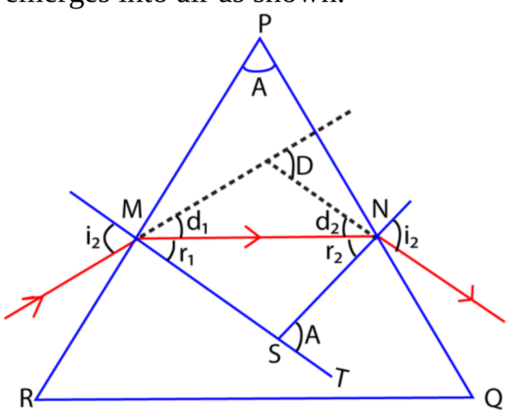
Deviation of light through a prism

The angle of deviation caused by the prism is the angle between the incident ray and the emergent ray.

Consider a ray of light incident in air on a prism of refracting angle **A** and finally emerges into air as shown.



From the diagram above, MS and NS are normal at the points of incidence and emergence of the ray respectively.

$\therefore \quad \angle MPN + \angle MSN = 180°$

Thus $\angle NST = \angle MPN = A$

Suppose $\mathbf{i_1, r_1}$ and $\mathbf{i_2, r_2}$ represents angles of incidence and refraction at faces M and N respectively, then

From geometry of Δ MNS, $r_1 + r_2 = A$ ------------------ (i)

Total deviation $D = d_1 + d_2$ where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\Rightarrow \quad D = i_1 - r_1 + i_2 - r_2$

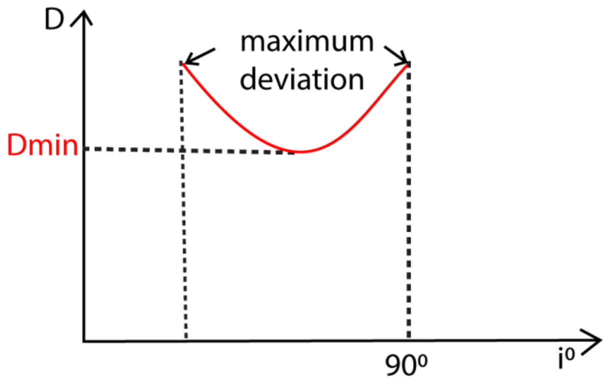
On simplifying, $D = i_1 + i_2 - (r_1+r_2)$ ------------- (ii)

Combining equation (i) and (ii) gives

$$\mathbf{D = i_1 + i_2 - A}$$

NOTE

Experiments show that as the angle of incidence **i** is increased from zero, the deviation **D** reduces continuously up to a minimum value of deviation $\mathbf{D_{min}}$ and then increases to a maximum value as the angle of incidence is increased as shown below:



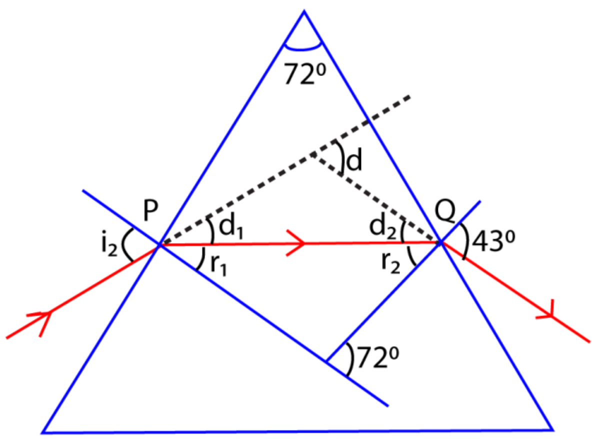
Example1

A ray of light is incident on a prism of refracting angle **72**° and refractive index of **1·3.** The ray emerges from the prism at **43**°**.**

Find

(i) the angle of incidence.

(ii) the deviation of the ray.



(i) At **P,** Snell's law becomes .

$$n_a \sin 43° = 1{\cdot}3 \sin r_2$$

$$\therefore \quad r_2 = 31{\cdot}64°$$

$$\text{But } r_1 + r_2 = 72°$$

$$\Rightarrow \quad r_1 = 72° - r_2$$

$$= 72° - 31{\cdot}64°$$

$$\therefore \quad r_1 = 40{\cdot}36°$$

At **Q**, Snell's law becomes

$$n_a \sin i_1 = 1{\cdot}3 \sin 40{\cdot}36°$$

$$\therefore \quad \mathbf{i_1 = 57{\cdot}34°.}$$

(ii) **Total Deviation D = $d_2 + d_1$**

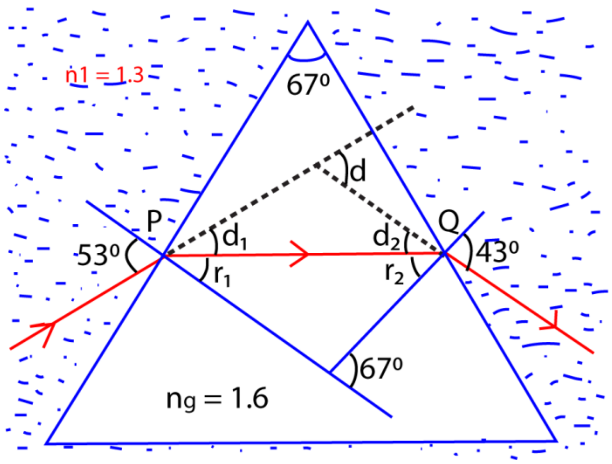
where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\Rightarrow$ D = (43° – 31·64°) + (57·34° – 40·36°)

$\therefore$ **D = 28·34°.**

Example 2

A prism of refracting angle 67° and refractive index of 1·6 is immersed in a liquid of refractive index 1·2. If a ray of light traveling through the liquid makes an angle of incidence of 53° at the left face of the prism, Determine the total deviation d, of the ray.



Total Deviation d = $d_2 + d_1$

where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\Rightarrow$ D = (53° – r_1) + ($i_2 - r_2$) ----------------------------------(i)

At P, Snell's becomes

$1{\cdot}2 \sin 53° = 1{\cdot}6 \sin r_1$

$\therefore$ $r_1 = 36{\cdot}8°$

But $r_1 + r_2 = 67°$

$\Rightarrow$ $r_2 = 67° - r_1$

$= 67° - 36{\cdot}8°$

$\therefore$ $r_2 = 30{\cdot}2°$

At Q, Snell's law becomes

$1{\cdot}6 \sin 30{\cdot}2° = 1{\cdot}2 \sin i_2$

$i_2 = 42{\cdot}12°$

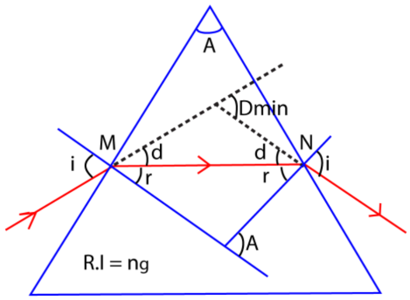
Substituting for r_1, i_2, and r_2 in equation (i) gives

d = (53° – 36·8°) + (42·12° – 30·2°)

d = 28·12°

Minimum deviation of light by a prism

At minimum deviation, light passes **symmetrically** through the prism. That is to say, the angle of incidence is equal to the angle of emergence.



