MATRICES

Summary:

1. *A matrix is a bracket with numbers in rows and columns. Thus* $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ *and* $\begin{pmatrix} 1 & 2 & 0 \\ -2 & 3 & 5 \end{pmatrix}$ *are matrices.*

2. *The order of a matrix with* ***m*** *rows and* ***n*** *columns is written as* $m \times n$ *and is called an* $m \times n$ *matrix.*

3. *The numbers in a matrix are called its elements or entries.*

4. (i) *To add and subtract matrices of the same order, add and subtract corresponding elements*

(ii) *Two matrices are equal if their corresponding elements are equal*

(iii) *A scalar* ***k*** *multiplied by a matrix* $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ *is treated as follows:*

$$kA = k\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix}$$

(iv) *Matrix multiplication is treated as follows:*

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix} = \begin{pmatrix} aw + by & ax + bz \\ cw + dy & cx + dz \end{pmatrix}$$

Matrix product $AB \neq BA$ *.*

Matrix product ***AB*** *can be done if the number of columns in* ***A*** *is equal to the number of row in* ***B.***

If a $\mathbf{2 \times 5}$ *matrix is multiplied by a* $\mathbf{5 \times 3}$ *matrix, then the resulting matrix has the outer dimensions* ***(****The new matrix is of order* $\mathbf{2 \times 3}$***)***

5. *An identity matrix* ***I*** *is a matrix with ones along the major diagonal and zeros elsewhere. Thus a* $\mathbf{2 \times 2}$ *identity matrix is given by* $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

6. If matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,*, then:*

*(i)Determinant of **A (Det A)*** $= ad - cb$

*(ii) Adjoint of matrix **A** =* $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

*(iii) The inverse of **A**,* $(A^{-1}) = \frac{1}{detA}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

(iv) The product $AA^{-1} = I$

(v) A matrix multiplied by an identity matrix remains unchanged

7. A singular matrix is the one whose determinant is zero and thus has no inverse.

EXAMPLES:

1. If matrix $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$,

*(a) State the order of matrix **A***

(b) Determine the:

*(i) determinant of **A***

*(ii) inverse of **A***

2. Given that matrix $P = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & -4 \\ 3 & 2 \end{pmatrix}$ *and* $R = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, *find:*

(i) P + Q (ii) Q – R (iii) 3P – 2Q + R (iv) PQ (v) QP (vi) QRP

(vii) P^2 *(viii)* Q^2 *(ix)* $(P+Q)^2$ *(x) 3P – 2I where **I** is a* 2×2 *identity matrix*

3. *Given that matrix* $A = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ *and* $B = \begin{pmatrix} 4 & 8 \\ 2 & 6 \end{pmatrix}$, *find* ***det (AB)***

4. *Given that matrix* $P = \begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$, $Q = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ *and* $R = P^2Q$, *find* R^{-1}

5. *Find the order of the resulting matrix when a* $\mathbf{3 \times 4}$ *matrix is multiplied by a* $\mathbf{4 \times 5}$ *matrix*

6. *If matrix* $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}$ *and* $B = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$,

(i) *determine the order of matrix* ***AB***

(ii) *find matrix* ***AB***

7. *If matrix* $P = \begin{pmatrix} 3 & 1 & 7 \\ -1 & 3 & 2 \end{pmatrix}$, $Q = \begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 5 & 2 \end{pmatrix}$ *and* $R = PQ$,

(i) *determine the order of matrix* ***R***

(ii) *find matrix* ***R***

8. *Given the matrix equation* ***AY = B,*** *use matrix inversion method to find:*

(i) *matrix* ***Y*** ***(ii)*** *matrix* ***A***

9. *If matrix* $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, *find matrix* ***B*** *such that* $AB = \begin{pmatrix} 17 & 13 \\ 21 & 14 \end{pmatrix}$

10. *If matrix* $B = \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$, *find matrix* ***A*** *such that* $AB = \begin{pmatrix} 10 & 4 \\ -5 & 9 \end{pmatrix}$

