Refraction at plane surfaces

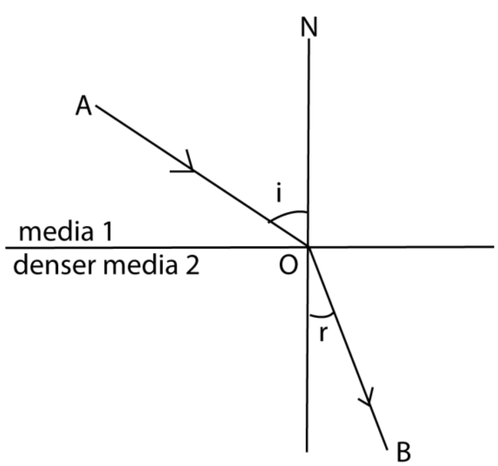
Refraction is the bending of light at an interface between two media of different optical densities or it is the change in speed of light when travelling from one medium to another of different optical density. Or it is the change in direction of light from one media to another of different optical densities.

NOTE ;

Light is refracted because it has different speeds in different media.

LAWS OF REFRACTION

Consider a ray of light incident on an interface between two media as shown.



Generally if light is incident from a less dense media to a denser media, it is refracted towards the normal at the point of incidence. From a denser media to a less dense media light rays bends away from the normal.

Laws of refraction

Law 1: the incidence ray, the normal and refracted ray at the point of incidence all lie in the same plane.

Law 2: the ratio of the sine of angle of incidence to the sine of angle of refraction is constant for a given pair of media (Snell's law)

That is $\frac{\sin i}{\sin r}$ = constant

The constant in Snell's law is known as **refractive index** of the two media

Refractive index, n

is the ratio of the sine of angle of incident to the sine of angle of refraction for a ray of light traveling from air in to a given medium.

OR

This is the ratio of the speed of light in a vacuum to speed of light in a medium.

Thus refractive index, n = $\frac{speed\ of\ light\ in\ a\ vacuum,\ c}{speed\ of\ light\ in\ a\ medium\ c_m}$

Where speed of light in a vacuum **$c = 3{\cdot}0 \times 10^8$ ms^{-1}.**

NOTE:

The refractive index, **n** for a vacuum is **1**. However if light travels from air to another medium, the value of **n** is slightly greater than **1**.For example, **n = 1·33 for** water and **n =1·5** for glass.

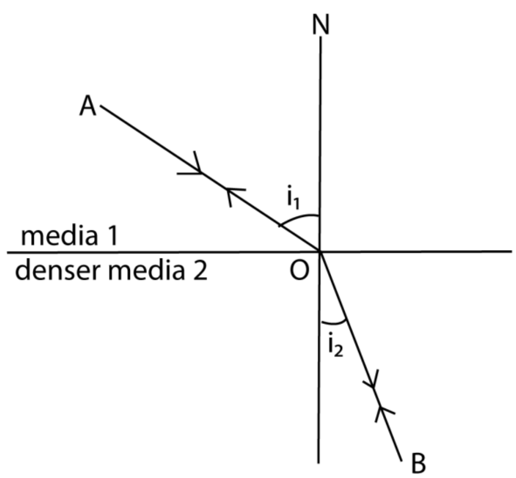
The principle of reversibility of light

It states that the paths of light rays are reversible. This means that a ray of light can travel from medium **1** to **2** and from **2** to **1** along the same path.

Relations between refractive indices

CASE I:

Consider a ray of light traveling from **medium 1 (air)** to **medium 2 (glass)** as shown.



Suppose i_1 and i_2 are the angles of incidence and refraction respectively in **medium 1** and **medium 2,** then

For light traveling from **(1)** to **(2)**, refractive index $_1n_2 = \frac{\sin i_1}{\sin i_2}$

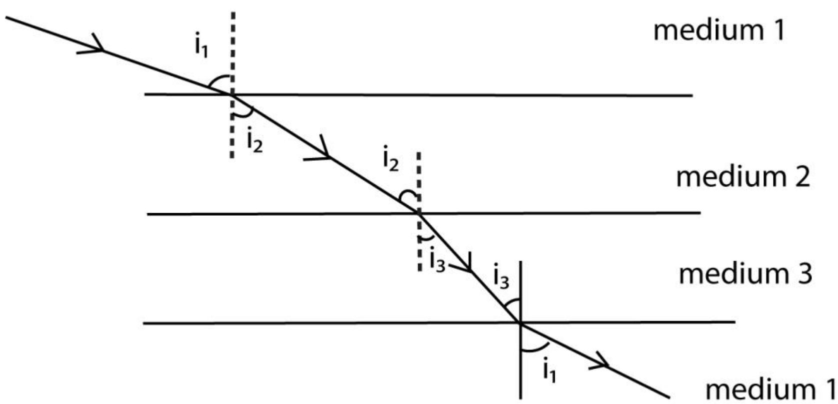
For light traveling from **(2)** to **(1)**, refractive index $_2n_1 = \frac{\sin i_2}{\sin i_1}$

It implies that $\frac{1}{_2n_1} = \frac{\sin i_1}{\sin i_2} = {_1n_2}$

Or $_1n_2 \times {_2n_1} = 1$

CASE II:

Consider a ray of light moving from **medium 1 (air)** through a series of media **2, 3** and then finally emerge into **medium 1 (air)** as shown.



At **medium 2-medium 3** interface, Snell's law gives.

$$_2n_3 = \frac{\sin i_2}{\sin i_3} = \frac{\sin i_2}{\sin i_1} \times \frac{\sin i_1}{\sin i_3} = {_2n_1} \times {_1n_3}$$

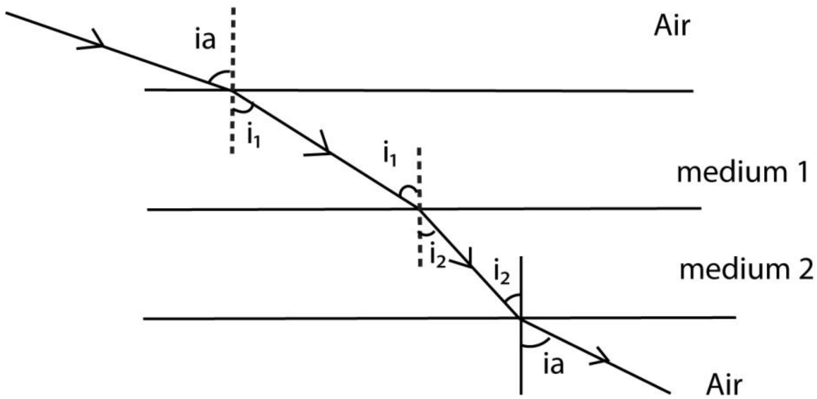
$\therefore \quad _2n_3 = {_2n_1} \times {_1n_3}$

Using the relation $_2n_1 = \frac{1}{1n2}$

$\Rightarrow \quad 1n3 = 1n2 \times 2n3 \quad$ OR $\quad 2n3 = \frac{1n3}{1n2}$

General relation between n and sin i

Consider a ray of light moving from air through a series of media **1, 2** and then finally emerge into air as shown.



At **air – medium 1** interface, Snell's gives $\frac{\sin ia}{\sin i_1}$ = n_1

⇒ **sin i_a** = $n_1 \sin i_1$ -- (i)

At **air - medium 2** interface, Snell's gives $\frac{\sin ia}{\sin i_2}$ = n_2

⇒ **sin i_a** = $n_2 \sin i_2$ -------------------------------------(ii)

Equating equation (i) and (ii) gives

$n_1 \sin i_1 = n_2 \sin i_2$.

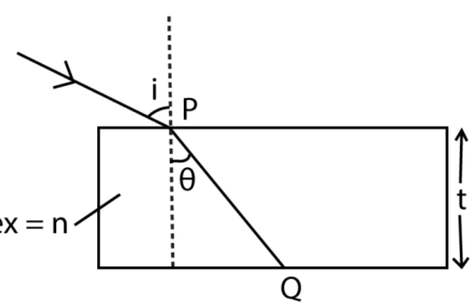
∴ **n sin i = a constant.**

Examples 1

Monochromatic light incident on a block of material placed in a vacuum is refracted through an angle θ. If the block has a refractive index **n** and is of thickness **t**, show that this light takes a time T = $\frac{nt\sec\theta}{c}$ to emerge from the block.

where **c** is the speed of light in a vacuum.

Solution:



Let T be the time taken by light to travel from point P to Q in the medium.

