## SIMILARITY AND ENLARGEMENT.

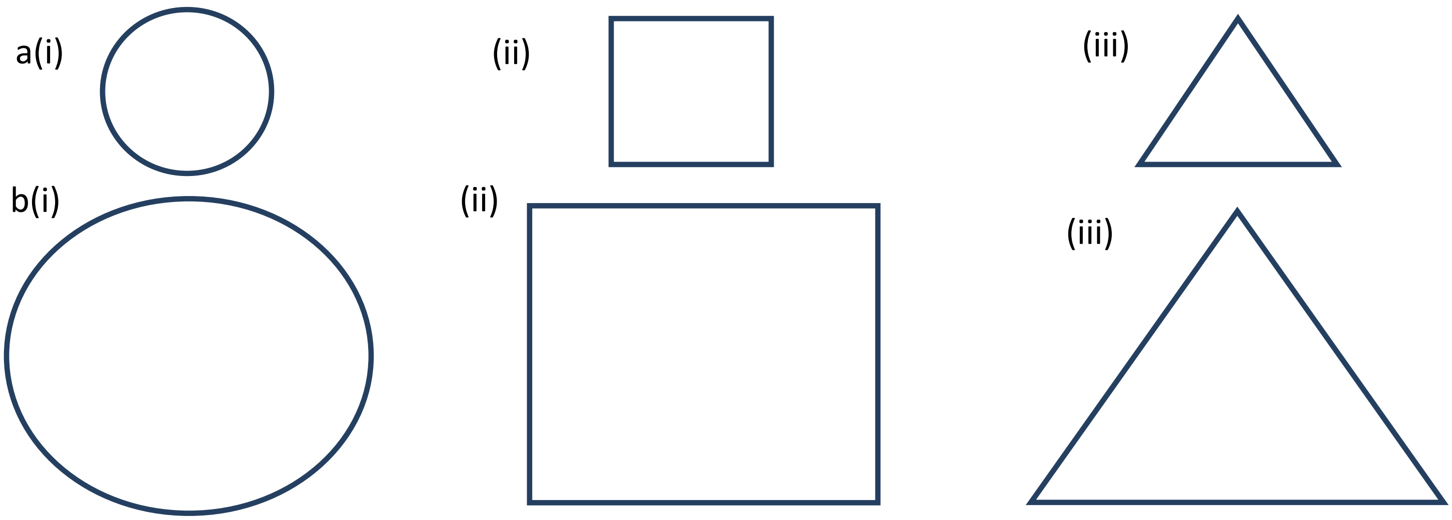
Similar figures and their properties.

Any two geometric figures are said to be similar if they have the same shapes even if their sizes are not the same.

For example if a photograph is enlarged, all the details of the small photograph are present in the enlarged copy. The dimensions of the features are increased proportionally.

The small photograph and the enlarged one are similar.

Any two circles, squares, equilateral triangles, regular polygons are similar.




All the details in (a) are also in (b).

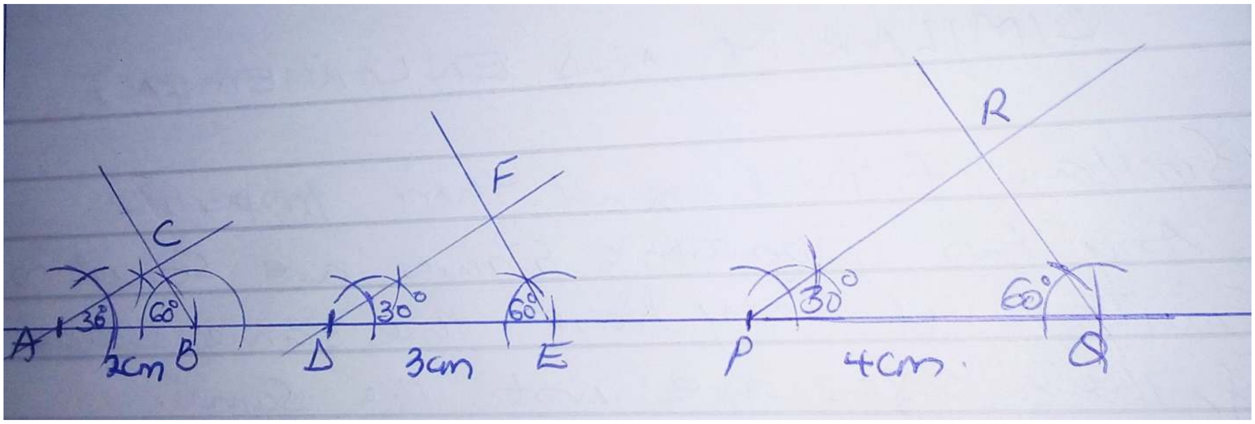
Construction of similar figures.