Consider a ray on one face of the prism at an angle $\mathbf{i_1}$ and leaves it at an angle $\mathbf{i_2}$ to the normal as shown

For minimum deviation, $i_1 = i_2 =$ **i** and $r_1 = r_2 =$ **r**. From the diagram, $D_{min} = d + d$

$D_{min} = 2d$ where $d = i - r$

$D_{min} = 2i - 2r$ ------------------------------ (a)

But $r + r = A$

$\Rightarrow 2r = A$ **OR** $r = \frac{A}{2}$ ------------------------------------- (b)

Combining equation (a) and (b) gives

$D_{min} = 2i - A$

$i = \frac{D_{mim}+A}{2}$ ---------------------------------(c)

At M Snell's law becomes

$n_a \sin i = n_g \sin r$

$n_g = \frac{n_a \sin i}{\sin r}$ ------------------------------(d)

Substituting equation (b) and (c) in (d) gives

$$n_g = n_a \frac{\sin\left(\frac{D_{min}+A}{2}\right)}{\sin A/2}$$

Since $n_a = 1$,

$$n_g = \frac{\sin\left(\frac{D_{min}+A}{2}\right)}{\sin A/2}$$

Note that

Equation (e) suggests that if the prism was surrounded by a medium of refractive index n_l, then at minimum deviation

$$n_g = n_1 \frac{\sin\left(\frac{D_{min}+A}{2}\right)}{\sin A/_2}$$

Examples 3

Calculate the angle of incidence at minimum deviation for light passing through a prism of refracting angle **70**° and refractive index of **1·65**.

Solution

Using $n_g = n_a \frac{\sin\left(\frac{D_{min}+A}{2}\right)}{\sin A/_2}$ where A = 70^0, Ng = 1.65, n_a = 1

$1.65 = \frac{\sin\left(\frac{D_{min}+70}{2}\right)}{\sin 70/_2}$

Dmin = 72^0

The required angle of incidence i $= \frac{D_{min}+A}{2} = \frac{72+70}{2} = 71^0$

Examples 3

An equilateral glass prism of refractive index **1·5** is completely immersed in a liquid of refractive index **1·3**. if a ray of light passes symmetrically through the prism, calculate the:

(i) angle of deviation of the ray.
(ii) (ii) angle of incidence

Analysis:

(a) For an equilateral prism, its refracting angle A =60°
(b) If the ray passes through the prism symmetrically, then the angle of deviation is minimum

Using $n_g = n_a \frac{\sin\left(\frac{D_{min}+A}{2}\right)}{\sin A/_2}$ where A = 760^0, Ng = 1. 5, n_1 = 1.3

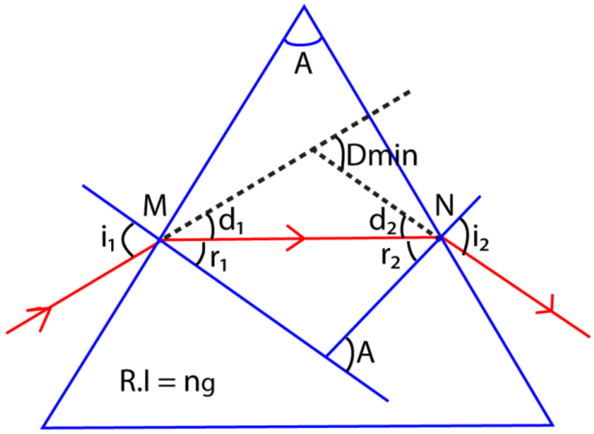
$1.5 = \frac{\sin\left(\frac{D_{min}+60}{2}\right)}{\sin 60/_2}$

Dmin = 10.5^0

The required angle of incidence i $= \frac{D_{min}+A}{2} = \frac{10.5+70}{2} = 35^0$

Deviation of light by a small angle prism

The small refracting angle of this prism causes the angle $\mathbf{i_1, r_1, r_2}$ and $\mathbf{i_2}$ to be small such that $\mathbf{\sin i_1 \approx i_1, \sin r_1 \approx r_1}$ $\mathbf{\sin r_2 \approx r_2}$ and $\mathbf{\sin i_2 \approx i_2}$.



From the diagram, $D = d_1 + d_2$

but $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\Rightarrow \quad D = (i_1 - r_1) + (i_2 - r_2)$

On simplifying $D = i_1 + i_2 - (r_1 + r_2)$

but $r_1 + r_2 = A$

$\therefore \quad D = i_1 + i_2 - A$----------------------(a)

At M Snell's law becomes.

$n_a \sin i_1 = n \sin r_1$

For small angles this gives $i_1 = nr_1$ --------------------------------(b)

Similarly at N Snell's law becomes $i_2 = nr_2$ ------------------------------(c)

Substituting equation (b) and (c) in (a) gives

$D = nr_1 + nr_2 - A$

$D = n(r_1 + r_2) - A$

but $r_1 + r_2 = A$

$\Rightarrow \quad D = nA - A$

$\therefore \quad \mathbf{D = (n - 1)A}$

∴ **The deviation produced by a small angle prism is independent of the magnitude of the small angle of incidence on the prism. (ie**: All rays entering a small-angle prism at small angles of incidence suffer the same deviation**)**

Note:

The result $\mathbf{D = (n - 1)A}$ will later be used in developing lens theory.

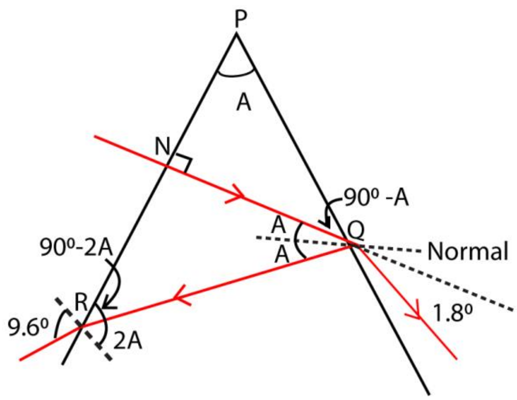
Examples 4

A ray of light that falls normally upon the first face of a glass prism of a small refracting angle under goes a partial refraction and reflection at the second face of the prism.

The refracted ray is deviated through an angle **1·8**° and the reflected ray makes an angle of **9·6**° with the incident ray after emerging from the prism through its first face. Calculate the refracting angle of the prism and its refractive index of the glass material.

Solution

Let **A** be the required refracting angle of the prism as shown



Consider the deviation suffered by the incident light

D = (n – 1) A where D = 1·8°

$\Rightarrow$ 1·8° = (n – 1) A ---------------------------------------(i)

From ΔPQN, $\angle PQN = 90° - A$

$\Rightarrow$ At Q, the angle of incidence = A

From ΔNQR, $\angle QRN = 90° - 2A$

$\Rightarrow$ At R, the angle of incidence = 2A

$\therefore$ At R, Snell's becomes $n_a \sin 9{\cdot}6° = n \sin 2A$

For small angles, **sin 9·6° ≈ 9·6° and sin A ≈ 2A**

$\Rightarrow$ 9·6° = 2nA ---(ii)

Equation (i) ÷ Equation (ii) gives

$$\frac{1{\cdot}8°}{9{\cdot}6°} = \frac{(n-1)A}{2nA}$$

$\Rightarrow$ 3·6°n = 9·6° (n – 1)

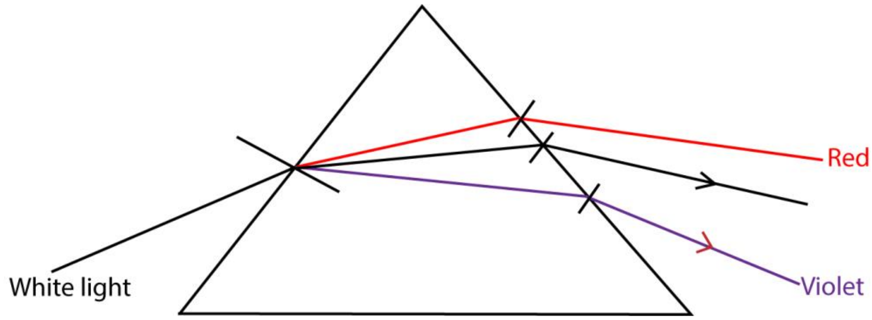
Thus **n = 1·6**

Equation (i) now becomes1·8° = (1·6– 1) A

$\therefore$ **A = 3°**

Dispersion of white light by a transparent medium

Dispersion of whit light is the separation of white light in to its component colors by a transparent medium due to their speed differences in the medium.