Thus, time T = $\frac{distance\ PQ}{speed\ of\ light\ in\ block,\ c_m}$

Where distance PQ $= \frac{t}{\cos\theta} = t\sec\theta$

$\Rightarrow$ T $= \frac{t\sec\theta}{c_m}$ -------------------------------------(i)

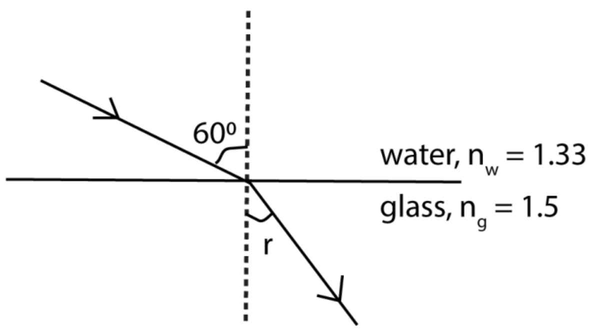
By definition, $n = \frac{c}{c_m}$

or cm = $\frac{c}{n}$ --(ii)

Substituting equation (ii) in to (i) gives, $T = \frac{nt\sec\theta}{c}$

Example 2

2. A monochromatic beam of light is incident at **60^0** on a water-glass interface of refractive index **1.33** and **1.5** respectively as shown



Calculate the angle of reflection **r.**

Solution:

Applying Snell's law gives

$n_w \sin 60° = n_g \sin r$

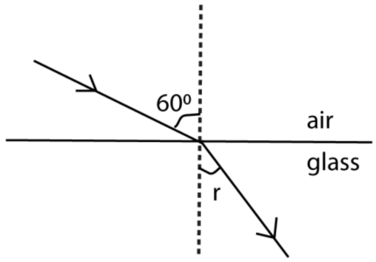
$1{\cdot}33 \sin 60° = 1{\cdot}5 \sin r$

Thus $\angle$ r = 50·2°.

Example 3

A ray of light propagating from air is incident on an air-glass interface at an angle of **60°.** If the refractive index of glass is **1·5**, calculate the resulting angle of refraction.

Solution



Applying Snell's law gives

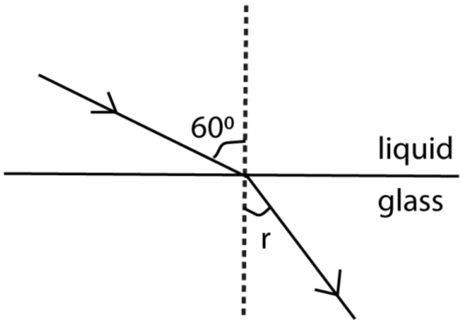
$n_a \sin 60° = n_g \sin r$ but $n_a = 1$, $n_g = 1{\cdot}5$

$\Rightarrow$ $1 \sin 60° = 1{\cdot}5 \sin r$

$\therefore$ $\angle$ **r = 35·6°.**

Example 4

A monochromatic ray of light is incident from a liquid on to the upper surface of a transparent glass block as shown.



Given that the speed of light in the liquid and glass is $\mathbf{2{\cdot}4 \times 10^8\ ms^{-1}}$ and $\mathbf{1{\cdot}92 \times 10^8\ ms^{-1}}$ respectively, find the angle of refraction, **r.**

Solution:

Applying Snell's law gives

$n_l \sin 60° = n_g \sin r$

$$\frac{c}{c_1} \sin 60° = \frac{c}{c_g} \sin r$$

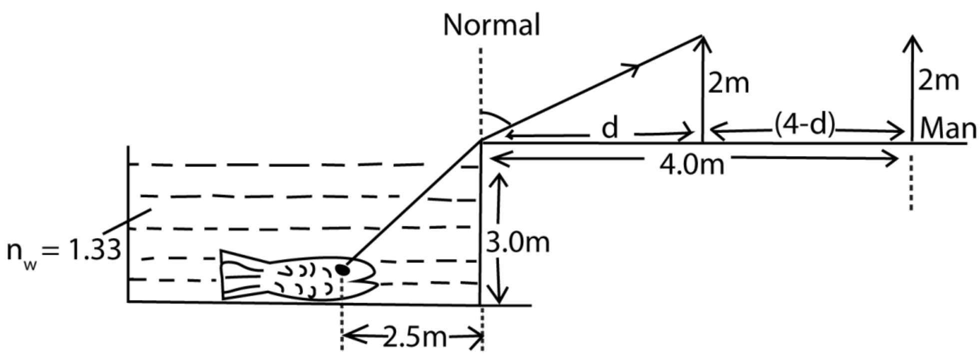
$\Rightarrow$ $\sin r = \frac{c_g}{c_1} \sin 60°$ but $c_g = 1{\cdot}92 \times 10^8\ ms^{-1}$, $c_l = 2{\cdot}4 \times 10^8\ ms^{-1}$

$$\therefore \sin r = \frac{1{\cdot}92 \times 10^8}{2{\cdot}4 \times 10^8} \sin 60°$$

$\Rightarrow$ $\angle$ **r = 43·9°.**

Example 5

A small fish is 3·0m below the surface of the pond and 2·5m from the bank. A man 2·0m tall stands 4·0m from the pond. Assuming that the sides of the pond are vertical, calculate the distance the man should move towards the edge of the pond before movement becomes visible to the fish. **(Refractive index of water = 1·33).**



From the diagram, $\tan i = \frac{2.5}{3}$ $\Rightarrow$ $\angle i = 39.81°$

Applying Snell's law at the edge of the pond gives

$n_w \sin i = n_a \sin r$

$1.33 \sin 39.81° = 1 \sin r$

$\Rightarrow \angle r = 58.4°$

Thus $\angle\theta = 90° - 58.4° = 31.6°$

From the diagram $\tan\theta = \frac{2}{d}$ $\Rightarrow$ $d = \frac{2}{\tan 31.6°}$.

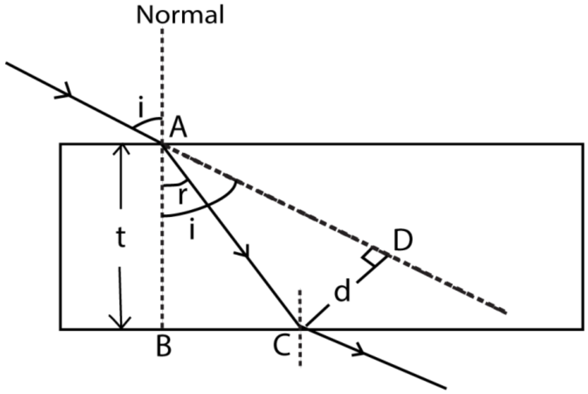
$\therefore$ d = 3·2m.

Thus required distance traveled = 4 – d
= 4– 3·2
= 0·8m

Sidewise displacement of light rays

When light travels from one medium to another, its direction is displaced sideways. This is called **lateral displacement.**

Consider a ray of light incident at an angle **i** on the upper surface of a glass block of thickness **t**, and then suddenly refracted through an angle **r** causing it to suffer a sidewise displacement **d**.



From Δ ABC, AC = $\frac{t}{\cos r}$ ---------------------------------(i)

From the diagram, ∠CAD = (i – r)

From Δ ACD, AC = $\frac{d}{\sin (i - r)}$ -------------------------(ii)

Equating equation (i) and (ii) gives

$$\frac{t}{\cos r} = \frac{d}{\sin (i - r)}$$

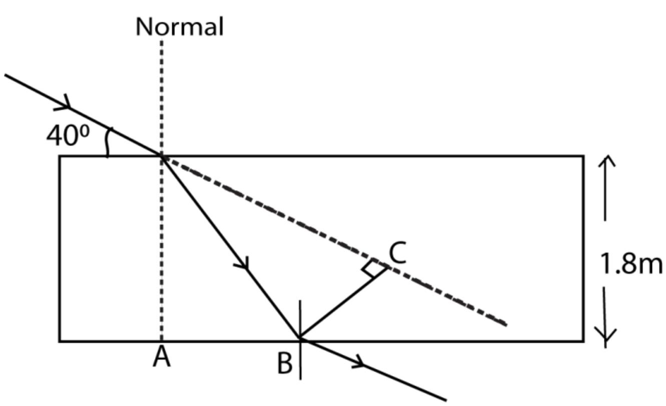
$$\Rightarrow \quad \mathbf{d} = \frac{\mathbf{t \sin (i - r)}}{\mathbf{\cos r}}$$

NOTE :

The horizontal displacement of the incident ray, **BC = t. tan r**

Example: 6

The figure below shows a monochromatic ray of light incident from a liquid of refractive index 1·33 onto the upper surface of a glass block of refractive index 1·42

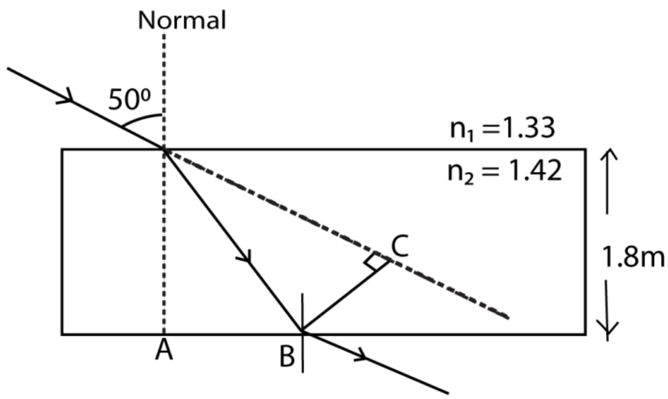


Calculate the;

(i) horizontal displacement AB.

