[L]^y[g]^z$

[LHS] = [T] = T

[RHS] = $M^xL^y[LT^{-2}]^z$

Equate powers of T, M,L

For T; -2z = 1

z = -½

For L; y + z =0

y – ½ = 0

y = ½

For M, x = 0

Therefore, T = $KL^{-\frac{1}{2}}$ x $g^{\frac{1}{2}}$ = $K\sqrt{\frac{L}{g}}$

T = $K\sqrt{\frac{L}{g}}$

Exercise

1. The sphere of radius, α, moving through a liquid of density, ρ, and velocity, v, experiences a retarding force given by F = $k\alpha^x\rho^yv^z$, where K is a non dimensional constant. Use dimensions to find the values of x, y and z. [Ans, y =1, z =2 , y = 2]

2. Use dimensional analysis to show how the process of velocity transverse process vibration of a stretched string depends on its length, L, mass, m, and the tension F of the string V = $KL^xM^yF^z$, where k is a non-dimensional constant. Find the values of x, y, z. [Ans, x = ½ , y = - ½ , z = 1/2]
3. A cylindrical vessel of cross section area, A, contains air of volume, V, atmospheric

pressure, ρ, trapped by frictionless down and released.

If the piston oscillates with simple harmonic motion, show that the frequency is given by f = $\frac{A}{2\pi}\sqrt{\frac{\rho}{MV}}$ and show that the expression is correction.

4. The equation for volume, V, of a liquid flowing through a pipe in time, t, under steady flow id given by, $\frac{V}{t} = \frac{\pi r^4 \rho}{8\eta l}$
   r = radius of the pipe
   ρ = pressure difference between the two ends of the pipe
   l = length of the pipe
   η =coefficient of viscosity of the liquid
   If the dimensions of η are $ML^{-1}T^{-1}$ show that the above equation is dimensionally consistent.
5. For streamline flow of a non-viscous incompressible fluid , the pressure, ρ, at a point is related to height, h. and velocity, V, by the equation
   $(p-a)=pg(h-b) + \frac{1}{2} p(v^2 - d)$
   Where a, b, and d are constant and p is the density of fluid and g is the acceleration due to gravity. Given that the equation is dimensionally consistent. Find the dimensions of a, b, and d.
   Solution
   Hint, we add or subtract quantities that have the same dimensions
   {ans. [a] has the same units as pressure = $ML^{-1}T^{-2}$, [b] has the same dimension as h = L, [d] has the same dimensions as $v^2 = L^2T^{-2}$}