Draw triangles ABC, DEF and PQR with the following measurements.

a) Triangle ABC, angle A=30°, angle B=60°, AB=2cm.

b) Triangle DEF, angle D=30°, angle E=60°, DE=3cm.

c) Triangle PQR, angle P=30°, angle Q=60°, PQ=4cm.




Note that sides AB, DE and PQ are corresponding.

Also AC, DF, and PR are corresponding.

Measure the remaining sides and complete the following

i) $\frac{DE}{AB} = \frac{3}{2}$ ; $\frac{EF}{BC} =$ ; $\frac{DF}{AC} =$

ii) $\frac{PQ}{AB} = \frac{4}{2} = 2$ ; $\frac{QR}{BC} =$ ; $\frac{PR}{AC} =$

iii) $\frac{PQ}{DE} = \frac{4}{3}$ ; $\frac{QR}{EF} =$ ; $\frac{PR}{DF} =$

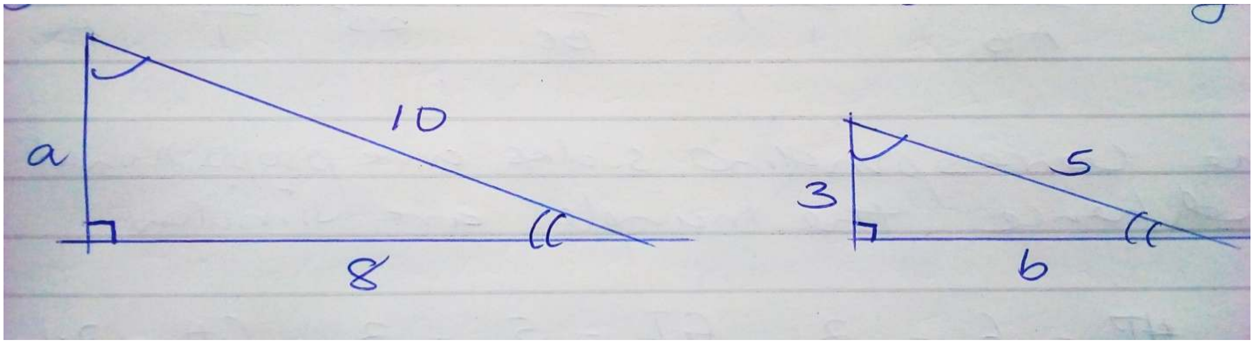
Triangle ABC is similar to DEF and PQR.

In, general, two plane figures are similar if

a) their corresponding angles are equal

b) their corresponding sides are in the same ratio.

<u>Example:1</u>The Triangles given below are similar. Find the numerical values of a and b.