11. *If matrix* $P = \begin{pmatrix} 4 & 3 \\ -2 & 1 \end{pmatrix}$, $Q = \begin{pmatrix} 11 \\ 7 \end{pmatrix}$ *and* $PR = Q$, *determine:*

(i) the order of matrix ***R***

(ii) matrix ***R***

12. *If matrix* $P = \begin{pmatrix} 3 & 1 \\ -1 & 3 \end{pmatrix}$, *find matrix* ***A*** *such that* $AP = I$, *where* ***I*** *is a* ***2*** $\times$ ***2*** *identity matrix.*

13. *If matrix* $A = \begin{pmatrix} x & -7 \\ 4 & 6y \end{pmatrix}$ *and* $B = \begin{pmatrix} 17 - y & -21 \\ 12 & 36 \end{pmatrix}$, *find the values* ***x*** *and* ***y*** *such that* ***3A = B***

14. *Find the values of* ***a*** *and* ***b*** *such that* $\begin{pmatrix} 3 & b \\ 4 & a \end{pmatrix}\begin{pmatrix} 7a \\ 2 \end{pmatrix} = \begin{pmatrix} 43 \\ 30 \end{pmatrix}$

15. *Find the values of* ***k*** *and* ***n*** *such that* $\begin{pmatrix} 2 & 4 \\ -3 & 3 \end{pmatrix} + k\begin{pmatrix} 3 & 1 \\ 0 & n \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -3 & -1 \end{pmatrix}$

16. *Find the values of* ***x*** *and* ***y*** *such that* $(1 \quad 3 \quad 2)\begin{pmatrix} 4 & 3 \\ x & 2 \\ 10 & y \end{pmatrix} = (39 \quad 25)$

17. *Find the values of* ***x*** *and* ***y*** *such that* $\begin{pmatrix} 4 & 1 \\ x & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

18. *Given that matrix* $A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$, *find the value of* λ *such that*

$A^2 + \lambda I = 5A$, *where* ***I*** *is a* ***2*** $\times$ ***2*** *identity matrix*

19. *Given that matrix* $A = \begin{pmatrix} x^2 & 3 \\ 1 & 3x \end{pmatrix}$ *and* $B = \begin{pmatrix} 3 & 6 \\ 2 & x \end{pmatrix}$, *find the possible values of* ***x*** *such that* ***AB = BA***

20. *Find the values of* ***x*** *for which the matrix* $\begin{pmatrix} x & 6 \\ 8 & 3x \end{pmatrix}$ *has no inverse*

21. *Find the values of* ***x*** *for which the matrix* $\begin{pmatrix} x & 3 \\ 4 & x-4 \end{pmatrix}$ *is singular*

22. *Find the values of* ***x*** *for which the matrix* $\begin{pmatrix} 2x & 3x \\ 2 & x \end{pmatrix}$ *is singular*

23. *Given that matrix* $M = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, *find the values of* λ *such that the matrix* $(M - \lambda I)$ *is singular, where* ***I*** *is a* 2×2 *identity matrix*

4. Given that $M = \begin{pmatrix} -1 & 2 \\ 2 & 4 \end{pmatrix}$, *show that* $\det(M^{-1}) = \dfrac{1}{\det M}$

7. Given that m $A = \begin{pmatrix} 2 & -1 \\ 0 & -1 \end{pmatrix}$ $B = \begin{pmatrix} 6 & 8 \\ 10 & -12 \end{pmatrix}$, *find* $(AB)^{-1}$ *and* $B^{-1}A^{-1}$

1. If $A = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$ *and* I *is a* 2×2 *identity matrix, prove that* $A^2 = 7A + 2I$

2.

