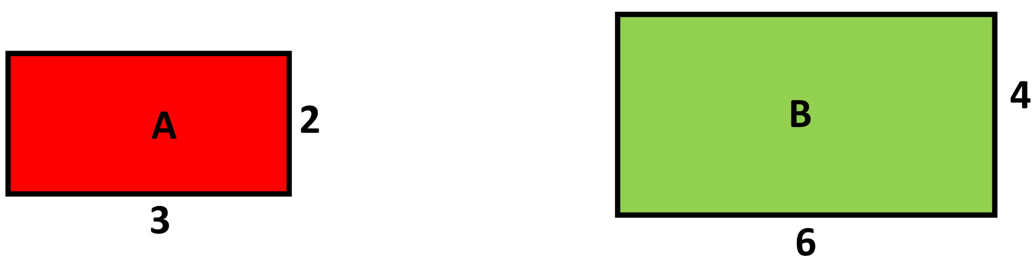
<u>LINEAR SCALE FACTOR, AREA SCALE FACTOR, AND VOLUME SCALE FACTOR.</u>

Linear scale factor (L.S.F)

Two plane figures or two solids are called similar if all corresponding angles are equal and if the ratio of any two corresponding lengths is constant.

The constant is called the linear scale factor. The linear scale factor is the ratio of any two corresponding lengths of similar figures.



A and B are two similar figures. The lengths of sides of figure B are twice the length of the corresponding sides of figure A.

$\therefore$ L.S.F $= \frac{4}{3} = \frac{2}{1} = 2$

Area Scale factor (A.S.F)

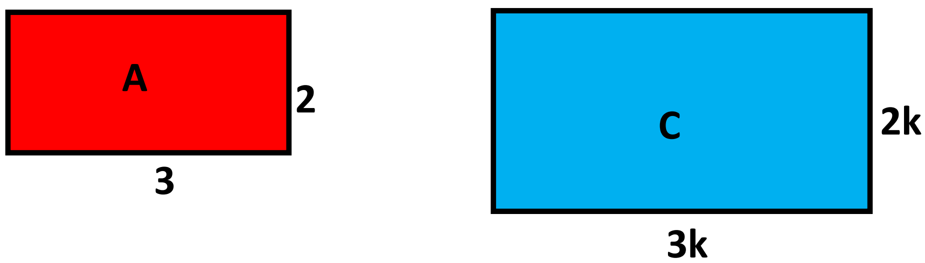
Area of A = 2x3 = 6 square units

Area of B = 6x4 = 24 square units

Ratio of their areas = 24:6 = 2^2

This is linear scale factor squared.

Let us consider rectangle A and similar rectangle C.



L.S.F = $\frac{3k}{3} = \frac{k}{1} = k$

Area scale factor = $\frac{3k \times 2k}{3 \times 2} = k^2$

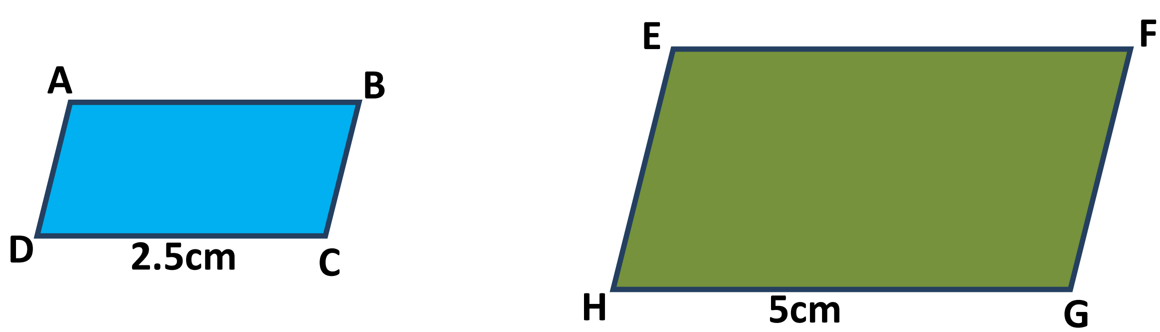
Note: if the L.S.F of two similar figures is k, then the area scale factor is k^2.

This rule applies to all similar figures.

In each of the following three examples, the two figures are similar.