(ii) lateral displacement BC of the emergent light.

Solution



(i) Applying Snell's law at the liquid- glass interface gives,

$$n_l \sin 50° = n_g \sin r.$$

$$1{\cdot}33 \sin 50° = 1{\cdot}42 \sin r$$

$$\Rightarrow \quad \angle r = 45{\cdot}8°$$

Horizontal displacement AB = $t \tan r$

$= 18 \tan 45{\cdot}8°$

= 18·51cm

(ii) Lateral displacement d $= \dfrac{t \sin(i - r)}{\cos r}$

$$d = \frac{18 \sin(50° - 45{\cdot}8°)}{\cos 45{\cdot}8°}.$$

d = 1·89cm.

Exercise

(1) What is meant by **refraction of light?**

(2) (i) State the laws of refraction of light.

(ii) State what brings about refraction of light as it travels from one medium to another.

(3) (i) What is meant by the **refractive index** of a material**?**

(ii) Light of two colors blue and red is incident at an angle γ from air to a glass block of thickness **t**. When blue and red lights are refracted through angles of θ_b and θ_r respectively, their corresponding speeds in the glass blocks are V_b and V_r**.**

Show that the separation of the two colors at the bottom of the glass block

$d = \frac{t}{c}\left[\frac{v_r}{\cos\theta_r} - \frac{v_b}{\cos\theta_{rb}}\right]$ where $\theta_r > \theta_b$ and c is the speed of light

(iii) Light consisting of blue and red is incident at an angle of $60°$ from air to a glass block of thickness **1.8cm.** If the speeds of blue and red light in the glassblock are $\mathbf{1\cdot 86 \times 10^{8}\ ms^{-1}}$ and $\mathbf{1\cdot 92 \times 10^{8}\ ms^{-1}}$ respectively, find theseparation of the two colors at the bottom of the glass block.

[**Answer: 0·54cm**]

(4) Show that when the ray of light passes through different media separated by plane boundaries, **n sinϕ = constant** where **n** is the absolute refractive index of a medium and ϕ is the angle made by the ray with the normal in the medium.

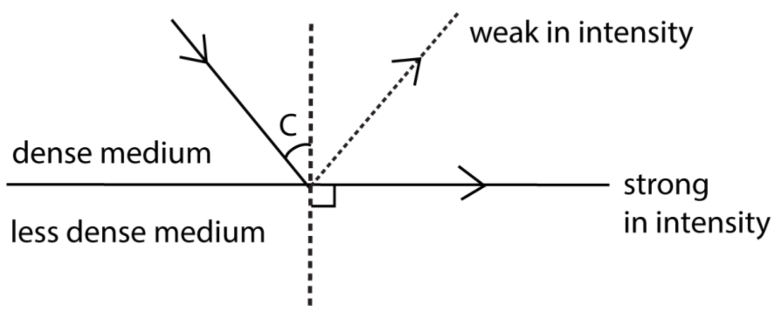
(5) Show that when the ray of light passes through different media **1** and **2** separated by plane boundaries, $_1n_2 \times {_2n_1} = 1$ where **n** is the refractive index of a medium.

(6). Show that when the ray of light passes through different media **1,2** and **3** separated by plane boundaries, $_1n_3 = {_1n_2} \times {_2n_3}$ where **n** is the refractive index of a medium.

(7) Show that a ray of light passing through a glass block with parallel sides of thickness **t** suffers a sidewise displacement $\mathbf{d} = \frac{t\sin(\Phi - \lambda)}{\cos\lambda}$ where ϕ is the angle of incidence and λ is the angle of refraction.

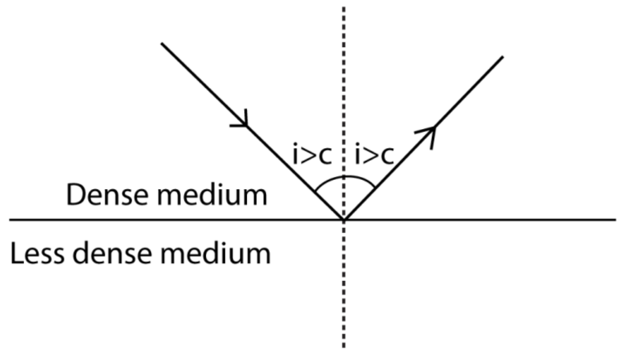
Critical angle and total internal reflection

Consider a ray of light incident on an interface between two media as shown.



Critical angle: is the angle of incidence in the dense medium which results into an angle of refraction of **90**° in the adjoining less dense medium.

If the angle of incidence is greater than the critical angle, all the incident light energy is reflected back in the dense medium and **total internal refection** is said to have occurred.



NOTE :

(a) At critical point Snell's law becomes.

$n_1 \sin c = n_2 \sin 90°$

$$\Rightarrow \quad \sin c = \frac{n_2}{n_1}$$

$$c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Where $\mathbf{n_1}$ and $\mathbf{n_2}$ are the refractive indices of the dense and the less dense medium respectively.

However if the less dens medium is air, then Snell's law becomes

$$n_1 \sin c = n_a \sin 90°$$

$$\Rightarrow \quad \sin c = \frac{1}{n_1}$$

(b) The conditions for total internal reflection to occur are;

(i) Light should travel from an optically denser to a less dense medium.

(ii) The angle of incidence at the boundary of the media should be greater than the critical angle.

Application of total internal reflection.

(i) It is responsible for the formation of a mirage.

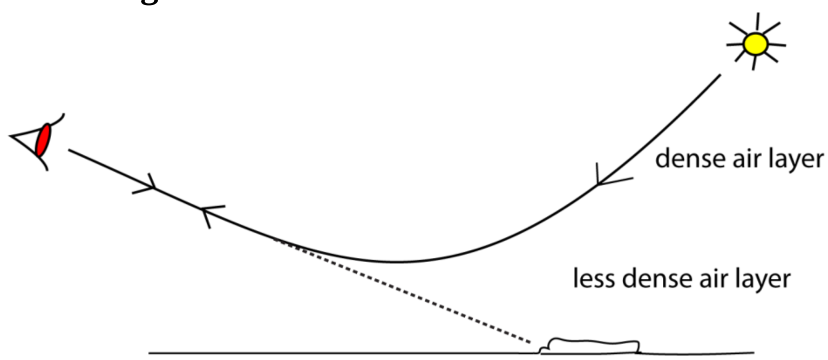
(ii) It is responsible for the formation of a rainbow.

(iii) It is responsible for the transmission of light in optical fibres.

(iv). It is responsible for the transmission of sky radio waves

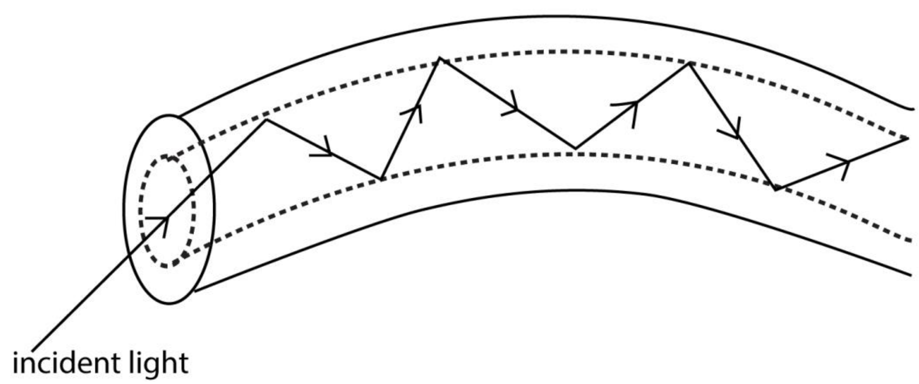
(v). It is responsible for the transmission of light in prism binoculars.

Formation of a mirage



On a hot day, The air layers near the earth's surface are hot and are less denser than the air layers above the earth's surface. Therefore as light from the sky pass through the various layers of air, light rays are continually refracted away from the normal till some point where light is totally internally reflected. An observer on earth receiving the totally internally reflected light gets an impression of a pool of water on the ground and this is the virtual image of the sky

An optical fibre

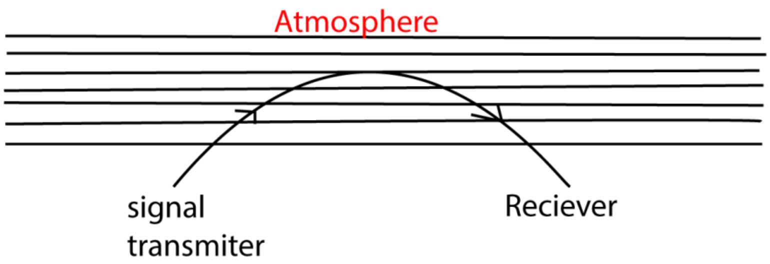


When a light ray enters in to the fibre, it bounces from one edge to another by total internal reflection. These successive total internal reflections enable the transmission of light in optical fibres.