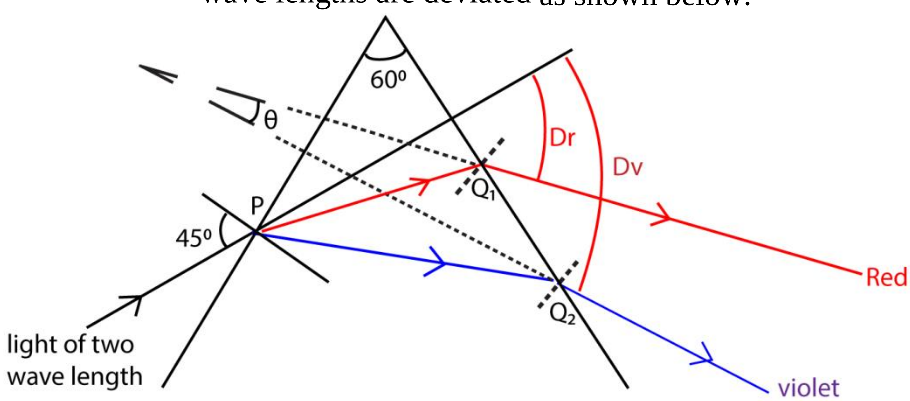
When white light falls on a transparent medium, its different component colors travel with different speeds through the medium. They are therefore deviated by different amounts on refraction at the surface of the medium and hence dispersion.

NOTE :

(i) White light is a mixture of various colors. This is called the spectrum of white light.

(ii) The spectrum of white light consists of red, orange, yellow, green, blue, indigo and violet light bands. On refraction, violet is the most refracted colour away from the normal (violet is the most deviated colour) while red is least deviated

(iii) When light of two wavelengths say red and blue light is incident at a small angle on a small angle prism of refracting angle A having refractive indices of n_r and n_b. for the two wave lengths respectively, then the two wave lengths are deviated as shown below.



The deviation of red and blue light is given by $d_r = (n_r - 1)A$

$d_b = (n_b - 1)A$.

The quantity $\theta = d_b - d_r$ is called the **Angular separation (Angular dispersion)** produced by the prism.

$\Rightarrow \theta = (n_b - 1) A - (n_r - 1)A$

on simplifying $\mathbf{\theta = (n_b - n_r) A}$

Examples 5

Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive index of 1·52 and 1·48 for the two wave lengths. Find the angular separation of the two wave lengths after refraction by the prism.

Solution

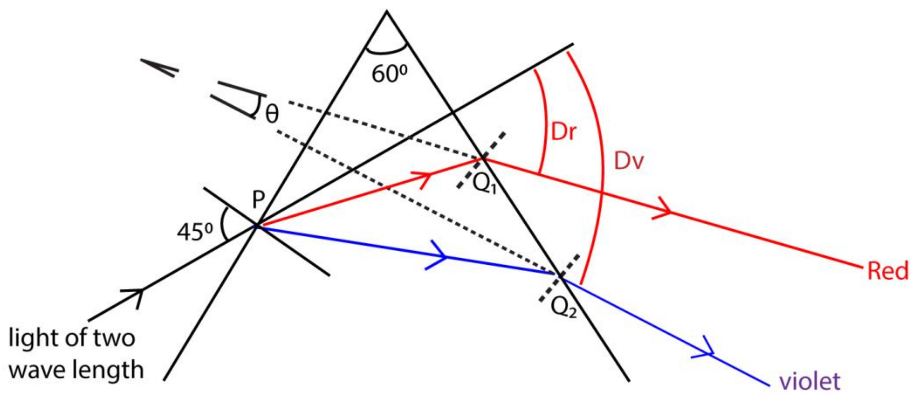
For a small prism, Angular separation $\theta = (n_1 - n_2) A$

$\theta = (1{\cdot}52 - 1{\cdot}48) \times 5°$

$\Rightarrow \theta = \mathbf{0{\cdot}2}\ °$

Examples 6

A glass prism with refracting angle 60° has a refractive index of 1·64 for red light and 1·66 for violet light. Calculate the angular separation of the red and violet rays which emerge from the prism when a ray of white light is incident on the prism at an angle of 45°



Case I:

Consider the deviation suffered by red light

At **P,** Snell's law becomes .

$n_a \sin 45° = 1{\cdot}64 \sin r_1$

$\therefore\ r_1 = 25{\cdot}54°$

But $r_1 + r_2 = 60°$

$\Rightarrow\ r_2 = 60° - r_1$

$= 60° - 25{\cdot}54°$

$\therefore\ r_2 = 34{\cdot}46°$

At $\mathbf{Q_1}$, Snell's law becomes

$n_a \sin i_2 = 1{\cdot}64 \sin 34{\cdot}46°$

$\therefore\ \mathbf{i_2 = 68{\cdot}13°}$.

Total Deviation $D_r = d_2 + d_1$ where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$\Rightarrow\ D_r = (45° - 25{\cdot}54°) + (68{\cdot}13° - 34{\cdot}46°)$

$\therefore\ \mathbf{D_r = 53{\cdot}13°}$.

Case II:

Consider the deviation suffered by violet light

At **P,** Snell's law becomes .

$$n_a \sin 45° = 1{\cdot}66 \sin r_1$$

$$\therefore \quad r_1 = 25{\cdot}21°$$

$$\text{But } r_1 + r_2 = 60°$$

$$\Rightarrow \quad r_2 = 60° - r_1$$

$$= 60° - 25{\cdot}21°$$

$$\therefore \quad r_2 = 34{\cdot}79°$$

At **Q**, Snell's law becomes

$$n_a \sin i_2 = 1{\cdot}66 \sin 34{\cdot}79°$$

$$\therefore \quad \mathbf{i_2 = 71{\cdot}28°.}$$

Total Deviation $D_v = d_2 + d_1$ where $d_1 = i_1 - r_1$ and $d_2 = i_2 - r_2$

$$\Rightarrow \quad D_v = (45° - 25{\cdot}21°) + (71{\cdot}28° - 34{\cdot}79°)$$

$$\therefore \quad \mathbf{D_v = 56{\cdot}28°.}$$

Thus required angular separation $\theta = D_v - D_r$

$$\theta = 56{\cdot}28° - 53{\cdot}13°$$

$$\Rightarrow \quad \theta = 3{\cdot}15°$$

Appearance of white light placed in water

Observation

A coloured spectrum is seen inside the water surface with violet on top and red down.

Explanation

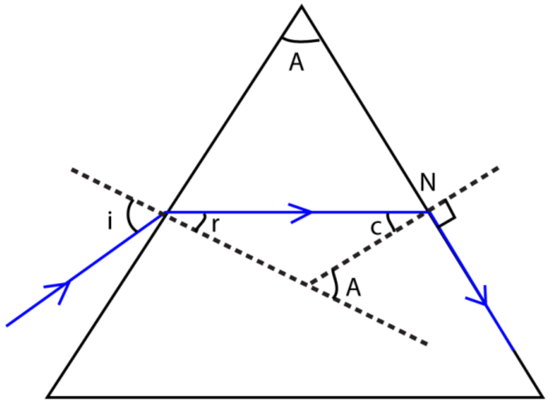
The different component colours of white light travel with different speeds through water.

They are deviated by different amounts on refraction at the water surface. Hence different coloured images are formed at different points inside the water surface with a violet coloured image on top.

Grazing property of light rays as applied to prisms.

If a ray of light is either such that the incident angle or the emergent angle is equal to 90° to the normal of the prism, then the ray is said to graze the refracting surface of the prism.

Consider a ray of light incident at an angle i on a glass prism of refracting angle A situated in air with the emergent light grazing the other refracting surface of the prism as shown.



From the diagram, r + c = A

∴ r = A – c---------------------------------- (a)

(a) At M Snell's law becomes

$n_a \sin i = n_g \sin r$-----------------------------(b)

(b) Substituting equation (a) in (b) gives

$\sin i = n_g \sin(A - c)$

⇒ sin i = ng (sin A cos C – sin C cos A)------------------- (c)

(c) At N, Snell's law becomes

$n_g \sin c = n_a \sin 90°$.