Example:1

Area of parallelogram ABCD is $5.5cm^2$. Find the area of EFGH



Solution:

L.S.F = $\frac{HG}{DC} = \frac{5}{2.5} = \frac{2}{1} = 2$

A.S.F = $(L.S.F)^2 = (2)^2 = 4$

Area of EFGH = 4 x Area of ABCD

= 4 x 5.5

=$22cm^2$

Example:2

A circle x has area of $12.0cm^2$. Find the area of circle y with

i) three times this radius

ii) half this radius

Solution:

i) L.S.F = $\frac{3}{1} = 3$

Area scale factor = $(3)^2 = 9$

Area of circle Y = 9 x 12.0 = $108cm^2$

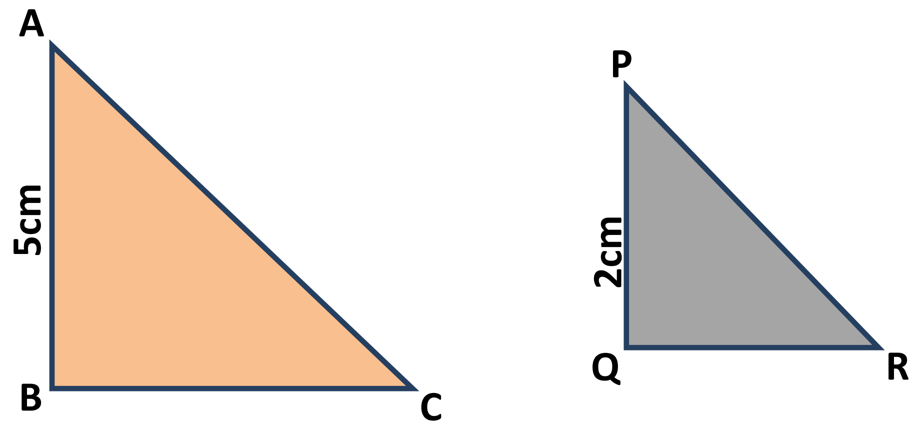
ii) L.S.F = $\frac{1}{2}/_1 = \frac{1}{2}$

Area scale factor is $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Area of Y = $\frac{1}{4}$ x 12 = $3cm^2$

Example:3

Find the area of the triangle ABC, Given the area of the triangle ABC = $150cm^2$



Area scale factor = $(L.S.F)^2 = \left(\frac{2}{5}\right)^2$

Area of PQR $= \left(\frac{4}{25}\right)^2 \times 150$

$= 24cm^2$

<u>Map scales</u>

Any shape on the map is similar to the actual shape on the ground. The scale of the map is the ratio of corresponding lengths (i.e. it is the linear scale factor).

A scale factor of 1:50,000 means 1 unit on the map is equivalent to 50,000 units on the ground.

L.S.F $\left(\frac{map}{ground}\right) = \frac{1}{50,000}$

Area scale factor = $\left(\frac{1}{50,000}\right)^2$

$\frac{Area\ on\ map}{Area\ on\ ground} = (scale)^2 = \left(\frac{1}{50,000}\right)^2$

Hence area of $1cm^2$ on map represents an area of $50,000 \times 50,000cm^2$ on the ground.

$1cm^2 = 50,000 \times 50,000cm^2$

$= 0.5 \times 0.5km^2$

$= 0.25km^2$

<u>Example:</u>

The scale on a map is 1:100,000. If the area of an island on the map is $200cm^2$

i) What is its actual area?

ii) What is its actual area on a map whose scale is 1:50,000?

i) Scale 1:100,000

1cm:100,000cm

$1cm:1km^2$

Area scale factor = $1cm^2 = 1km^2$

$200cm^2 = 200 \times 1km^2$

$= 200km^2$

ii) Scale = 1:50,000

1cm: 50,000

Area scale factor $1cm^2$:(50,000)

$1cm^2 = 0.25km^2$

Let the area on the map be y

$1cm^2$ rep $0.25km^2$

yrep $200km^2$ (from (i) above)