NOTE

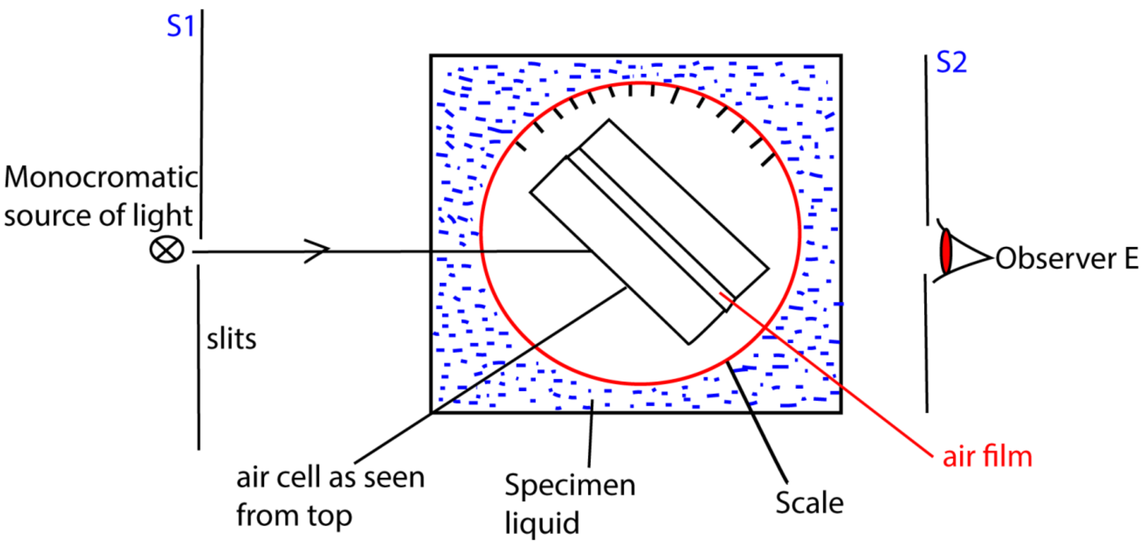
- **(i)** An optical fibre finds a practical application in an endoscope, a device used by Doctors to inside the human body.
- **(ii)** Optical fibres are used in telecommunication systems (i.e. Telephone or TV signals are carried along optical fibers by laser light).

SKY RADIO WAVES



- Radio are refracted like light waves
- Radio waves from a radio station on earth's surface are directed to atmosphere, a region of ionized gas formed at very high altitudes.
- As the optical density of the atmosphere decreases with height, the waves are totally internally reflected and can be received at the other side of the earth.

Estimation of refractive index of a liquid by an air-cell method

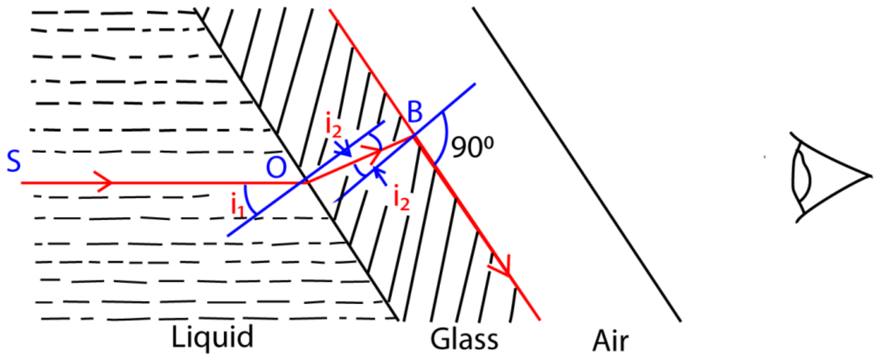


- The air cell is immersed in a liquid under test.
- A beam of monochromatic light is directed onto the air cell and then observed through the cell from the opposite side at **E**.
- The cell is then rotated on one side until light is suddenly cut off and the angular position $\boldsymbol{\theta_1}$ is noted.
- The cell is again rotated in the opposite direction until light is suddenly cut off and the angular position $\boldsymbol{\theta_2}$ is noted.
-
- The refractive index of the liquid can then be calculated from

$$\mathbf{n} = \frac{1}{\sin\boldsymbol{\theta}} \quad \text{where } \boldsymbol{\theta} = \frac{\boldsymbol{\theta_1} + \boldsymbol{\theta_2}}{\mathbf{2}}$$

NB:
The source of light should be monochromatic so that the extinction of light is sharp since monochromatic light does not undergo dispersion, as it is with white light.

Theory of the air-cell method



Ray **OS** is refracted along **OB** in glass. However, at **B** total internal just begins. Suppose i_1 is the angle of incidence in the liquid, i_2 is the angle of incidence in the glass while **n** and n_g are the corresponding refractive indices, Then applying the relation **n sin i = a constant** gives

$n \sin i_1 = n_g \sin i_2 = n_a \sin 90°$

$\therefore \quad n \sin i_1 = n_a \sin 90°$

$\Rightarrow n \sin i_1 = 1$

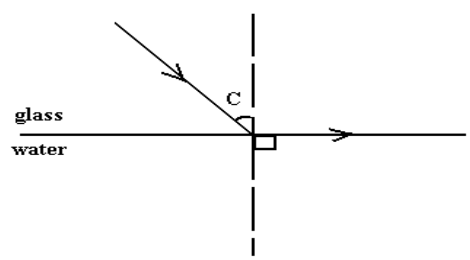
Thus $n = \dfrac{1}{\sin i_1}$

But $\angle i_1 = \dfrac{\theta_1 + \theta_2}{2}$ Hence $n = \dfrac{1}{\sin \theta}$

Examples 7

The critical angle for water-air interface is 48˚ 42[1] and that of glass-air interface is 38˚ 47[1]. Calculate the critical angle for glass-air interface.

Solution



Applying Snell's law gives $n_g \sin c = n_w \sin 90^o$--------------------(i).

Given that for water-air interface $c_w = 48^o42^1$.

$\therefore n_w \sin c_w = 1$

$n_w \sin 48^o42^1 = 1$

$\Rightarrow \quad n_w = 1{\cdot}33$---(ii)

Also for glass-air interface $c_g = 38^o47^1$

$n_g \sin c_g = 1$

$\therefore \quad n_g \sin 38^o47^1 = 1$

$\Rightarrow \quad n_g = 1{\cdot}67$---(iii)

Substituting equation (ii) and (iii)into equation (i) gives

$1{\cdot}67 \sin c = 1{\cdot}33 \sin 90°$

$\Rightarrow \quad c = 52{\cdot}8°$

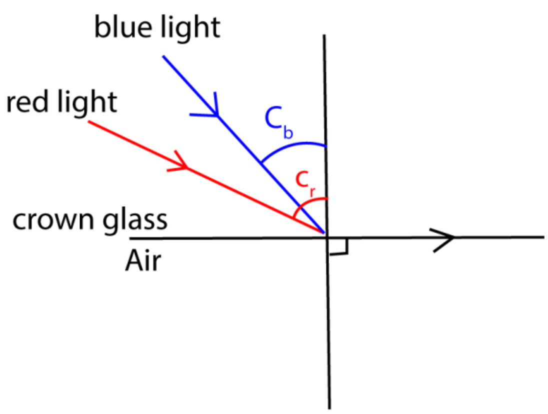
Note that $1^0 = 60^1$

Examples 8

The refractive index for red light is 1·634 of crown glass and the difference between the critical angles of red and blue light at the glass-air interface is $0°56^1$.What is the refractive index of crown glass for blue light

Solution

Analysis the critical angle between two media for red light is greater than that for any other light colour. This gives rise to the ray diagram below



Since $c_r > c$---(i)

Applying Snell's law to red light gives

$n_r \sin c_r = 1$

$1{\cdot}63 \sin c_r = 1$

$\Rightarrow \quad c_r = 37{\cdot}73°$

Equation (i) now becomes

$c_b = c_r - 0°56^1$

$c_b = 37{\cdot}73° - 0°56^1$

$c_b = 36{\cdot}8°$

Applying Snell's law to blue light gives

$n_b \sin c_b = 1$

$n_b \sin 36{\cdot}8° = 1$

$\Rightarrow \quad n_b = 1{\cdot}67$

Examples 9

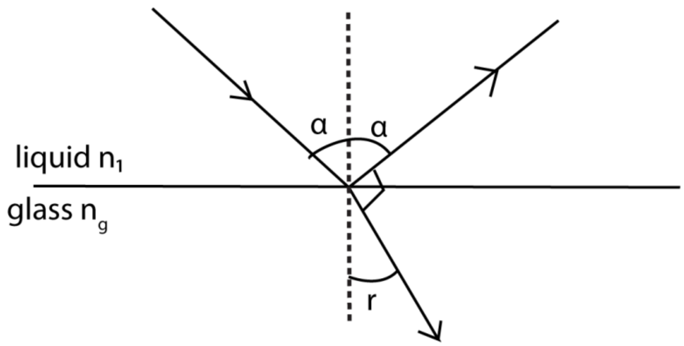
(i) A glass block of refractive index n_g is immersed in a liquid of refractive index n_l. A ray of light is partially reflected and refracted at the interface such that the angle between the reflected and the refracted ray is 90°.