$\therefore \sin c = \frac{1}{n_g}$

$$\cos c = \sqrt{(1 - sin^2\ c)} = \sqrt{(1 - \frac{1}{n_g^2})} = \frac{\sqrt{(n_g^2 - 1)}}{n_g}$$

Substitution sin c and cos c in equation

$$\text{Sini} = n_g\left(\sin A\ x\ \frac{\sqrt{(n_g^2-1)}}{n_g} - \frac{1}{n_g}\cos A\right)$$

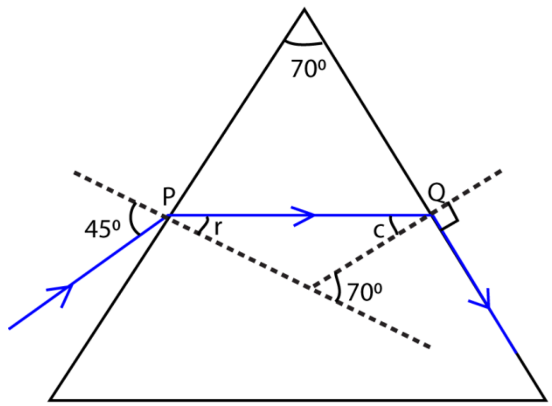
Simplifying

$$\sqrt{(n_g^2 - 1)} = \frac{\sin i + \cos A}{\sin A}$$

Knowing the angles i and A, the refractive index n_g of a material of a prism can be determined.

Examples 7

Monochromatic light is incident at an angle of **45°** on a glass prism of refracting angle **70°** in air. The emergent light grazes the other refracting surface of the prism. Find the refractive index of the glass material.



At P, Snell's law becomes

$n_a \sin 45° = n_g \sin r$ --------------------------------(a)

From the diagram, $r + c = 70°$

$\Rightarrow r = 70° - c$ -------------------------------(b)

Substituting equation (b) in (a) gives

$\sin 45° = n_g \sin(70° - c)$ -----------------------------------(c)

At Q, Snell's law becomes

$n_g \sin c = n_a \sin 90°$

$\Rightarrow n_g = \frac{1}{\sin c}$ − − − − − − − − − − − − − −(d)

Substituting equation (d) in (c) gives

$$\sin 45^0 = \frac{(70^0 - c)}{\sin c}$$

$$\sin 45° \sin c = \sin 70° \cos c - \sin c \cos 70°$$

$$\Rightarrow \quad (\sin 45° + \cos 70°)\sin c = \sin 70° \cos c$$

Dividing cos c throughout gives

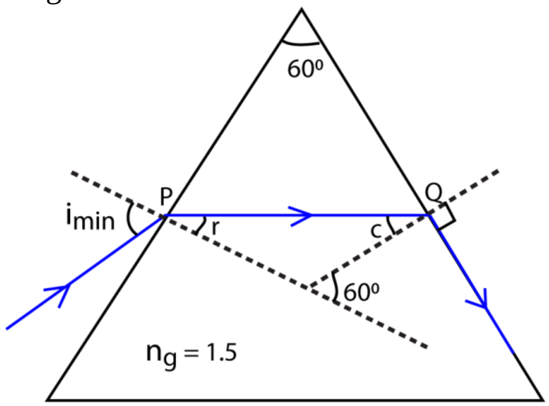
$$\tan c = \frac{\sin 70^0}{\sin 45^0 + \cos 70^0}$$

$$\Rightarrow c = 41 \cdot 9^{\circ}$$

Equation (d) now becomes $n_g = \frac{1}{\sin 41.9^0} = 1.5$

Examples 8

A ray of light is incident on one refracting face of a prism of refractive index 1·5 and refracting angle 60°. Calculate the minimum angle of incidence for the ray to emerge through the second refracting face.



Analysis

for minimum angle of incidence, the emergent ray grazes the second refracting face.

At **Q**, Snell's law becomes

$1{\cdot}5\sin c = n_a \sin 90°$

$\Rightarrow$ c = 41·8°

But r + c = 60°

$\Rightarrow$ r = 60° – c

= 60° – 41·8°

$\therefore$ r = 18·2°

At **P**, Snell's law becomes

$1{\cdot}5 \sin 18{\cdot}2° = n_a \sin i_{min}$

$\therefore i_{min} = 28°$

NOTE

For grazing condition the following condition may be used $n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$

$$1.5 = \sqrt{1 + \left(\frac{\sin i_{min} + \cos 60^0}{\sin 60^0}\right)^2}$$

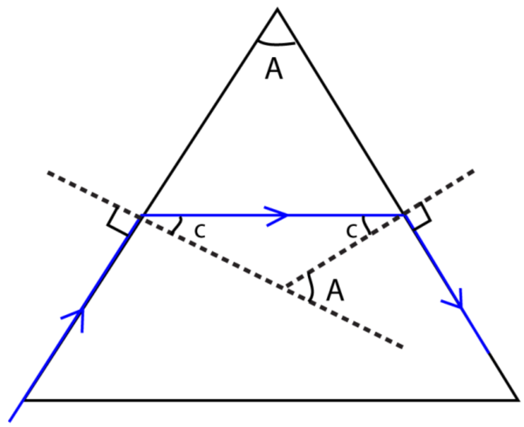
$$= 28^0$$

+ º

Limiting angle of the prism

This is the maximum refracting angle of the prism for which the emergent ray grazes the second refracting surface.

Suppose the incident ray grazes the first refracting surface then the limiting angle A is given by A = 2c where **c** is the critical angle of the glass air interface as shown.



Exercise:

1. (i) Obtain an expression relating the deviation of a ray of light by the prism to the refracting angle and the angles of incidence and emergence.

(ii) The deviation of a ray of light incident on the first face of a **60**° glass prism at an angle of **45**° is **40**°**.** Calculate the angle of emergence of a ray on the second face of the prism.

[Ans $\mathbf{i_2}$ = 65°]

(iii) A prism of refractive index **1·64** is immersed in a liquid of refractive index **1·4**. A ray of light is incident on one face of the prism at an angle of **40**°**.** If the ray emerges at an angle of **29**°**,** determine the angle of the prism.

[**Answer: 57·7**°]

2. (i) For a ray of light passing through the prism, what is the condition for minimum deviation to occur?

(ii) Derive an expression for the refractive index of a prism in terms of the refracting angle, **A**, and the angle of minimum deviation **D**.

(iii) A glass prism of refractive index n and refracting angle ,**A**, is completely immersed in a liquid of refractive index $\mathbf{n_i}$. If a ray of light that passes symmetrically through the prism is deviated through an angle θ**,** Show that

$$\frac{n_1}{n} = \frac{\sin A/2}{\sin(\frac{\theta + A}{2})}$$

3.(a) A glass prism with refracting angle **60°** is made of glass whose refractive indices for red and violet light are respectively **1·514** and **1·530.** A ray of white light is set incident on the prism to give a minimum deviation for red light.

Determine the:

(i) angle of incidence of the light on the prism.
(ii) angle of emergence of the violet light.
(iii) angular width of the spectrum.

(b) A certain prism is found to produce a minimum deviation of **51°.** While it produces a deviation of **62·8°** for a ray of light incident on its first face at an angle of **40·1°** and emerges through its second face at an angle of **82·7°.**

Determine the:

(i) refracting angle of the prism.
(ii) angle of incidence at minimum deviation.
(iii) refractive index of the material of the prism.

[**Ans (i) 60°** **(ii) 55·5°** **(iii) 1·648**]

4. (i) A ray of monochromatic light is incident at a small angle of incidence on a small angle prism in air. Obtain the expression **D = (n – 1)A** for the deviation of light by the prism.

(ii) A glass prism of small angle, **A**, and refractive index $\mathbf{n_g}$ and is completely immersed in a liquid of refractive index $\mathbf{n_L}$. Show that a ray of light passing through the prism at a small angle of incidence suffers a deviation given by

$$D = \left(\frac{n_g}{n_L} - 1\right) A$$

5. Explain why white light is dispersed by a transparent medium.

6. Light of two wave length is incident at a small angle on a thin prism of refracting angle 5° and refractive index of 1·52 and 1·48 for the two wave lengths. find the angular separation θ of the two wave lengths after refraction by the prism.