$\frac{a}{3} = \frac{10}{5}$

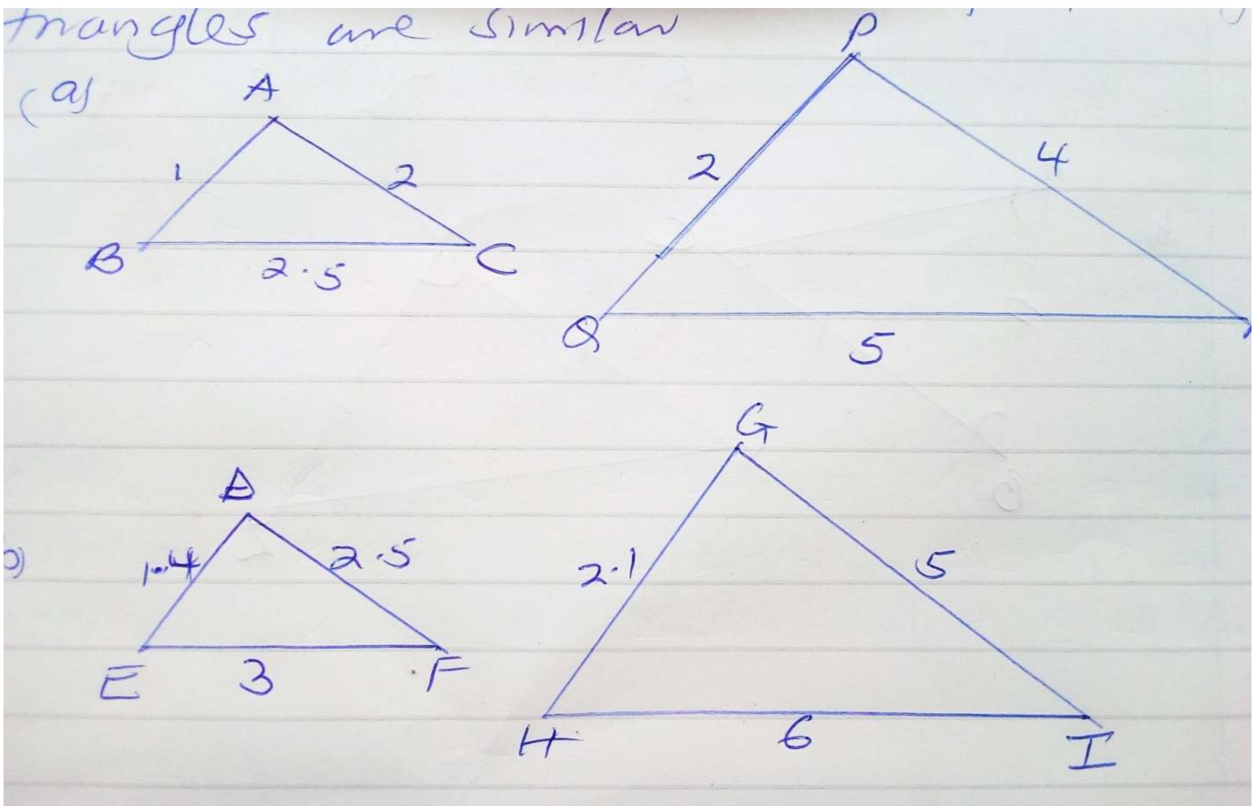
a = $\frac{10\times3}{5} = 6$

$\frac{b}{8} = \frac{5}{10}$

b = = $\frac{5\times8}{10} = 4$

<u>Example:2</u>

Determine whether the following pair of triangles are similar.



solution:

a) $\frac{PQ}{AB} = \frac{2}{1}, \ \frac{QR}{BC} = \frac{2}{2.5} = \frac{2}{1}, \ \frac{PR}{AC} = \frac{4}{2} = \frac{2}{1}$