Show that $\mathbf{n_g = n_l \tan} \alpha$ where α is the angle of incidence from the liquid to glass.

(ii) When the procedures in **(i)** above are repeated with the liquid removed, the angle of incidence increases by **8**°.

Given that $\mathbf{n_l = 1{\cdot}33}$, find $\mathbf{n_g}$ and the angle of incidence at the liquid-glass interface. Find angle r.

Solution



Applying Snell's law at the liquid-glass interface gives

$n_g \sin r = n_l \sin \alpha$

But $r + 90° + \alpha = 180°$

$\Rightarrow r = 90° - \alpha$

$\therefore n_g \sin(90° - \alpha) = n_l \sin \alpha.$

From trigonometry, $\sin(90° - \alpha) = \cos \alpha$

$\Rightarrow n_g \cos \alpha = n_l \sin \alpha.$

Dividing $\cos \alpha$ on both sides gives

$\mathbf{n_g = n_l \tan \alpha.}$

(ii) When the liquid is removed.

From $n_g = n_l \tan \alpha$---(i)

$\Rightarrow n_g = n_a \tan(\alpha + 8°)$ but $n_a = 1$

$$\therefore n_g = \frac{\tan\alpha + \tan 8^0}{1 - \tan\alpha\tan 8^0}$$

n_g (1– tan α tan 8°) = tan α + tan 8°

$n_g – n_g$ tan α tan 8° = tan α + tan 8°--------------------------------(ii)

from equation (i) tan α = $\frac{n_g}{n_1}$

$$n_g - \frac{n_g}{n_1}tan8^0 = \frac{n_g}{n_1} + \ tan8^0$$

but n_l = 1·33.
∴ $n^2_g – 2.340\ n_g + 1·326 = 0$ ----------------------------(iii)

Equation (iii) is quadratic in n_g and solving it gives
n_g = 1·39 **or** n_g not physically possible.

Using equation (i)

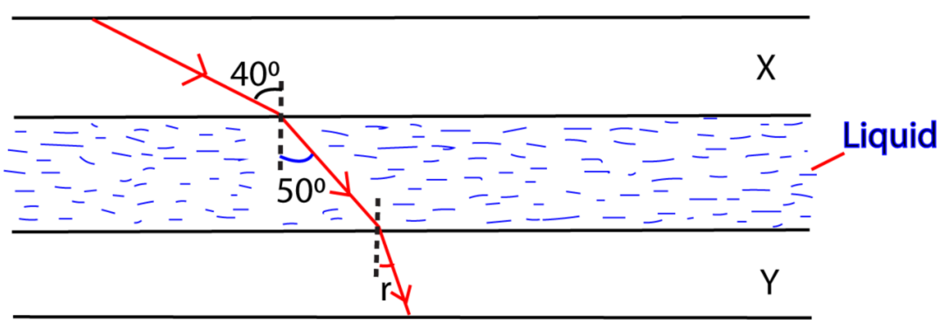
$$\tan\alpha = \frac{n_g}{n_l} = \frac{1·39}{1·33}$$

$\alpha = \tan^{-1}(1.045) = 46·3°$

The required angle of incidence = α + 8°
= **54·3°**

Examples 10

The figure below shows a liquid layer confined between two transparent plates **X** and **Y** of refractive index **1·54** and **1·44** respectively.



A ray the interface between media **X** and the liquid is refracted through an angle of **50**° by the liquid.

Find the
(i) refractive index of the liquid.
(ii) angle of refraction ,**r** in the medium **Y.**
(iii) minimum angle of incidence in the medium **X** for which the light will not emerge from medium **Y**.

Solution

(i) Applying Snell's law at the plate **X** – liquid interface gives

$$n_x \sin 40° = n_l \sin 50°$$
$$1{\cdot}54 \sin 40° = n_l \sin 50°$$
$$n_l = \frac{1{\cdot}54 \sin 40°}{\sin 50°}$$
$$\therefore \; \mathbf{n_l = 1{\cdot}29}$$

(ii) Applying Snell's law at the liquid – plate **Y** interface gives

$$n_l \sin 50° = n_y \sin r$$
$$1{\cdot}29 \sin 50° = 1{\cdot}44 \sin r$$
$$\Rightarrow \; \angle \mathbf{r} = \mathbf{43{\cdot}3°}$$

(iii) For light not to emerge from plate Y, it grazes the liquid – plate Y interface.

$$\Rightarrow \; \angle r = 90°$$

Applying Snell's law at the liquid – plate Y interface gives

$$n_l \sin i_l = n_y \sin 90°$$
$$1{\cdot}29 \sin i_l = 1{\cdot}44 \sin 90°$$
$$\sin i_l = \frac{1{\cdot}44}{1{\cdot}29} \text{ --------------------------------(i)}$$

Moreover, applying Snell's law at the plate X – liquid interface gives

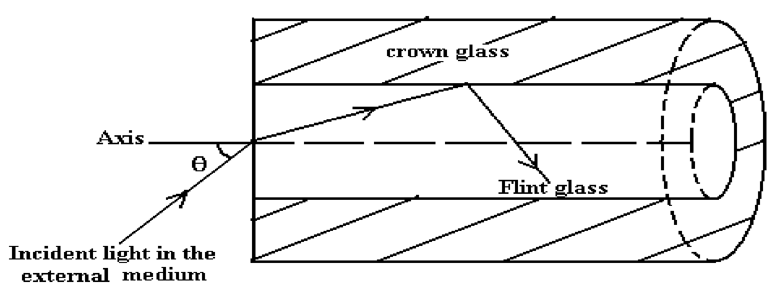
$$n_x \sin i_x = n_l \sin i_l$$
$$1{\cdot}54 \sin i_x = 1{\cdot}29 \sin i_l \text{ ----------------------------(ii)}$$

Substituting equation (i) in (ii) gives

$$1{\cdot}54 \sin i_x = 1{\cdot}29 \times \frac{1{\cdot}44}{1{\cdot}29}$$
$$\Rightarrow \; \angle \mathbf{i_x} = \mathbf{40{\cdot}5°}$$

Examples 11

The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.

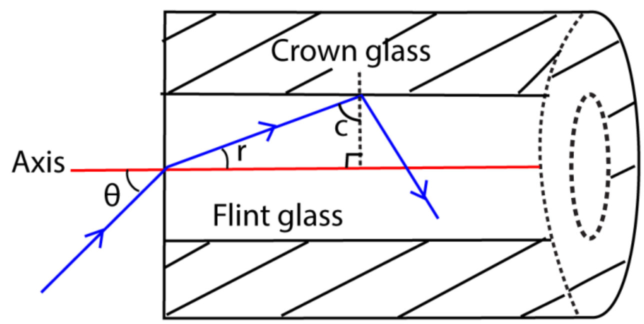


The refractive indices for flint glass, crown glass and the external medium are n_1, n_2 and n_3 respectively. Show that a ray that enters the pipe is totally reflected at the flint-crown

Glass interface provided $\sin\theta = \frac{\sqrt{[n_1^2 - n_2^2]}}{n_3}$ where θ is maximum angle of incidence in the external medium

Solution

Analysis for light to be totally reflected, it must be incident at a critical angle on the flint-crown glass interface



Applying Snell's law at the external medium-flint glass interface gives

$n_3 \sin\theta = n_1 \sin r$

but $r + c = 90°$

$\therefore \quad r = 90° - c$

$\Rightarrow \quad n_3 \sin\theta = n_1 \sin(90° - c)$

$\therefore \quad n_3 \sin\theta = n_1 \cos c$

$\Rightarrow \quad \cos c = \frac{n_3 \sin\theta}{n_1}$ --------------------------------(i)

Applying Snell's law at the flint-crown glass interface gives

$n_1 \sin c = n_2 \sin 90°$

$\Rightarrow \quad \sin c = \frac{n_2}{n_1}$ ---(ii)

Using the trigonometric relation $\sin^2 c + \cos^2 c = 1$, then

$$\left(\frac{n_2}{n_1}\right)^2 + \left[\frac{n_3 sinA}{n_1}\right]^2 = 1$$

Thus, $\sin\theta = \frac{\sqrt{n_1^2 - n_2^2}}{n_3}$

EXERCISE

1. Explain the term total internal reflection and give three instances where it is applied.

2. With the aid of suitable ray diagrams, explain the terms critical angle and total internal reflection.

3. Show that the relation between the refractive index **n** of a medium and critical angle **c** for a ray of light traveling from the medium to air is given by **n** = $\frac{1}{\sin c}$

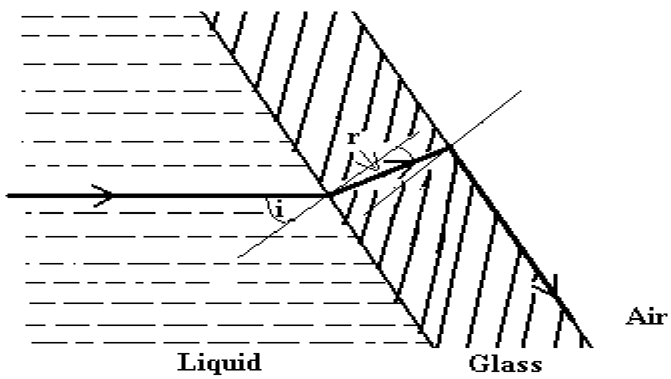
4.Show that the critical angle, **c** at a boundary between two media when light travels from medium 1 to medium 2 is given by **sin c** =$\frac{n_2}{n_1}$ where **n**1and **n2** are the refractive indices of the media respectively.