6. Given the matrices $A = \begin{pmatrix} 4\cdot 5 & 1 \\ 0 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$**,** find matrix **M** such that

$3M - 2I = 2A - B$, where ***I*** is a $\mathbf{2 \times 2}$ identity matrix

SOLUTION TO SIMULTANEOUS EQUATIONS BY MATRIX METHOD

Summary:

The following steps apply in solving simultaneous equation using matrix method:

(i) Write the equations in matrix form

(ii) Find the inverse of the **2 ´ 2** matrix

(iii) Pre multiply both sides of the matrix equation by the inverse matrix

EXAMPLES:

1. Use matrix method to solve the following simultaneous equations:

(i) $x - y = 5$
$3x + 2y = 5$

(ii) $2x - 5y + 14 = 0$
$4x + 3y - 11 = 0$

(iii) $4x + 3y = 24$
$2y - 3y = -1$

(iv) $x + y = 15$
$\frac{x}{3} + \frac{y}{9} = 3$

2. Find the inverse of $\begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}$, hence solve the simultaneous equations

$3x + 2y = 12$
$4x + 5y = 23$

3. Tom bought **3** pens and **2** books at **Shs 4,800.** Bob bought **5** pens and **4** books from the same shop at **Shs 9,000.**

(i) Form two equations to represent the above information

(ii) Use matrix method to find the cost of each pen and that of each book

(iii) How much would Ben pay for **10** pens and **6** books

4. Shs **4000** can buy **10** bans and **5**cakes or **4**bans and **10**cakes.

(i) Form two equations to represent the above information

(ii) Find by matrix method the cost of each ban and that of each cake.

MATRIX WORD PROBLEMS

1. Tom, Bob and Ben went to a supermarket for shopping .
Tom bought **3** pens and **5** books and **4** rulers
Bob bought **4** pens and **3** books and **2** rulers
Ben bought **6** pens and **3** rulers
The cost of a pen is **Shs 500,** a book is **Shs 800** and a ruler is **Shs 1500.**

(a) Write down:

(i) a **3 ´ 3** matrix for the items bought by the three boys.

(ii) a **3 ´ 1** cost matrix for each item

(b) Use matrix multiplication to find the amount of money spent by each boy

2. In a swimming competition, **7** points were awarded for each first-place finish, **4** points for second and **2** points for third.
Senior one had **4** first place finishes, **7** second place finishes and **3** third place finishes.

Senior two had **8** first place finishes, **9** second place finishes and **1** third place finish.

Senior three had **10** first place finishes, **5** second place finishes and **3** third place finishes.

*Senior four had **3** first place finishes**,** **3** second place finishes and **6** third place finishes.*

***(a)** Write down:*

*(i) a **4 ´ 3** matrix for the number of finishes each class had.*

*(ii) a **3 ´ 1** matrix for the points awarded for each finish*

***(b)**Use matrix multiplication to determine the winner of the competition*

***3.** Shops **A, B, C,** and **D** ordered for balls, bats and gloves as follows:*

	Balls	Bats	Gloves
Shop A	70	30	50
Shop B	60	20	25
Shop C	40	15	10
Shop D	50	40	30

*The balls cost **Shs 5,000** each, bats **Shs 3,000** each and gloves **Shs 2,000** each*

***(a)** Write down:*

*(i) a **4** × **3** matrix for the items ordered by each shop.*

*(ii) a **3** × **1** cost matrix for each item*

***(b)** By matrix multiplication**,** find the total cost of the items for each shop*

***(c)** If the supplier had to pay a tax of **20%** of the cost of the items sold, find his expenditure on the order.*

EER:

1. *Given that **I** is an identity matrix of order **2 × 2** and matrix* $A = \begin{pmatrix} 2 & -1 \\ -2 & -5 \end{pmatrix}$,

*find matrix **B = A+2I***

2. *Find the inverse of matrix* $P = \begin{pmatrix} -2 & 3 \\ -4 & 5 \end{pmatrix}$

3. *Use matrix method to solve the simultaneous equations:*

$$\frac{x}{2} + \frac{y}{3} = 5$$
$$\frac{x}{3} + \frac{y}{4} = 1$$

8. *A hotel rents double rooms at **Shs 40,000** per day and single rooms at **Shs 25,000** per day. If **14** rooms were rented one day for a total of **Shs 470,000***