Cross multiply

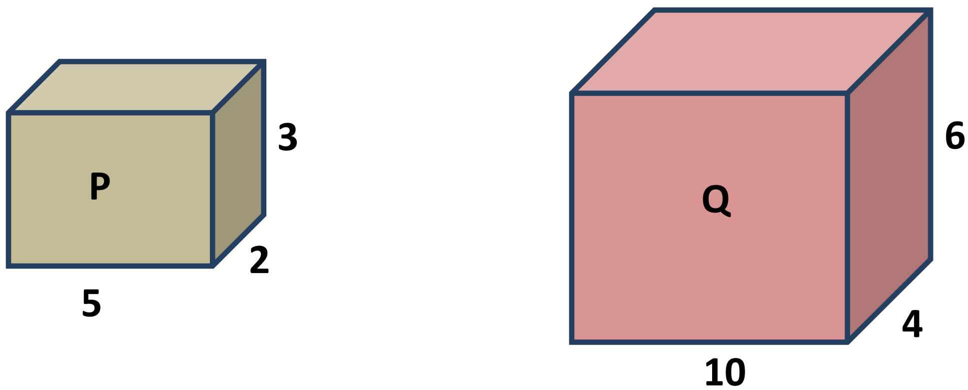
$y \times 0.25km^2 = 1cm^2 \times 200km^2$

y = $\frac{1cm^2 \times 200km^2}{0.25km^2}$

y = 800cm^2

<u>Volume scale factor</u>

Consider two similar cuboids P and Q.



L.S.F = $\frac{4}{2} = \frac{2}{1} = 2$

A.S.F = $\frac{4 \times 6}{2 \times 3} = 2 \times 2 = (2)^2 = 4$

This is true for each corresponding faces.

Volume scale factor (V.S.F) = $\frac{10 \times 4 \times 6}{5 \times 2 \times 3}$

= 2x2x2 = $(2)^3$ = 8

∴ V.S.F = (L.S.F)^3

In general, if L.S.F of two similar solids is K, then the V.S.F is K^3

<u>Example:</u>

Two similar cylinders are such that the height of the larger one is three times that of the smaller one. The smaller one has surface area of $27cm^2$ and volume $6cm^3$.

What are the area and volume of the larger one?

Solution:

L.S.F = 3

A.S.F = $(3)^2$ = 9

V.S.F = $(3)^3$ = 27

Area of the larger one = 9 x Area of smaller one

Area of the larger one = 9 x 27 = $243cm^2$

Volume of larger one = 27 x volume of small one

= 27 x 6 = $162cm^3$

Example:

Two beakers of similar shapes hold $\frac{1}{4}$ litre and 2 litres respectively.

i) The smaller is 4cm high. What is the height of the larger one?

ii) If the Larger one has surface area of $400cm^2$, what is the area of the smaller one?

Solution:

i) The volumes are 2 litres and $\frac{1}{4}$ litres

V.S.F = $\frac{2}{1/4}$ = 8 = $(2)^3$

L.S.F = $\sqrt[3]{V.S.F} = \sqrt[3]{2^3} = 2$

Height of the larger one is 4 x 2 = 8cm.

ii) A.S.F = $(L.S.F)^2 = 2^2 = 4$

Area 0f small one = $\frac{400}{4}$ = $100cm^2$

Exercise:

1. A 1-litre beaker is 15cm high. Find the capacity of a 7.5cm high similar beaker.
2. Two similar rectangular cartons have bases 10cm long and 30cm. The smaller has volume $1200cm^3$. Find the volume of the larger one.
3. The volume of two cubes are $12.5cm^3$ and $100cm^3$. Find their
   i) Linear ratio.
   ii) Area ratio.
4. Two triangles whose bases are 7.5cm and 2.5cm are similar. What is the area of the smaller triangle if the larger one has area $81cm^2$
5. A model of an aero plane constructed to a scale of 1:600 is 56cm long. What is the length of the original aero plane?
6. Two similar solids have linear scale factor 3. If the larger one has a surface area of $36cm^2$ and volume of $12cm^3$, what are the surface area and volume of the other?
7. On a map of scale 1:200 000, a town is represented by an area of $4cm^2$. Calculate in km^2 the area covered by the town on ground?

8. The photograph of a house actually 8m high is 10cm in height.

   a)The front door of the house is 2.4m high. How high is that door on the photograph?

   b) If the area of a window on the photograph is $1.4cm^2$, what is the area of the actual window? (Give your answer to the nearest $0.1m^2$)

9. A scale model of a ship is one two-hundredth as long as the ship itself.

   i) Find the height of the mast and the area of the deck of the ship if the height of the mast and the deck area of the model are 10cm and $900cm^2$ respectively

   ii) If the volume of the hold in the ship is $1600m^3$, find the volume of the hold in mm^3, in the model.

<u>Stay Home</u>

<u>Stay Safe</u>