5. Explain how a mirage is formed.

6. Explain briefly how sky radio waves travel from a transmitting station to a receiver.

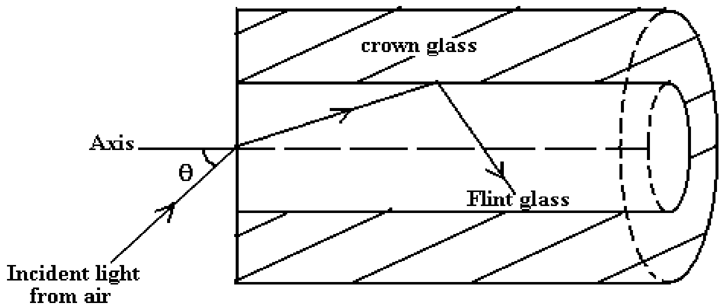
7. Describe how you would determine the refractive index of the liquid using an air cell.

8)



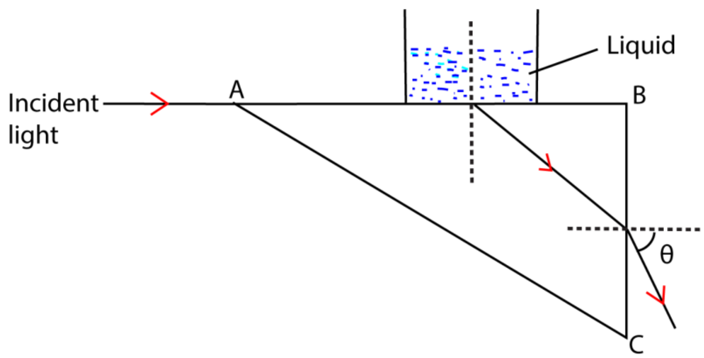
In the figure above, a parallel sided glass slide is in contact with a liquid on one side and air on the other side. A ray of light incident on glass slide from the liquid emerges in air along the glass-air interface. Derive an expression for the absolute refractive index **,n,** of the liquid in terms of the angle of incidence **i** in the liquid-medium.

(9) The diagram below shows a cross-section through the diameter of the light pipe with an incident ray of light in its plane.



The refractive indices for flint glass and crown glass are $\mathbf{n_1}$ and $\mathbf{n_2}$ respectively. Show that a ray which enters the pipe is totally reflected at the flint-crown glass Interface provided $\sin\theta = \sqrt{n_1^2 - n_2^2}$ where θ is the maximum angle of incidence at the air-flint glass interface.

10.A liquid of refractive index n_L is tapped in contact with the base of a right-angled prism of refractive index n_g by means of a transparent cylindrical pipe as shown.

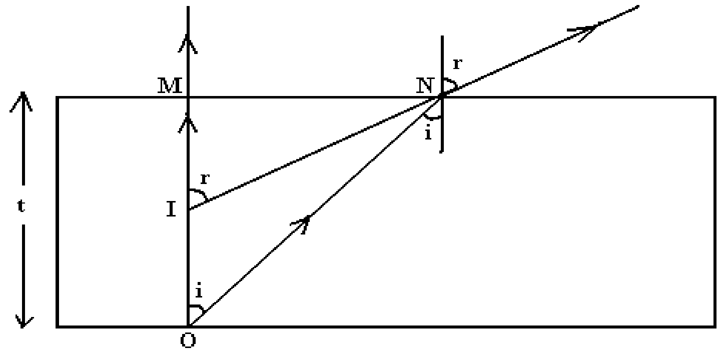


Show that a ray of light which is at a grazing incidence on the liquid-glass interface emerges in to air through face **BC** at an angle θ below the horizontal provided

$n_L = \sqrt{(n_g^2 - sin^2\theta)}$. Hence find n_L if $n_g = 1.52$ and θ = 47.40.

Real and apparent depth

Consider an object **O** viewed normally from above through a parallel-sided glass block of refractive index $\mathbf{n_g}$ and thickness **t** as shown.



A ray from an object **O** normal to the glass surface at **M** passes un deviated. While the ray **ON** inclined at a small angle **i** to the normal is refracted at **N**. The observer above the glass block sees the image of the object **O** at **I.**

Applying Snell's law at **C** gives

n_g sin i = n_a sin r-----------------------------------(i).

From the diagram, $\sin i = \frac{MN}{ON}$ and $\sin r = \frac{MN}{IN}$

Equation (i) now becomes $n_g \frac{MN}{ON} = n_a \frac{MN}{IN}$

$$n_g = \frac{ON}{IN}$$

Since angle **i** is very small, then ON ≈ OM and IN ≈ IM

$$n_g = \frac{OM}{IM}$$

From the diagram**, OM = real depth** and **IM = apparent depth**.

Hence $\mathbf{n_g} = \frac{\textit{real depth}}{\textit{apparent depth}}$.

If the apparent displacement of the object **OI = d,** then **IM = (t – d).**

So that ng $= \frac{OM}{IM} = \frac{t}{t-d}$

Thus, apparent displacement, d $= \left[1 - \frac{1}{n_g}\right] t$

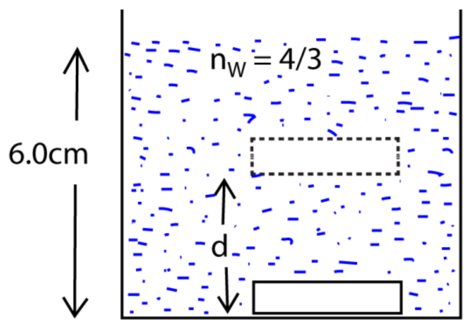
NOTE

(i) The apparent displacement **d** of an object **O** is independent of the position of **O** below the glass block. Thus the same expression above gives the displacement of an object which is some distance in air below a parallel-sided glass block.

(ii) If there are different layers of different transparent materials resting on top of each other, the apparent position of the object at the bottom can be found by adding the separate displacements due to each layer.

Examples 12

An object at a depth of **6·0cm** below the surface of water of refractive index $\frac{4}{3}$ is observed directly from above the water surface. Calculate the apparent displacement of the object

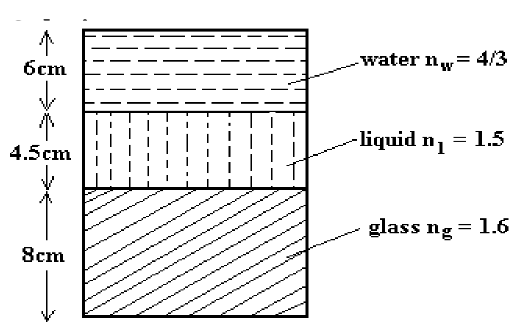


Using the relation $d = \left[1 - \frac{1}{n_g}\right] t$ gives;

$$d = \left[1 - \frac{3}{4}\right] x\, 6 = 1.5 \text{ cm}$$

Examples 13

A tank contains a slab of glass **8cm** and refractive index **1·6**. Above this is a depth of **4·5cm** of a liquid of refractive index **1·5** and upon these floats **6cm** of water of refractive index $\frac{4}{3}$ Calculate the apparent displacement of an object at the bottom of the tank to an observer looking down wards directly from above.



Using the relation d $= \left[1 - \frac{1}{n_g}\right] t$

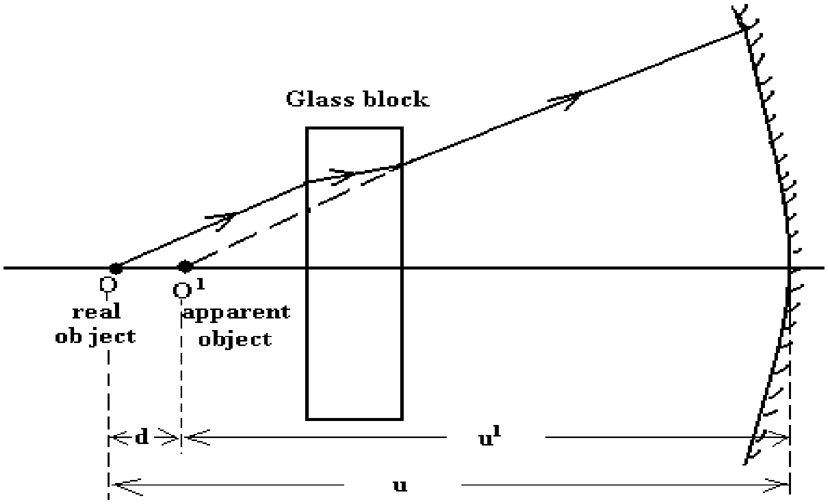
$$d = \left[1 - \frac{1}{n_w}\right] t + \left[1 - \frac{1}{n_1}\right] t + \left[1 - \frac{1}{n_g}\right] t$$

$$d = \left[1 - \frac{4}{3}\right] t + \left[1 - \frac{1}{1.5}\right] t + \left[1 - \frac{1}{1.6}\right] t$$

d = 1·5 + 1·5 + 3 d = 6cm.