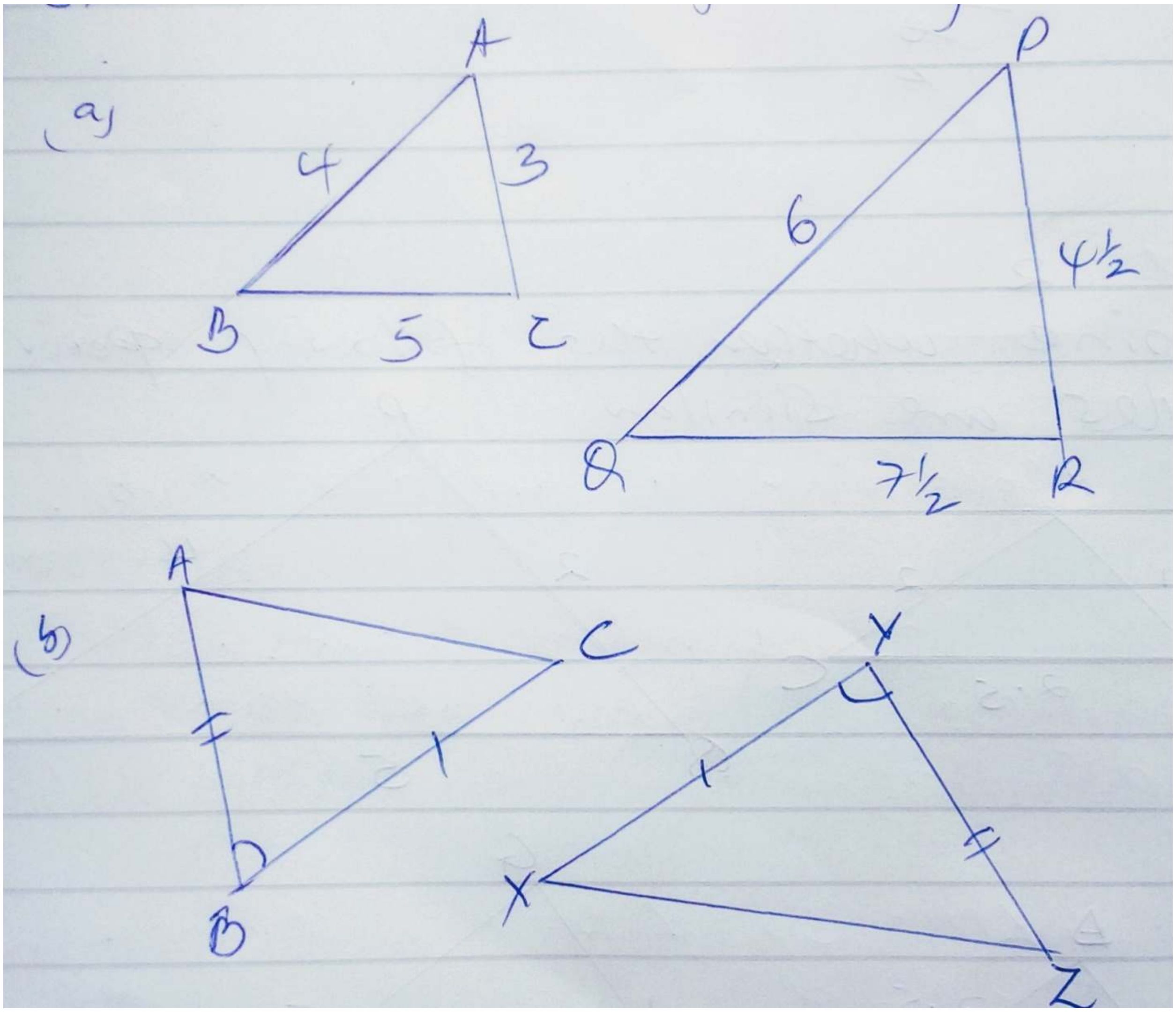
The corresponding sides are proportional and hence the triangles are similar.

b) $\frac{HP}{AB} = \frac{6}{3}, \ \frac{GI}{DF} = \frac{5}{2.5} = \frac{2}{1}, \ \frac{GH}{DE} = \frac{2.1}{1.4} = \frac{3}{2}$

The ratio of the corresponding sides is not equal hence the triangles are not similar.

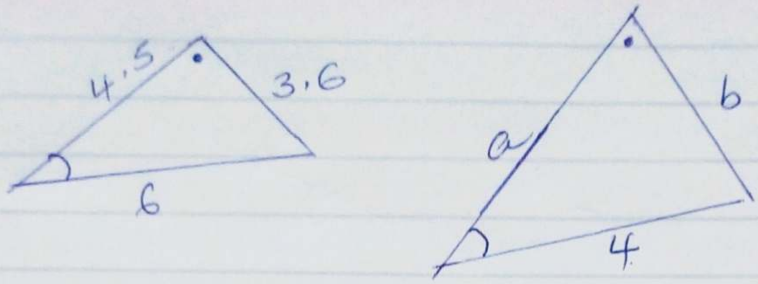
## Exercise:

1) State whether the following are similar

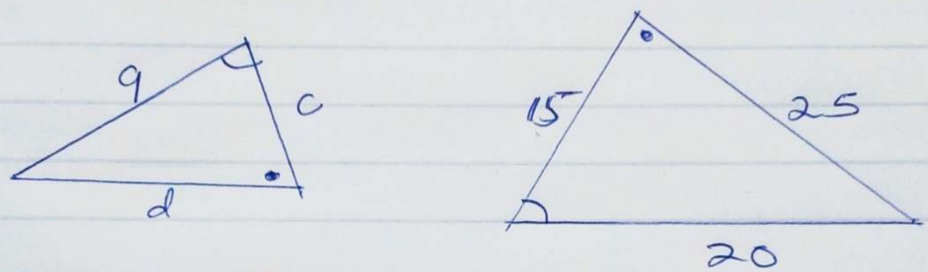



2) Calculate the length of the sides marked with a letter. (All the measurements are in cm.)