[Ans θ = 0·2°]

7.A point source of white light is placed at the bottom of a water tank in a dark room. The light from the source is observed obliquely at the water surface. Explain what is observed.

8. Monochromatic light is incident at an angle ϕ on a glass prism of refracting angle, A, situated in air. If the emergent light grazes the other refracting surface of the prism, Show that the refractive index, n_g, of the prism material is given by

$$n_g = \sqrt{1 + \left(\frac{\sin i + \cos A}{\sin A}\right)^2}$$

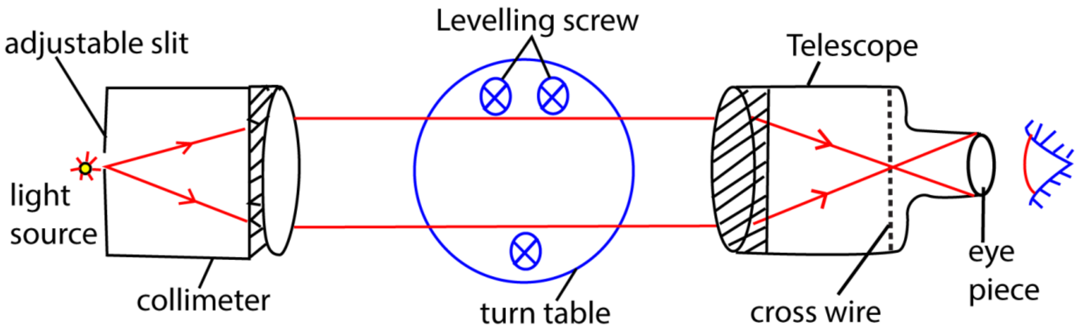
9. A ray of light is incident at angle of 30° on a prism of refractive index 1·5 .calculate the limiting angle of the prism such that the ray does not emerge when it meets the second face.

[Ans A = 61·3°]

A spectrometer

It is an instrument used to measure accurate determination of deviation of a parallel beam of light which has passed through a prism. This provides a mean of studying optical spectra and measurement of refractive indices of glass in form of a prism.

It consists of a collimator, a telescope, and a turn table on which the prism is placed as shown.



Before the spectrometer is put in to use, 3 adjustments must be made onto it and these include,

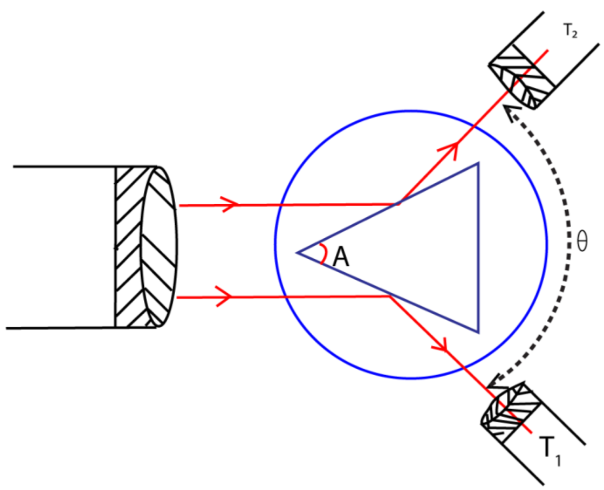
(i) The collimator is adjusted to produce parallel rays of light.
(ii) The turntable is leveled.
(iii) The telescope is adjusted to receive light from the collimator on its cross wire.

Application of spectrometer

(a) Measure of refractive angle of a prism
(b) Measurement of the angle of minimum deviation
(c) Measurement of refractive index of prism
(d) Measurement of spectra of light.

Measurement of the refracting angle "A" of the prism

The collimator is adjusted to produce parallel rays of light.
The turn table is leveled.
The telescope is adjusted to receive light from the collimator on its cross wire.
The prism is placed on the turn table with its refracting angle facing the collimator as shown.

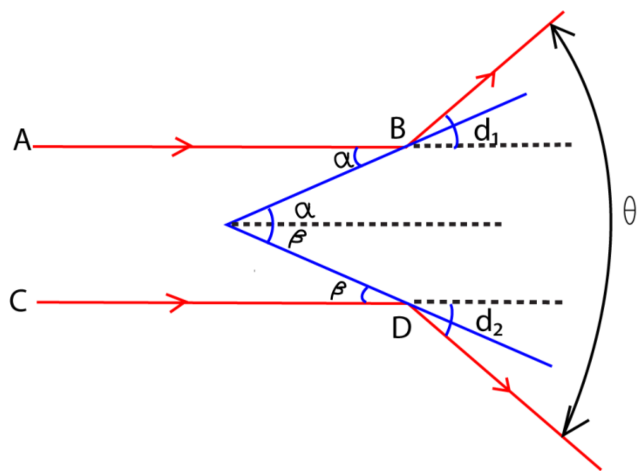


With the table fixed, the telescope is moved to position T_1 to receive the light from the collimator on its cross wire. This position T_1 is noted and the telescope is turned in to a new position T_2 to receive light on its cross wire. The angle θ between T_1 and T_2 is measured.

The prism angle A is given by $\mathbf{A} = \frac{\theta}{2}$

Proof of the relation

Consider a parallel beam of light incident on to a prism of refracting angle A making glancing angles α and β as shown.



From the geometry, α + β = A-----------------------------.(i).

Deviation d_1 of ray AB = 2α

Deviation d_2 of ray CD =

2β. Total deviation θ = d_1

+d_2

= 2α + 2β

= 2 (α + β)--------------------------

(ii) Combining equation (i) and (ii) gives

θ = **2A.**

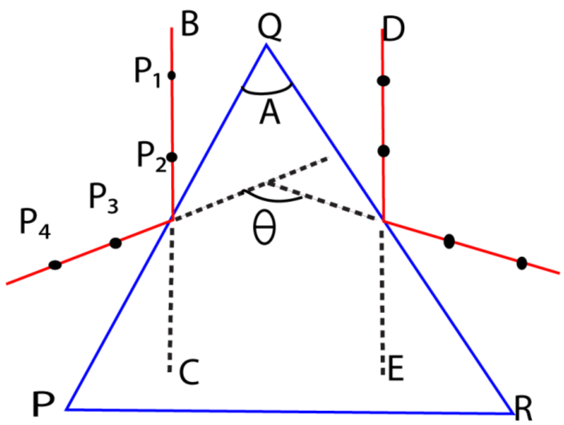
NOTE

It is now clear from the geometry that the angle θ turned through in moving the telescope from T_1 to T_2 is given by θ = 2A

Thus $A = \frac{\theta}{2}$

Method 2: Using optical pins

A white paper is stuck to the soft board using top-headed pins. Two parallel lines AB and DC are drawn on the paper and the prism is placed with its apex as shown.

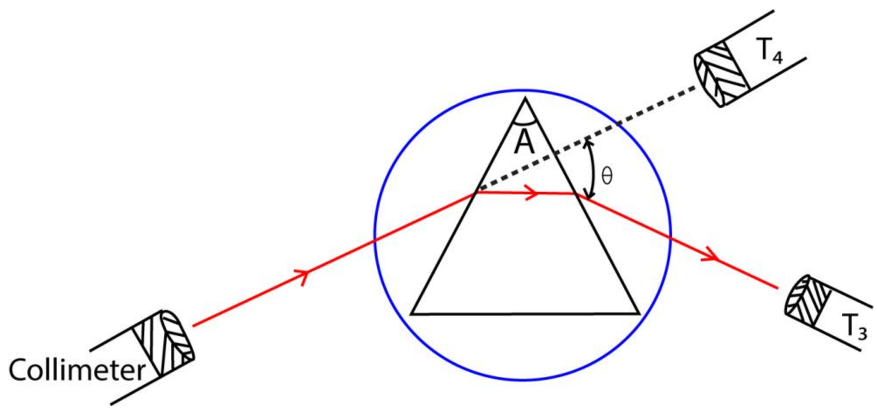


Two optical pins P_1 and P_2 are placed along AB and pins P_3 and P_4 are placed such that they appear to be in line with the images of P_1 and P_2 as seen by reflection from face
PQ .The procedure is repeated for face QR. The prism is removed and angle θ is measured.