(i) *Form two equations to represent the above information*

(ii) *Find by matrix method how many rooms of each kind were rented.*

4. *In the morning, **5** breads and **8** cakes were bought.*

*In the afternoon, **7** breads and **6** cakes were bought.*

*The cost of a bread is **Shs 4000** and a cake is **Shs 1200***

(a) *Write down:*

(i) *a **2 × 2** matrix for the bought items*

(ii) *a **2 × 1** cost matrix for each item*

(b) *Use matrix multiplication to find the expenditure in each case.*

17. Given that matrix $A = \begin{pmatrix} 1 & 1 \\ x & y \end{pmatrix}$, *find the values of **x** and **y** such that*

$A^2 = I$, *where **I** is a **2 × 2** identity matrix*

4. Given that $P = \begin{pmatrix} 6 & -4 \\ 2 & -1 \end{pmatrix}$ *and* $PQ = \begin{pmatrix} 16 & -18 \\ 6 & -5 \end{pmatrix}$, *find:*

*(i) the inverse of **P**.*

(ii) matrix $Q = P^{-1}[\,PQ\,]$.

*5. Find the values of **x** for which the matrix* $\begin{pmatrix} x & x+9 \\ 2 & x+5 \end{pmatrix}$ *has no inverse*

*19. Find the values of **x** for which the matrix* $\begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$ *is singular*

*19. Find the values of **x** for which the matrix* $\begin{pmatrix} x & 2x \\ x-1 & x+1 \end{pmatrix}$ *is singular*

*19. Find the values of **x** for which the matrix* $\begin{pmatrix} x-5 & 3 \\ -2 & x \end{pmatrix}$ *is singular*

*19. Find the values of **x** for which the matrix* $\begin{pmatrix} x & 4 \\ 1 & x-3 \end{pmatrix}$ *is singular*

21. Given that matrix $M = \begin{pmatrix} 2 & -1 \\ -6 & 1 \end{pmatrix}$, *find the values of **k** such that the matrix* $(kI - M)$ *is singular, where **I** is a **2 × 2** identity matrix*

21. Given that matrix $A = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$, *find the values of* λ *such that the matrix* $(A - \lambda I)$ *is singular, where **I** is a **2 × 2** identity matrix*

*16. Find the values of **x** and **y** such that* $(1 \quad 3)\begin{pmatrix} 4 & y \\ x & 2 \end{pmatrix} = (7 \quad 7)$

2. Given that matrix $P = \begin{pmatrix} x+7 & x \\ 3 & 0 \end{pmatrix}$, $Q = \begin{pmatrix} x-1 & 0 \\ 2 & 2 \end{pmatrix}$ *and* $R = P + Q$, *find the value of **x** for which the determinant of **R** is **2***

21. Given that matrix $P = \begin{pmatrix} 4x+1 & 3x \\ 2x+1 & 2x \end{pmatrix}$, *find the values of **x** for which the determinant of **P** is **6***

*6. **Shs 244,000** can buy **5** bans and **6**cakes, while **Shs 356000** can buy **7** bans*

*and **9**cakes. Find by matrix method the cost of each ban and that of a cake.*

*7. Find the values of **y** for which the matrix* $\begin{pmatrix} 2y & 5 \\ 4 & y+\frac{1}{y} \end{pmatrix}$ *is singular*

8. Given that matrix $A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$, *find the value of* λ *such that*

$A^2 + \lambda I = 5A$, *where **I** is a* 2×2 *identity matrix*

9. Given the matrix $A = \begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$,, *find the values of **x** and **y** such that*

$xA + yI = A^2$, *where **I** is a* 2×2 *identity matrix.*

9. Given the matrix $A = \begin{pmatrix} 0 & 3 \\ 1 & -2 \end{pmatrix}$,, *find the values of **x** and **y** such that*

$$A^2 = \begin{pmatrix} x & -6 \\ -2 & y \end{pmatrix}$$

10. *Bob and Ben went to a supermarket for shopping.*

*Bob bought **2 kg** of sugar**, 4** bars of soap**, 5** counter books and one bottle of cooking oil.*

*Ben bought **5 kg** of sugar**, 3** bars of soap and a dozen of counter books.*

*The cost of sugar per kg was **Shs 1,500,** a bar of soap was **Shs 1,000,** a counter book was **Shs 3,000** and a bottle of cooking oil was **Shs 2,000.***