Examples 14

A small object is placed **20cm** in front of a concave mirror of focal length **15cm.** A parallel-sided glass block of thickness **6cm** and refractive index **1·5** is then placed between the mirror and the object. Find the shift in the position and size of the image



Consider the action of a concave mirror in the absence of a glass block u = 20cm and f = 15cm

Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v = \frac{fu}{u-f} = \frac{15 \ x \ 20}{20-15} = 60cm$$

Thus in the absence of a glass block**, image distance = 60cm**

In this case, magnification, m = $\frac{v}{u} = \frac{60}{20} = 3$

Consider the action of a glass block

Apparent displacement $= \left[1 - \frac{1}{n_g}\right] t = \left[1 - \frac{1}{1.5}\right] x \ 6 = 2$m

Thus in the presence of a glass block, object distance **u^1 = (20 – 2) cm =18cm**

”The object is displaced and it appears to be 18cm in front of the mirror”

Consider the action of a concave mirror in the presence of a glass block

u_1 = 18cm and f = 15cm

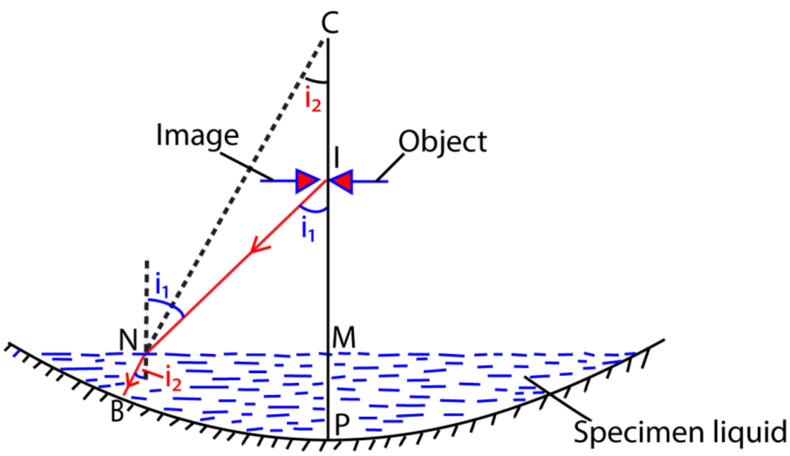
Using the mirror formula $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ gives

$$v_1 = \frac{fu_1}{u_1 - f} = \frac{15 \ x \ 18}{18 - 15} = 90cm$$

∴ **The required shift in the image position = v_1 – v = (90 – 60) cm = 30cm**

In this case, magnification, $m_1 = \frac{v_1}{u_1} = \frac{90}{18} = 5$

Determination of refractive index of a liquid using a concave mirror method.



A small quantity of the liquid under test is poured into a concave mirror of known radius of curvature **r**. An object pin is moved along the principal axis of the mirror until it coincides with its image at **I.** The distance **IP** is noted. The required refractive index $n_1 = \frac{r}{IP}$

PROOF

For refraction at **N,** $n_l \sin i_2 = n_a \sin i_1$ --------------------------- (i)

From the diagram, $\sin i_2 = \frac{NM}{MN}$ NC and $\sin i_1 =$ NI

Equation (i) now becomes

$$\mathbf{n_1} \frac{NM}{NC} = \frac{NM}{NI}$$

.

On simplifying, $\mathbf{n_l} = \frac{NC}{NI}$

But **N** is very close to **M** hence $NC \approx MC$ and $NI \approx MN$

$\Rightarrow$ $\mathbf{n_l} = \frac{MC}{MI}$

Also for a small quantity of the liquid, **M** is close to **P** $\Rightarrow MC \approx CP =$ **r**, and $MI \approx IP$.

Thus $\mathbf{n_l} = \frac{\mathbf{r}}{\mathbf{IP.}}$

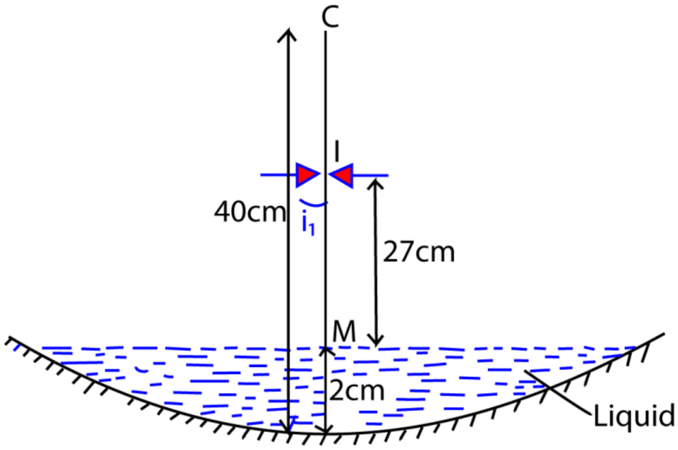
NOTE :

(i) If the specimen liquid is of reasonable quantity, then its depth **d** can not be ignored. In this case, $\mathbf{n_l} = \frac{\mathbf{MC}}{\mathbf{MI}} = \frac{\mathbf{r - d}}{\mathbf{MI}}$

(ii) if the radius of curvature **r** of the concave mirror is not known, first determine it using the method discussed in the previous section.

Examples 15

A liquid is poured in to a concave mirror to a depth of **2·0cm**. An object held above the liquid coincides with its own image when its **27·0cm** above the liquid surface. If the radius of curvature of the mirror is **40·0cm**, calculate the refractive index of the liquid.



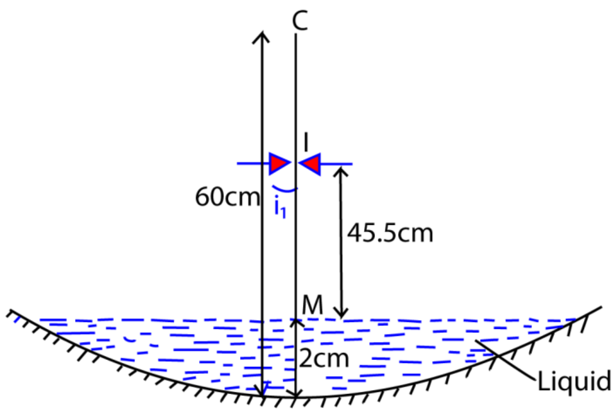
Using the relation $\mathbf{n_l} = \dfrac{\mathbf{r - d}}{\mathbf{MI}}$

$= \dfrac{40 - 2}{27}$

$\therefore \quad \mathbf{n_l = 1{\cdot}4}$

Examples 16

A liquid is poured into a concave mirror to a depth of **2·0cm**. An object held above the liquid coincides with its image when it is **45·5cm** from the pole of the mirror. If the radius of curvature of the mirror is **60·0cm**, calculate the refractive index of the liquid.



Using the relation $\mathbf{n_l} = \dfrac{\mathbf{r - d}}{\mathbf{MI}}$

$= \dfrac{60 - 2}{45{\cdot}5 - 2}$

$\therefore \quad \mathbf{n_l = 1{\cdot}33}$

Examples 17

A small concave mirror of focal length **8cm** lies on a bench and a pin is moved vertically above it .At what point will the pin coincide with its image if the mirror is filled with water of refractive index **4**/**3.**

Solution

ANALYSIS

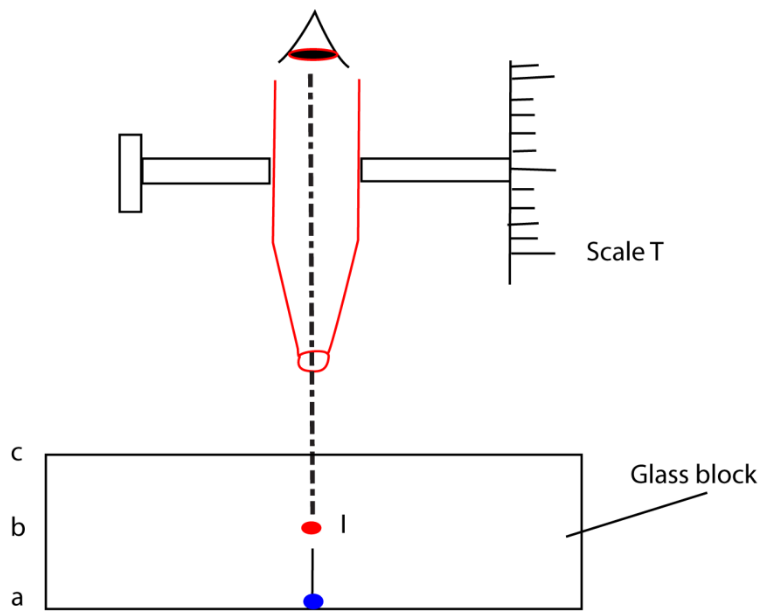
For a small concave mirror, the quantity of water is small that its depth **d** can be ignored

Using the relation $n_l = \frac{r}{IP}$ Where **r = 2f = 2 × 8 = 16cm**

$\Rightarrow \quad IP = \frac{r}{n_1} = \frac{16}{4/3} = 12cm$

Therefore the pin coincided with its image at a height of **12cm** above the mirror

Measurement of refractive index of a glass block by apparent depth method.