The required refracting angle $A = \frac{\theta}{2}$

Measurement of minimum deviation "Dmin" of the prism

The collimator is adjusted to produce parallel rays of light.
The turn table is leveled.
The telescope is adjusted to receive light from the collimator on its cross wire The prism is placed on a turn table with its refracting angle facing away from the collimator as shown.



The telescope is turned in the direction of the base of the prism until light can be seen. With light kept in view, both the telescope and the table are turned until light movesin the opposite direction. Position T_3 of the telescope is noted.

The table is then fixed and the prism is removed so that the telescope is turned to a new position T_4 to receive the un deviated light. The angle between T_3 and T_4 is determined and this is the angle of minimum deviation D_{min}.

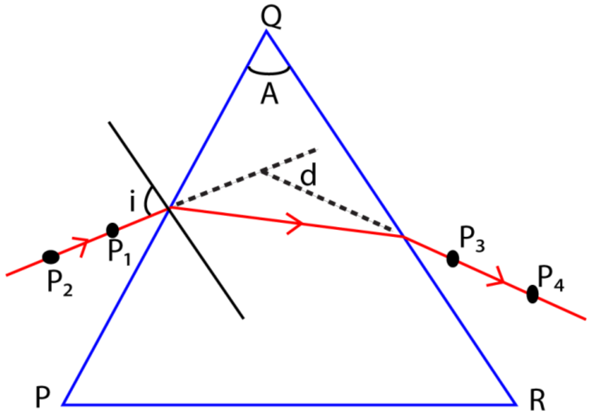
NOTE

(i) Position T_3 is noted because, in the position of minimum deviation light viewed through the telescope moves in the opposite direction.

(ii)The refractive index of a glass prism of known refracting angle A can be determined using a spectrometer from the relation

$$n = \frac{\sin\frac{(D_{min}+A)}{2}}{\sin\frac{A}{2}}$$

Method 2: Using optical pins



Two optical pins P_1 and P_2 are placed along the lines that make different angles of incidence **i.** Pins P_3 and P_4 are placed such that they appear to be in line with the images of P_1 and P_2 as seen through the prism. The angles of deviation **d** are measured for different angles of incidence. A graph of **d** against **i** is plotted to give a curve whose angle of deviation at its turning point is the angle of minimum deviation D_{min} of the prism.

Uses of a glass prism

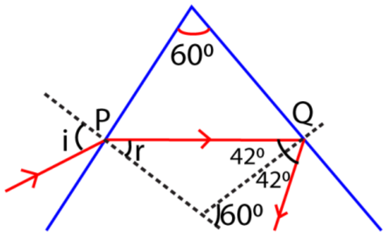
1. They enable the refractive index of a glass material to be measured accurately.
2. They are used in the dispersion of light emitted by glowing objects.
3. They are used as reflecting surfaces with minimal energy loss.
4. They are used in prism binoculars.

Examples 9

A ray of monochromatic light is incident on one face of a glass prism of refracting angle 60° and is totally internally reflected at the next face.

(i) Draw a diagram to show the path of light through the prism.

(ii) Calculate the angle of incidence at the first face of the prism if its refractive index is 1·53 and the angle of incidence at the second face is 42°.



From the diagram, r + 42° = 60°

$\therefore$ r = 18°

At P, Snell's becomes

$n_a \sin i = 1{\cdot}53 \sin 18°$

$\therefore \quad i = 28{\cdot}2°$

NOTE

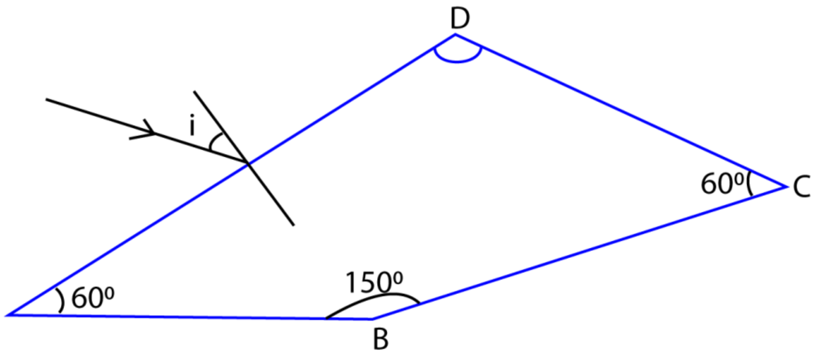
For n_g =1·53, then the critical angle c for the above glass material is given by the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1.53} = 40.8°$$

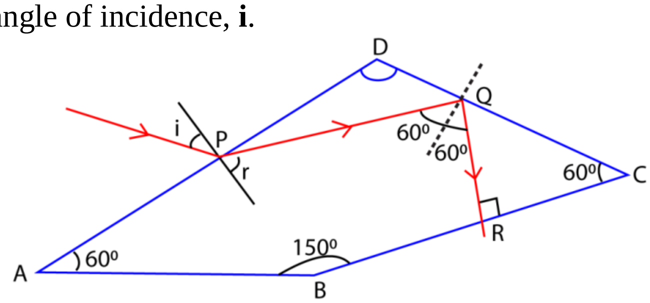
Thus total internal reflection occurs at **Q** since the angle of incidence is greater than the critical angle **c**

Examples 10

7. A ray of light is incident on the face AD of a glass block of refractive index 1·52 as shown.



If the ray emerges normally through face BC after total internal reflection, Calculate the angle of incidence, **i**.



Breakdown

(i) Its after a total internal reflection at Q that the ray emerges through face BC

(ii) At R, there is no refraction. Therefore, Snell's law does not hold at this point.

From ΔRCQ, $\angle RQC + 60° + 90° = 180°$

$\therefore \angle RQC = 30°$

$\Rightarrow$ At Q, the angle of reflection = 60°

Hence at Q, the angle of incidence = 60°

Solving ΔQDP gives $\angle QPD = 60°$

Hence at P, the angle of refraction **r** = 30°

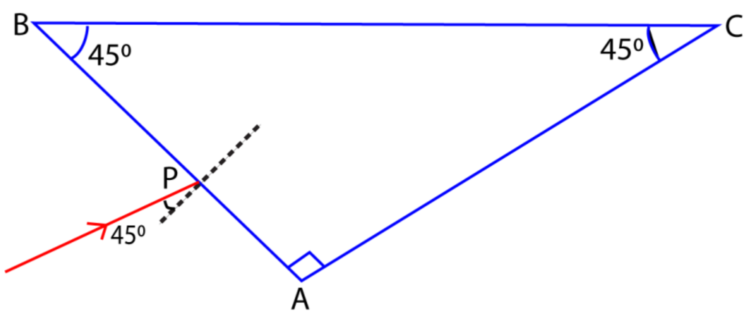
$\Rightarrow$ At P, Snell's law becomes

$$n_a \sin i = 1{\cdot}52 \sin 30°$$

$$\therefore \quad i = 49{\cdot}5°$$

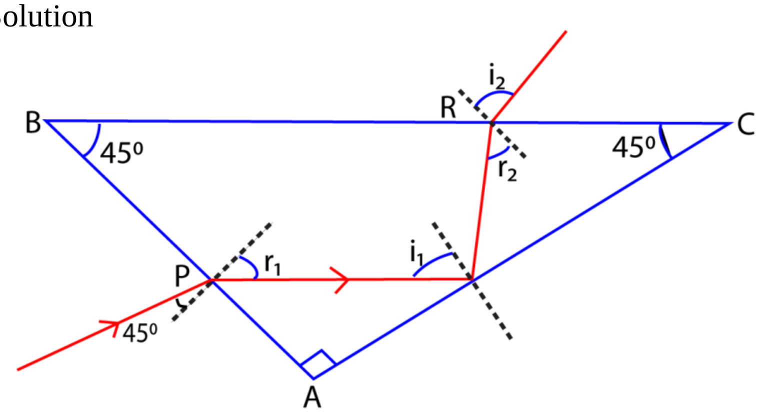
Examples 11

8. A ray of light is incident at 45° on a glass prism of refractive index 1·5 as shown.



Calculate the angle of emergence and sketch the ray diagram.

