RATIO AND PROPORTION.

A) RATIO.

The word ratio' is used to describe a fraction. If the ratio of a girl's height to his father's height is 2:3, then he is 2/3 as tall as his father.

Example 1

Change the ratio 4:5 into the form

a) 1: p b) q: 1

a) $4:5 = 1:{}^{5}/_{4}$ b) $4:5 = {}^{4}/_{5}:1$

$= \underline{1:1.25}$ $= \underline{0.8:1}$

Example 2

Divide 60 Oranges between two people A and B in the ratio 5:7.

Consider 60 Oranges as 12 equal parts (i.e. 5+7). Then A receives 5 parts and B receives 7 parts.

$\therefore$ A receives $\frac{5}{12}$ of 60 Oranges = 25 Oranges

B receives $\frac{7}{12}$ of 60 Oranges = 35 Oranges

Example 3.

Divide 400 kg in the ratio 1:3:4. The parts are ${}^{1}/_{8}$, ${}^{3}/_{8}$ and 4/8 (of 400 kg).

i.e 50 kg, 150 kg and 200kg.

Exercise .

Express the following ratios in the form 1 :p

1). 5:6 2). 4:24 3). 3:150

4. A man and a woman share a Coca Cola prize of Shillings 1million between them in the ratio 2:3. The woman shares her part between herself, her mother and her daughter in the ratio 3:2:1.

   How much does her daughter receive?

5. A man and his wife share a sum of money earned in a competition in the ratio 4:1. If the sum of money is trebled, in what ratio should they divide it so that the man doubles his dividend?
6. If x: 4 = 16:x, calculate the positive value of x.
7. If y: 9 = 32:2y, calculate the positive value of y.
8. Shillings 400,000 is divided between Aggie, Bob and Cissy so that Aggie has twice as much as Bob and Bob has three times as much as Cissy. How much does Bob receive?
9. A cake weighing 1100 g has three ingredients: flour, sugar and raisins. There is twice as much flour as sugar and one and a half times as much sugar as raisins. How much flour is there?

B) PROPORTION

The majority of problems where proportion is involved are usually solved by finding the value of a unit quantity.

Example 4.

If a wire of length 2 metres Shillings 10,000, find the cost of a wire of length 35cm.

200 cm costs Sh.10, 000.

1 cm costs $\frac{10,000}{200} = \text{Sh.}50$

35 cm costs 50 x 35 = <u>Sh.1750</u>

Example 5.

Ten men can dig a trench in 5 hours. How long will it take five men to dig the same size trench?

10 men take 5 hours

1 man would take 10 x 5 = 50 hours

5 men would take $^{50}/_{5}$ hours = 10 hours.

Example 6.

40 machines can produce 2400 identical bottle tops in 5hours.At this rate how many bottle tops would 15 machines produce in 9hours?

1 machine would produce (2400/40) = 60 bottle tops in 5 hours.

1 machine would produce (60/5) = 12 bottle tops in 1hour.

15 machines would produce (15 x 12) = 180 bottle tops in 1 hour.

15 machines would produce (180 x 9) = <u>1620</u> bottle tops in 9 hours.

Exercise

1. Six men build a wall in 20 days. How long would it take ten men?
2. Seven milk bottles contain $3^1/_2$ litres of milk between them. How much do four bottles hold?
3. A car uses 15 litres of petrol in 80km.How much fuel does one need to travel from Kampala to Gayaza and back, if Gayaza is 16km away.
4. If it takes 12 men 8days to dig a hole 6 feet deep, how long will it take 16men to dig a hole 10 feet deep?
5. A ship has sufficient food to supply 750 passengers for 4 weeks.
   (i) How long would the same amount of food last for 1000 people?
   (ii) If after 1 week of travel 150 additional passengers boarded the ship, for how long would the remaining food last?

C) DIRECT AND INVERSE PROPORTION.

<u>Direct proportion.</u>

'is proportional to' may be written as '**varies directly with'** or '**is directly proportional to'**. Quantities that vary directly are such that as one increases the other also increases.

Examples.

y is proportional to x implies y = kx ,where k is a constant.

Y is proportional to the square of x implies $y = kx^2$.

i.e when y =kx ,then y/x =k.

<u>Question</u>: y is proportional to the square root of x. Given that y =7.7 when x =17.9,find:

(a) y when x = 22.9

(b) x when y = 10.2.

<u>Answer</u>: $y = k\sqrt{x}$. But since y=7.7 when x=17.9 ,then

$7.7 = k\sqrt{17.9}$.
$k = 7.7/(\sqrt{17.9})$
$k=1.820$.
(a) $y = k\sqrt{x}$, $y = 1.820 \times \sqrt{22.9}$
$= \underline{8.7094}$.

(b) $\sqrt{x} = y/k$, $x = (y/k)^2$
$= (10.2/1.820)^2$
$= \underline{31.4093}$.