(a) *Write down:*

(i) *a 2×4 matrix for the items bought by the two people.*

(ii) *a 4×1 cost matrix for each item*

(b) *Calculate the:*

(i) *expenditure of each person by matrix multiplication*

(ii) *total expenditure of both Bob and Ben*

(c) *How much did Ben spend than Bob*

3. *A charity organization donated Ball pens, exercise books, graph books and table books to senior four, three and two Classes of a school as below;*

*Senior four students got **2** ball pens, **12** exercise books, **3** graph books and **1** table book each.*

*Senior three students got **2** ball pens, **8** exercise books, **1** graph books and **1** table book each.*

*Senior two students got **1** ball pens**, 6** exercise books and **1** table book each*

*There are **100** students in senior four, **120** in senior three and **130** students in senior two.*

The organization bought the items at the following rates:

*Ball pens at **Shs500** each, Exercise books at **Shs1500** each, graph book at **Shs 2000** each and table books at **Sh.6000** each.*

***(a)** Write down*

***(i) 1 ´ 3** matrix for the number of students.*

***(ii) 3 ´4** matrix for the items*

***(iii) 4 ´1** cost matrix.*

***(b)**By matrix multiplication, determine the*

***(i)** number of items of each type distributed.*

***(ii)** total amount spent by the organization in acquiring the items.*

***(c)** If the organization had to pay **5%VAT** on the items bought, determine the total amount spent.*

10. *If matrix* $B = \begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix}$, *find matrix **A** such that* $AB = \begin{pmatrix} 17 & 13 \\ 21 & 14 \end{pmatrix}$

9. *If matrix* $P = \begin{pmatrix} 3 & -4 \\ -2 & 3 \end{pmatrix}$, *find matrix **A** such that* $AP = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

15. If $\begin{pmatrix} x & -2 \\ -1 & y \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 18 & 0 \\ 0 & 18 \end{pmatrix}$, *find x and y*

16. Given that $\begin{pmatrix} 3 & x \\ 5 & 4 \end{pmatrix}\begin{pmatrix} 1 & y \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ 13 & 7 \end{pmatrix}$*, find x and y.*

1. *Given that* $A = \begin{pmatrix} 2 & -7 \\ 1 & -4 \end{pmatrix}$*,* $B = \begin{pmatrix} 8 & 3 \\ 0 & 2 \end{pmatrix}$ and $C = BA$, find;
2. *i)* $C + 3B$
3. *ii)* C^{-1}

Give that $P = \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$*,* $Q = \begin{pmatrix} -1 & 1 \\ 3 & 2 \end{pmatrix}$ *and* $R = \begin{pmatrix} 4 & 6 \\ 10 & 15 \end{pmatrix}$*, find matrix T such*

$T = P^2 + 3Q - R$

4. *If* $M = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$*;*

a) Determine ; *i)* M^2 *ii)* M^3

b) identify matrix M^2

12. Four Secondary schools football teams of Ntare H.S, Layibi College, Mvara S.S and Kitende S.S qualified for a football tournament , which was played in two rounds with other teams.

First round

Ntare H.S won three matches, drew one and lost one match.

Layibi college won two matches, drew one and lost two matches

Mvara S.S won one month , drew three and lost one match.

Kitende S.S won four matches , drew one and lost no match.

Second round:

Ntare H.S won three matches , drew two and lost no match.

Layibi college won two matches, drew two and lost one match.

Mvara S.S won no match, drew three and lost two matches.

Kitende S.S won three matches, drew two and lost no match.

a) Write down:

(i) a 4 x 3 matrix to show the performance of the four teams in each of the two rounds. (02 marks)

(ii) a 4 x 3 matrix which shows the overall performance of the teams in both rounds. (02 marks)

b) If three points are awarded for a win, one point for a draw and no point for a loss, use matrix multiplication to determine which school won the tournament.

(03 marks)

c) Given that MTN donated sh. 3, 450,000 to be shared by the four teams according to the ratio of their points scored in the tournament, find how much money each team got.
(05 marks)