Solution



At P, Snell's becomes $n_a \sin 45° = 1{\cdot}5 \sin r$

∴ $r_1 = 28{\cdot}1°$

At P, $\angle APQ + r_1 = 90°$ where $r_1 = 28{\cdot}1°$

⇒ $\angle$**APQ = 61·9°**

From ΔAPQ, $\angle PQA + 61{\cdot}9° + 90° = 180°$

∴ $\angle PQA = 28{\cdot}1°$

⇒ At Q, the angle of incidence **i = 61·9°**

Testing for total internal reflection a t Q using the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1.5} = 41.8°$$

Thus light is totally reflected at **Q** since **i** > **c**.

⇒ $\angle$**PQA = $\angle$RQC = 28·1°**

From ΔRQC, $28{\cdot}1° + 45° + 90° + r_2 = 180°$

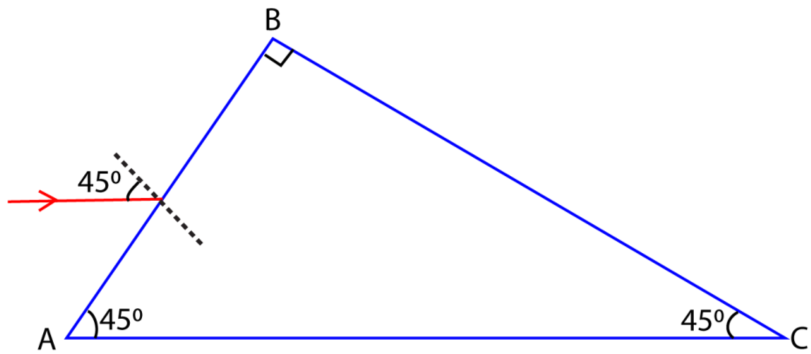
$r_2 = 16{\cdot}9°$

At R, Snell's becomes $1{\cdot}5 \sin 16{\cdot}9° = n_a \sin i_2$

Thus i_2 = 25·15°

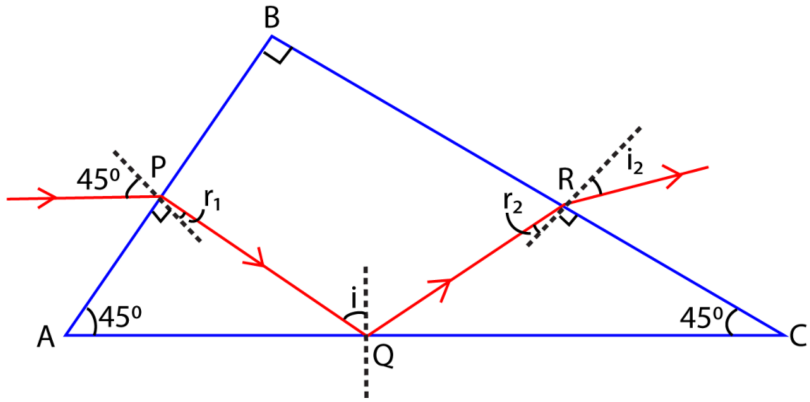
Examples 12

A ray of light is incident at 45° on a glass prism of refractive index 1·5 as shown.



Calculate the angle of emergence and sketch the ray diagram.

Solution



At P, Snell's becomes $n_a \sin 45° = 1{\cdot}5 \sin r$

$\therefore \quad r_1 = 28{\cdot}1°$

From ΔAPQ, $\angle PQA + 45° + 90° + r_1 = 180°$ where $r_1 = 28{\cdot}1°$

$\therefore \quad \angle PQA = 16{\cdot}9°$

$\Rightarrow$ At Q, the angle of incidence **i = 73·1**°

Testing for toat internal reflection at Q using the relation

$$\sin c = \frac{1}{n_g} = \frac{1}{1.5} = 41.8°$$

Thus light is totally reflected at **Q** since **i** > **c**.

$\Rightarrow$ $\angle$**PQA = $\angle$RQC = 16·9**°

From ΔRQC, $16{\cdot}9° + 45° + 90° + r_2 = 180°$

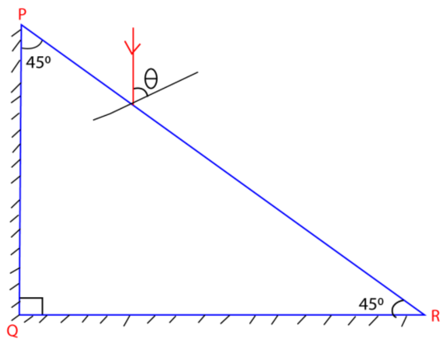
$r_2 = 28{\cdot}1°$

At R, Snell's becomes $1{\cdot}5 \sin 28{\cdot}1° = n_a \sin i_2$

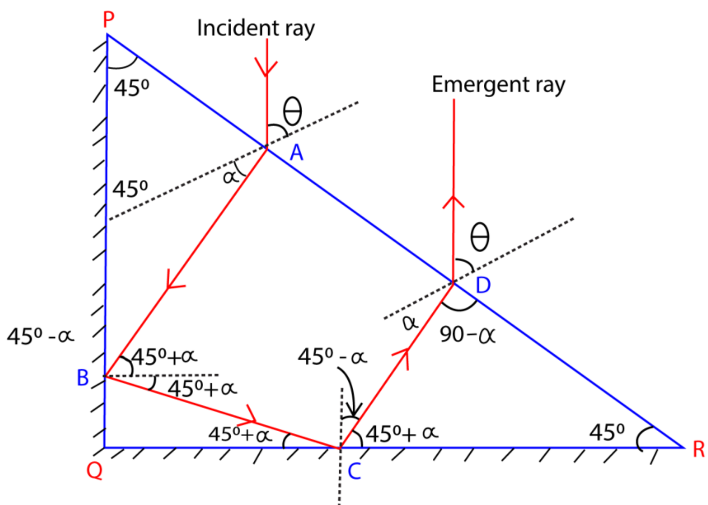
Thus i_2 = 45°

Examples 13

The diagram in the figure below shows a cross section of an isosceles right angled prism sides PQ and QR are coated with a reflecting substance. A ray of light is incident on PR at an angle θ as shown



(i) Draw a diagram to show the path of light through the prism.
(ii) Show that the ray leaving the prism is parallel to the incident ray.

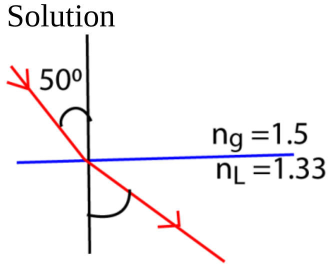


From geometry of the figure above the angle of emergence at D is the same as the incident angle θ at A. Hence the emergent ray is parallel to the incident ray.

Example 14

(a) Define the term refraction and refractive index.
(b) Derive an expression for apparent displacement of an object when viewed through a parallel- sided block.
(c) A ray of light is incident on a glass-water interface at an angle of 500. If the refractive indices of water and glass are1.33 and 1.5 respectively, determine the angle made by the refracted with the normal at point of incidence.