A vertically traveling microscope having a graduated scale **T** besides it is focused on lycopodium particles placed at **O** on a sheet of white paper. The scale reading **a** on **T** is noted. A glass block whose refractive index is required is placed on the lycopodium particle on a paper and the microscope is raised until the particles are refocused at **I**. The scale reading **b** is again noted. Lycopodium particles are then poured on top of the glass block and the microscope is re-raised until the particles are again refocused. The new scale reading **c** is then noted. The refractive index of the block can then be calculated from

$$\boldsymbol{n = \frac{Real\ depth}{Apparent\ depth} = \left[\frac{c-a}{c-b}\right]}$$

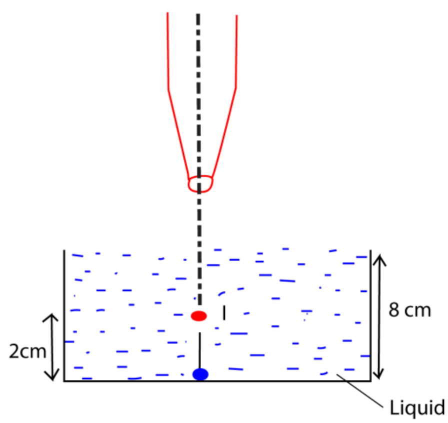
NOTE

The refractive index of a liquid can be found by focusing on a sand particle at the bottom of an empty container and the scale reading is noted. The specimen liquid is then put in the container and the traveling microscope is refocused on the sand giving a scale reading b. Finally the traveling microscope is focused on the liquid surface giving a scale reading c.

Thus $\mathbf{n_L} = \left[\frac{c-a}{c-b}\right]$

Examples 18

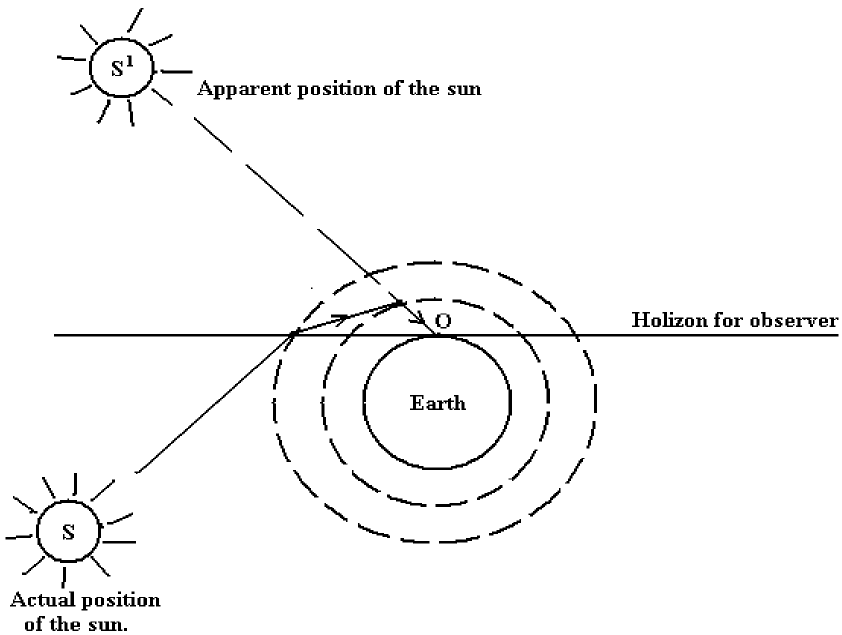
A microscope is focused on a mark at the bottom of the beaker. Water is poured in to the beaker to a depth of **8cm** and it is found necessary to raise the microscope through a vertical distance of **2cm** to bring the mark again in to focus. Find the refractive index of water.



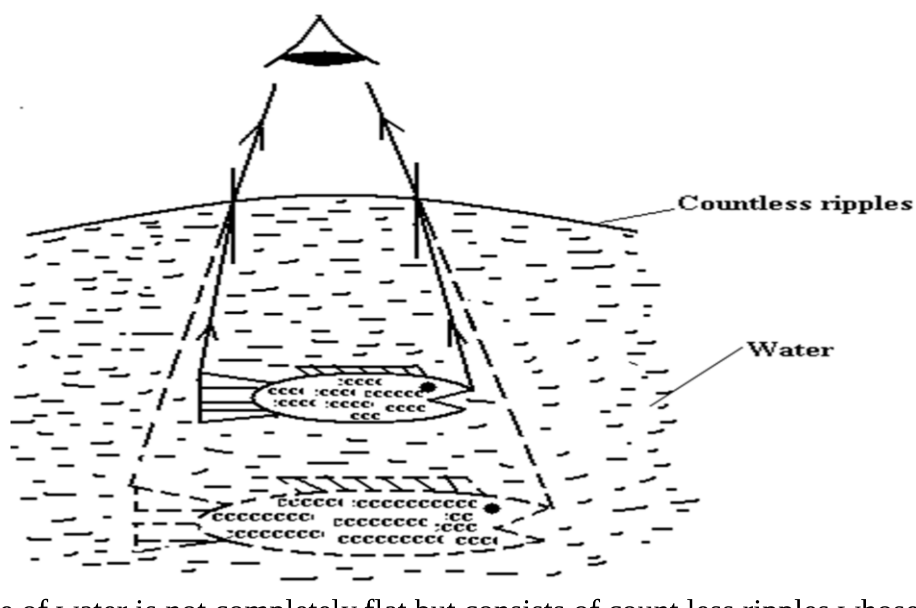
$$\boldsymbol{n_g = \frac{Real\ depth}{Apparent\ depth} = \left[\frac{8}{8-2}\right]}$$

= 1.33 (2 decimal point)

Observation of sunlight situated below the horizon



The aparent size of a fish situated in water.



A large surface of water is not completely flat but consists of count less ripples whose convex surface on air acts as a convex lens of long focal length. In consequence the fish is with in the focal length of the lens hence it appears magnified to an observer viewing it from above.

EXERCISE

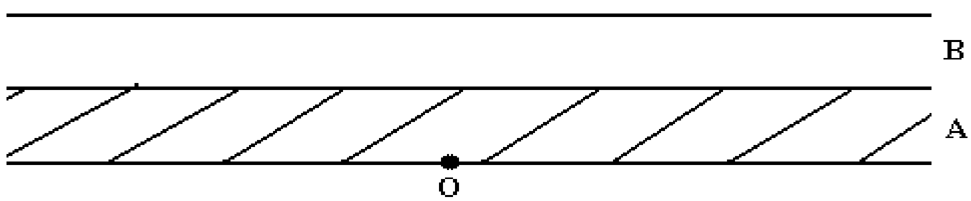
1. Show that for an object viewed normally from above through a parallel sided glass block , the refractive index of the glass material is given by

$$n_g = \frac{\text{real depth}}{\text{apparent depth}}$$

2. Derive an expression for the apparent displacement of an object when viewed normally through a parallel sided glass block.

3. A vessel of depth **2d cm** is half filled with a liquid of refractive index μ_1,and the upper half is occupied by a liquid of refractive index μ_2. Show that the apparent depth of the vessel, viewed perpendicularly is $d\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right)$

4.Two parallel sided blocks **A** and **B** of thickness **4·0cm** and **5·0cm** respectively are arranged such that **A** lies on an object **O** as shown in the figure below



Calculate the apparent displacement of **O** when observed directly from above,

if the refractive indices of **A** and **B** are **1·52** and **1·66**.

5. A tank contains liquid **A** of refractive index **1·4** to a depth of **7·0cm**. Upon this floats **9·0cm** of liquid **B**. If an object at the bottom of the tank appears to be **11·0cm** below the top of liquid **B** when viewed directly above from, calculate the refractive index of liquid **B**.

6. Describe how the refractive index of a small quantity of a liquid can be determined using a concave mirror.

7. Describe how the refractive index of a glass block can be determined using the apparent depth method.

8. A small liquid quantity is poured into a concave mirror such that an object held above the liquid coincides with its image when it is at a height **h** from the pole of the mirror. If the radius of curvature of the mirror is **r**, show with the aid of a suitable illustration, that the refractive index of the liquid. $n = \frac{r}{h}$

9.Explain how light from the sun reaches the observer in the morning before the sun appears above the horizon

10. Explain the apparent shape of the bottom of a pool of water to an observer at the bank of the pool.

11. Explain why a fish appears bigger in water than its actual size when out of water.