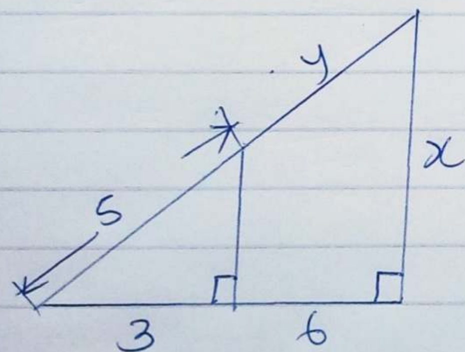
(a)



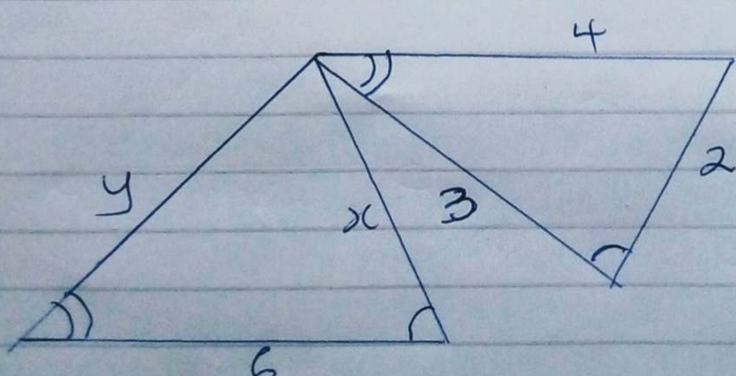

(b)




(c)



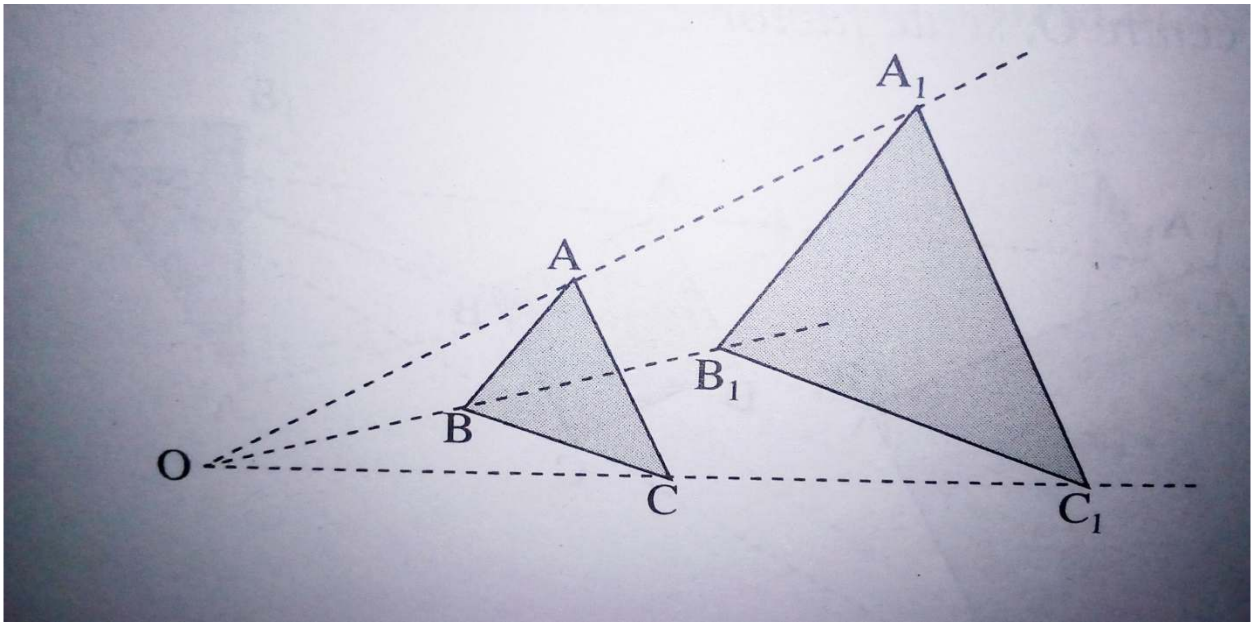

(d)