Solution



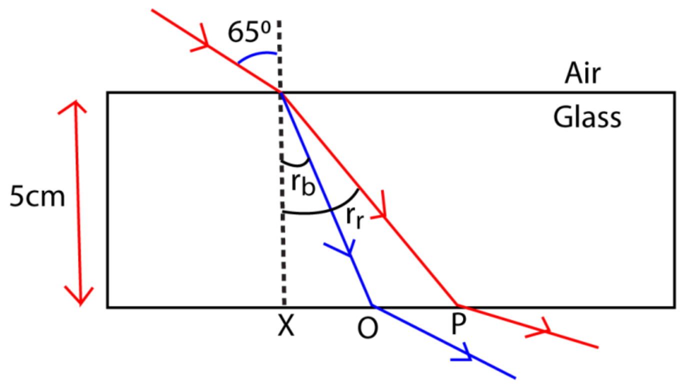
nsini = constant

1.5sin 50^0 = 1.33 sin r

Sin r = $\frac{1.5\sin 50^0}{1.33}$

r = 60^0

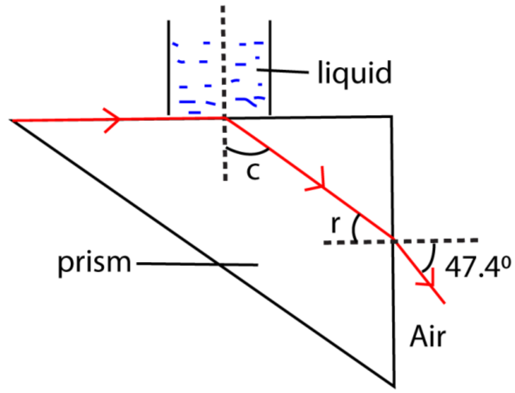
(d) A light ray consisting of blue and red light is incident from air to a glass block. The two colors emerge from the block into air at point O and P respectively as shown below.



The speeds of blue and red light respectively in glass are 1.88 x 10^8ms^{-1} and 1.84x10^8ms^{-1}. Find the distance OP. (Answer 0.17cm)

Example 15

The following diagram shows the direction followed by a ray of monochromatic light through a right angled triangular prism of refractive index 1.52. the light emerges in air. Calculate the refractive index of liquid.



Solution

Note: A ray on the boundary is grazing ray.

Let the refractive index of the liquid and air be and c is the refractive index of glass.

From

n_g sini = na sin 47.4^0

1.52 sin r = 1sin 47.4^0

r = 29^0

r + c = 90^0

c = 61^0

$n_L$sin 90^0 = ng sin c

$n_L = 1.52 \sin 61^0 = 1.33$

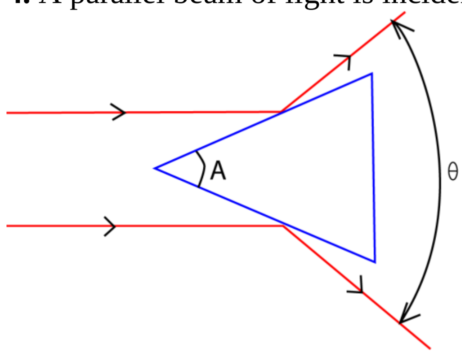
EXERCISE

1. Draw a labeled diagram of a spectrometer and State the necessary adjustments that must be made on to it before put in to use.

2. Describe how the refracting angle of the prism can be measured using a spectrometer.

3. You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the refracting angle **A** of the prism

4. A parallel beam of light is incident on to a prism of refracting angle, **A**, as shown



Show that θ **= 2A**

5. Describe how the minimum deviation, **D**, of a ray of light passing through a glass prism can be measured using a spectrometer.

6. You are provided with pins, a white sheet of paper, a drawing board and a triangular prism. Describe how you would determine the angle of minimum deviation, **D,** of a ray of light passing through a glass prism.

7. Describe how the refractive index of a material of a glass prism of known refracting angle can be determined using a spectrometer.

8. Describe briefly two uses of glass prisms

9. A ray of light is incident on prism surrounded by a liquid of refractive index 4/3 at an angle 30^0. If the refractive index of the prism is 1.5, find the angle at which the ray emerges from the opposite face of the prism if its refractive angle is 60^0. [Answer 38.5^0].

10. A ray of light is incident on a prism in air along its face. If the ray from opposite face emerge at an angle 30^0 and the refractive index of the prism is 1.52, find the refractive angle of the prism. [Answer 60.3^0]

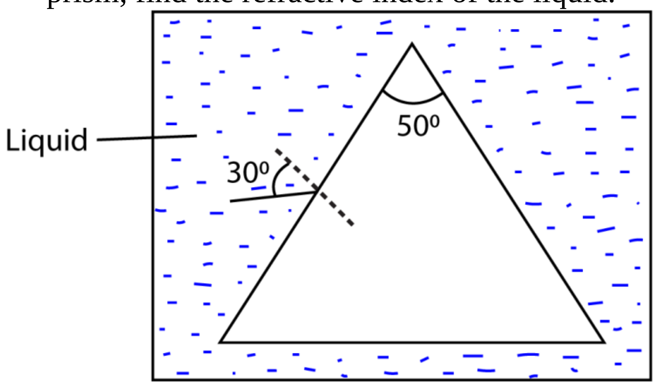
11. (a) Define limiting angle of a prism.
This is the maximum angle of the prism for which light incident on the prism at 90^0 emerges from the opposite face.

(b) Light incident to a prism at such an angle that it emerges from the opposite face of the prism grazing the surface. If the angle of the prism is 60^0 and the refractive index of the prism is 1.52, calculate the angle of incidence of light. [Answer = 29.5^0]

12. Monochromatic light is incident on the refracting face of a prism of refracting angle 60^0 made of glass of refractive index 1.5. Calculate the minimum angle of incidence for the ray to emerge through the second refractive face. [Answer 27.9^0)

13. Monochromatic light is incident at an angle of 45^0 on a glass prism of refractive angle 70^0 in air. The emergence light grazes the other surface of the prism. Find the refractive index of glass. [Answer 1.5].

14. A ray of light propagating in a liquid is incident on the prism of refracting angle 50^0 and refracting index 1.6 as shown below. If the ray passes symmetrically through the prism, find the refractive index of the liquid.

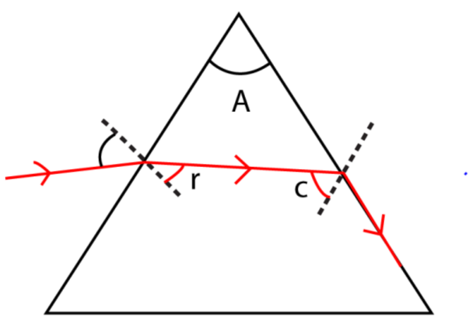


15. Monochromatic light is incident at an angle of 28^0 on a glass prism of refractive index 1.5. The emergent light grazes the surface of the prism as shown below

(a) Calculate the angle of refraction, r.

(b) Find the critical angle, c for the glass-air interface.

(c) Find the refracting angle, A, of the prism. [Answer: r= 24.2^0, C = 41.8^0, A = 66^0].



16. A ray of light is incident on a prism of refractive index 1.3 and refractive angle 72^0. The ray emerges from the prism at angle 43^0. Find

(a) the angle of incidence,

(b) the deviation of they ray [Answer 57.4^0, d = 28.4^0]

17. A glass prism of refractive index 1.5 and refracting angle 60^0 is completely immersed in a liquid of refractive index 1.3. If a ray of light passes symmetrically through the prism, calculate the

(i) angle of incidence

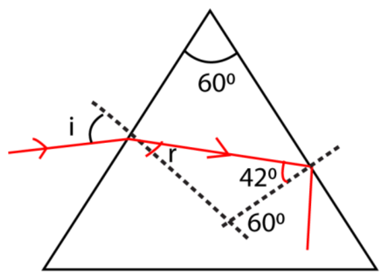
(ii) angle of deviation [Answer: i = 35.2^0, d= 10.4^0]

18. A ray of monochromatic light enters one face of a 60^0 glass prism and is totally internally reflected at the other face

(i) draw a ray diagram to show the path of light through the prism.

(ii) Calculate the angle of incidence at the first face if the refractive index of glass prism is 1.53 and the angle of incidence at the second face is 42^0.

(i)



(ii) Answer 28.2^0

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