<u>Inverse proportion</u>.
'Is inversely proportional to' may also be written as **'varies indirectly with'.**
Quantities that vary inversely are such that as one increases the other decreases.
Examples.
Y is inversely proportional to the square root of x implies
$Y = \frac{k}{\sqrt{x}}$.
y is inversely proportional to the cube of x implies $y = \frac{k}{x^3}$.
i.e when $y = k/x$, then $yx = k$.

<u>Question</u>: If p varies inversely as q and p = 7 when q = 6
(a) Find p when q =3.
(b) Find q when p =10.
<u>Answer</u>**(a)** $p = k/q$, then $k = pq$
$= 7 \times 6$
$= 42$. Hence $p = 42/3$
$= \underline{14}$.
(b) $q = k/p$, thus $q = 42/10$
$= 4.2$.

<u>Exercise</u>:
1) If p varies directly as q, and p = 6 when q = 2. Find
   (a) p when q = 3/2,
   (b) q when p = 10.
2) Given that x is inversely proportional to y ,and x = 4 when y = 5.
   Find (a) x when y = 2.5,
   (b) y when x = 1.25.
3) The electrical resistance of a wire varies as the square of the diameter of the cross-section .If the resistance is 2.4 ohms when the diameter is 0.1cm, find the diameter of a wire whose resistance is 0.65 ohms.

D).FOREIN EXCHANGE (CURRENCY CONVERSION)

The buying rate for the forex bureau is the selling rate for the customer and the selling rate is the buying rate for the customer.

Example:

A certain forex bureau on the 14th January offered the following rates for a pound sterling.

Buy at Ush.3,165 and Sell at Ush.3,200.

a) How many pounds would a businessman get for 9 million shillings?

b) How many Uganda shillings would a tourist get for £500?

(a) The trader will buy pounds at Ush.3200.(The bureau's selling rate)

Ush.3, 200 buy £ 1.

Ush.1 would buy £ 1/3,200.

Ush. 9,000,000 buy $\frac{9,000,000}{3,200}$ = £ 2,812.5

= £ 2,812 and 50 pence.

(b) The tourist must sell his pounds at Ush.3,165.(The buying rate for the Bureau).

£ 1 costs Ush.3,165

£ 500 cost 500 x 3,165 = Ush.1, 582,500.

Exercise.

1. You are required to read the forex lists on the business page of today's monitor news paper and select a forex bureau of your choice. Use the rates for the pound sterling to answer the questions below.

   a) How many pounds would you get for:-
      i) 30,000/= ii) 450,000/= iii) 20 millions?

   b) How many Uganda shillings would you get for:-
      i) £200 ii) £10,000 iii) £500,000?

2. An aid worker promises to donate a mattress worth shs.89, 000 and its cloth cover costing Shs. 25,500 to a family in the war ravaged area in northern Uganda.
   Given that 1U.S dollar ($), = Ush 1,822/= and £ 1 = U.S $ 1.76, advise her on the cost in pounds sterling of the:
   i) mattress
   ii) total cost of the mattress and its cover

3. Convert U.S $ 400 (US dollars) to pounds sterling, (£), if U.S $1 = Ush. 1,825 and £1 = Ush 3,210.

E). Map Scales.

When a map is drawn to a scale of 1:50,000,
1cm on the map represents 50,000cm on the land.

Example.

A map is drawn to a scale of 1:20,000.Calculate:
(a) the length of a river which appears as 5cm long on the map,
(b) the length on the map of a lake which is 8km long,
(c) the dimensions of a farm which appears as 10cm by 23cm on the map,
(d) the area on the map of a village which is 2.4 km^2.

(a) 1cm represents 20,000cm

5cm represents 5 x 20,000 = 100,000cm.

The river is 100,000cm long = 1km.

(b) 20,000cm are represented by 1cm

8km = 8 x 100,000 =800,000cm

800,000cm are represented by $\frac{800,000}{20,000} = 40$cm.

(c) 10cm = 10 x 20,000 = 200,000cm = 2km.
23cm = 23 x 20,000 = 460,000cm = 4.6 km.
The farm is of dimensions 2km by 4.6km.

(c) 1km = 100,000cm
$1km^2 = (100,000)^2cm^2 = 10^{10}\ cm^2$.
$2.4\ km^2 = 2.4 \times 10^{10}\ cm^2$.
But the scale is 1cm to 20,000cm
$1cm^2$ to $(20,000)^2\ cm^2 = 4 \times 10^8\ cm^2$
Since $4 \times 10^8\ cm^2$ on land are represented by $1cm^2$ on the map.
$2.4 \times 10^{10}\ cm^2$ are represented by $\frac{2.4 \times 10^{10}}{4 \times 10^8}\ cm^2 = 60cm^2$.
The area of the village will be $60cm^2$ on the map.

Exercise.

1) The scale of a map is 1:10,000.Calculate:
   (a) the length of a road which appears as 11.5cm on the map,
   (b) the length on the map of a wall stretching 1 km long,
   (c) the area of a school which is represented by a rectangle on the map measuring 8.5cm by 10cm.
   (d) the area on the map of a farm whose actual area is $5km^2$.