## ENLARGEMENT

An enlargement is a transformation under which an object is made bigger or smaller

i) **Positive enlargement**




O is the center of enlargement, not necessarily the origin. A'A, C'C and B'B produced must pass through O. The enlargement is positive if both, the image and the object are on the same side of the center of enlargement (coe) .

The image is further away from O than the object if the linear scale factor (lsf) is greater than 1 and it is between the centre and the object if lsf is less than 1. The linear scale factor(lsf or k) is given by the ratio of any corresponding lengths.

$$k=\frac{OA'}{OA}=\frac{OB'}{OB}=\frac{OC'}{OC}$$

OR

$$k = \frac{A'C'}{AC} = \frac{C'B'}{CB} = \frac{B'A'}{BA}$$

Enlargement preserves angles and the ratio of the corresponding sides of the figures before and after enlargement i.e. if AB is twice BC then A'B' will be twice B'C'.

**Note:**

Enlargement is not an Isometry since it does not preserve the size of the of the object.

- Enlargement is specified by the center of enlargement and the linear scale factor are known.

-If the center of enlargement is on the object then that point is invariant.

**Procedure for enlarging the figure.**

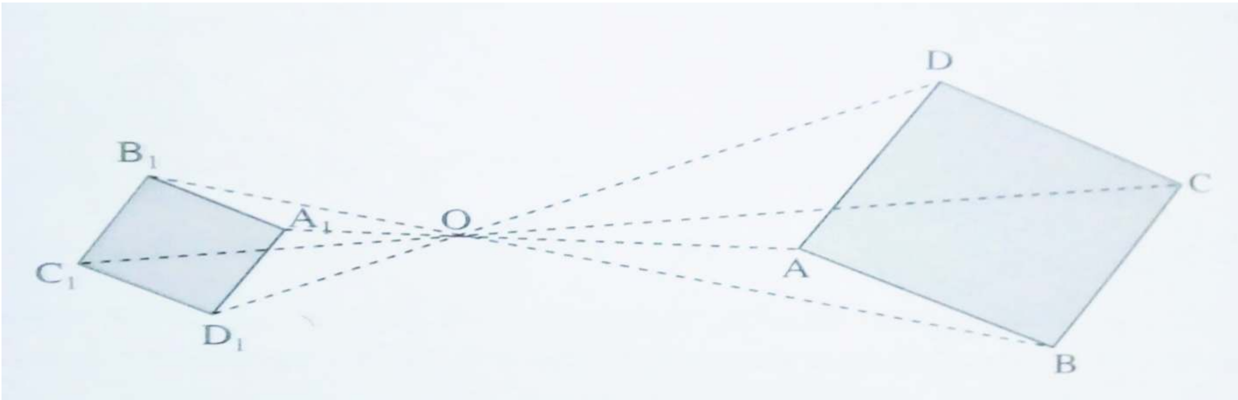
-Join each point to the center of enlargement and produce the line in a particular direction.

-Use the scale factor to locate the image.

**ii) Negative enlargement**

The enlargement is negative if the coe is betweenthe image and the object.

In this case, after joining a point to the center of enlargement, produce the line in the same direction of the center of enlargement. Using the coe, locate the image.




*Fractional scale factor, if the linear scale is less than 1, then the image is smaller than the object. On the other hand, if the linear scale is greater than 1, the image is larger than the object.

In case of a linear scale factor which is positive but less than 1, the image is always nearer to the center of enlargement.

If the linear scale factor is negative and less than 1, the image will be smaller on the opposite side of the center of enlargement.

**Example:**

Plot point A(0, 0), B(9, 0), C(9, 9) and D(0, 9). Find the co-ordinates of A'B'C' and D' where A'B'C'D' is the image of ABCD under an enlargement of linear scale factor $\frac{1}{3}$, center (3,3)

Procedure:

-Join every point to the center of enlargement.

-Multiply the length OA, OB, OC and OD according to **the scale factor, i.e$\frac{1}{3}$.** Hence join the image points to form the image.

**Note that O is the center of enlargement (O is (3,3))**

**Example:**

The vertices of triangle OAB have coordinates O(0, 0), A(0, 2), B(4, 2). Find the coordinates of the triangle O'A'B' after an enlargement, scale factor 2, center O(0,0)

**Example:**

Given, triangle OAB where O is (0, 0), A is (0, 2) and B(4, 2) and an enlargement of scale factor of -2, center (3, 3). Find the co-ordinates of O',A', and B' the vertices of the image under transformation.

**Example:**

Triangle ABC, with coordinates A(6, 2), B(2, 2) and C(5, 4) is mapped into A'B'C' by an enlargement with scale factor $\frac{1}{2}$, center O(0, 0). Find the coordinates of A', B' and C'.

**Finding the center of enlargement and the scale factor.**

**Procedure:**

-The center of enlargement can be found by joining each object point to its image and producing these lines until they intersect. This is the center of enlargement.

-The linear scale factor is given by

K=$\frac{OA'}{OA} = \frac{OB'}{OB}$

**EXERCISE:**

~The coordinates of A, B and C are (2,0), (3,2) and (1,3) respectively. Triangle ABC is mapped onto triangle A' B'C' by an enlargement center O(0,0). Find the coordinates of A', B' and C' if the scale factor is

i) 3 ii) -2 iii) $\frac{1}{2}$

~Square ABCD with coordinates, A(1,1), B(2,1), C(2,2) and D(1,2) mapped onto A'B'C'D' with coordinates A'(3,3), B'(5,3), C'(5,5) and D'(3,5). If the transformation is an enlargement, find the scale factor and the center of enlargement.

~Triangle PQR with coordinates P(3,1), Q(6,1) and R(6,2) maps onto P'Q'R' with coordinates P'(3,4), Q'(-3,4), and R'(-3,2) under an enlargement. Find the center of enlargement and the scale factor.

~On the same axes, plot the squares P(2,2), Q(4,2), R(4,4), S(2,4) and P'(1,1), Q'(2,1), R'(2,2), and S'(1,2). Find the center of enlargement and the linear scale factor by using the graph.

## COMBINARTION OF TRANSFORMATIONS

Two or more transformations may be combined successively (applied one after another) to the object. If a transformation X maps a point P onto P' then P' is denoted by X(P). If a second translation Y is applied to the result of the first then the proceeding image P" is denoted by YX(P)

Note that YX(P) means that X is applied first and then Y.

Denoted by "X followed by Y".

Example:

If Q represents a positive quarter turn about the origin and T the translation given by vector $\binom{5}{3}$. Find the image of triangle ABC under the combined transformation QT, given that the vertices of triangle ABC are A(-2,-1), B(-2,-2), and C(-3,-3). What single translation is equivalent to QT?

**Solution:** Draw the three triangles on the same Cartesian plane and compare triangle ABC with A”B”C”.

Exercise

1. $R_1$ represents a reflection in the x-axis, $H_1$ is a half turn about the origin. Find $H_1R_1$(T) where T is the triangle with vertices at (2,1), (4,1) and (4,2). Describe a single transformation that maps T onto $H_1R_1$(T)

2. Triangle XYZ has coordinates at X(3,2), Y(4,2) and Z(3,4). Its image X’Y’Z’ under an enlargement has vertices at X’(4,3), Y’(6,3) and Z’(4,7). Find the center and scale factor of enlargement.

3. On squared paper, draw triangle ABC where A(1,3), B(1,5) and C(2,5). On the squared paper let X vary from -3 to 6 and Y vary from -6 to 6. Q is the reflection in line x=3. R is a negative quarter turn about the point (5,2).
ABC is mapped onto $A_1B_1C_1$ by P, $A_1B_1C_1$ is mapped onto $A_2B_2C_2$ by Q and $A_2B_2C_2$ is mapped onto $A_3B_3C_3$ by R.
Draw the triangles in your diagram

i) Describe the transformation H where H=HP

ii) Describe the transformation K which maps ABC onto $A_3B_3C_3$.