## S. 2 ,S. 3 \& S. 4 WORK <br> 11. INEQUALITIES

## DEFINITION;

- AN Inequality is statement that two numbers or expressions are not equal.
- Recall that an equation will always have equal signs and at least one unknown (letter).
- Examples of equations:
$\mathrm{A}+\mathrm{X}=5 ; \quad 2 \mathrm{x}-10=8$ e.t.c.
- We use inquality symbols to write inqualities.
- Just like with equations, the solutions of inqualities are values that the the inqualities true.
OBJECTIVES:
By end of this topic students should be able to:

1. Determine real number solutions of inqualities.
2. Graph solutions of inqualities on a number line.
3. Write inqualities using the inquality symbols.
4. Describe the graph of the solution an inquality on a number line.
5. Be.familiar with the notations $\mathrm{a}>\mathrm{b}$ and $\mathrm{a}<\mathrm{b}$ e.t.c.
6. Identify the effect of adding, subtracting, multiplying, and dividing by positive and negative numbers.
7. Use the addition, subtracion, multiplication, and divition properties of inqualities to solve inqualities.
8. Solve inqualities using two or more operations.
9. Solve and graph compound inqualities.
10. Translate and solve problems using inqualities.

## POINTS TO NOTE:

- For any real numbers a and b , either $\mathrm{a}=\mathrm{b}, \mathrm{a}<\mathrm{b}$, or $\mathrm{a}>\mathrm{b}$
- The grah of inquality is the set of all real numbers that make the inquality true.
- Adding or subtracting a positive number from both sides of an inquality does not change the direction of the inquality symbol.
- Multiplying or dividing both sides of an inquality by a positive real number does not change the direction of the inquality symbol.
- Multiplying or divid ing both sides of the inquality by a negative real number reverses the direction of the inquality symbol.
SOLVING INQUALITIES USING TWO OR MORE OPERATIONS:
- TO solve inqualities with two or more operations, combine like terms where necessary.
Then use the inquality properties to get the variable terms on one side of the inquality symbol.
Apply the distributive property if the inquality contains brakects.
SOLVING COMPOUND INQUALITIES:
- A compound inquality written with and is a conjunction. The solution of a conjunction includes all the real numbers that are in the solution of both parts of the inquality.
- A compound inquality written with or is a disjunction. The solution of a disjunction includes all real numbers that are in the solution of either part of the inquality.
- Recall that an equation will always have an equal sign and at least one unknown (letter).
- Examples: $\mathrm{a}+2=5,2 \mathrm{x}-5=10$ e.t.c
- An inequality has signs such as $<,>, \leq$, or $\geq$ symbols of inequalities

| Symbol | Meaning. |
| :--- | :--- |
| $<$ | Less than. |
| $\leq$ | Less than or equal to. |
| $>$ | Greater than. |
| $\geq$ | Greater than or equal to. |

- Illustration of the meaning of inequalities.
- We may use a number line to illustrate the above suppose x is such that.
(i) $x>5$
(ii) $x<7$
(iii) $\mathrm{x} \leq 3$
(iv) $x \geq-2$
- We represent possible values of $x$ on number lines
i.e
(i)

for
(ii)

for
(iii)

for


NB: (i) $\quad \mathrm{x}$ can take all valves shown by the line and the solid $\operatorname{dot}(\mathrm{s})$

or
(ii) x cannot take the values represented by hollow $\operatorname{dot}(\mathrm{s})$ $\qquad$ or
or $\square$

| Symbol | type of dot |
| :--- | :---: |
| $<$ | $\bigcirc$ |
| $>$ | $\bigcirc$ |
| $\leq$ | 0 |
| $\geq$ | 0 |

- Worked examples

1. Represent all the values of x for which $-1 \leq \mathrm{x}<3$ on a number line.

## Solution:

- at -1 we shall have a solid dot ie
- at 3 we shall have a hollow dot ie $\bigcirc$


3. Represent $x>0$ on a number line

## Solution:

- at zero we shall have a hollow dot ie $\bigcirc$


4. Represent $-2 \leq x \leq 3$ on a number line.

## Solution:

- at -2 we shall have a solid dot
- at 3 we shall also have a solid dot


5. State the inequalities represented by the following:


## Solution:

Since we have a hollow dot at 2 , and the line is on left hand side of 2 , therefore the values of x must be less than 2 . Hence $\mathrm{x}<2$ is the inequality.
6. State the inequality represented by the following


## Solution:

- at -4 we have a hollow dot and the line is on the right hand side of -4 , therefore 4 must be less than the values of $x$.
- at 2 we have a solid dot, implying that 2 is included among the values of x . therefore $-4<\mathrm{x} \leq 2$ is the inequality.

7. State the inequality represented by the following.


$$
\begin{array}{llllllllllllll}
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7
\end{array}
$$

## Solution:

- at -3 we have a solid dot, implying that -3 is included among the values of $x$.
- at 4 we also have a solid dot, implying that 4 is included among the values of x . therefore $-3 \leq \mathrm{x} \leq 4$ is the inequality.
- $\quad$ Solving inequalities.
- The rules for solving inequalities are same as those for equations with the following exceptions:
(i) when multiplying or dividing an inequality by a negative number, the sign must be reversed.
(ii) When interchanging the LHS and RHS the inequality sign must be reversed.
- Illustrations of the above exceptions:

1. Solve: $-2 \mathrm{x} \leq 4$

## Solution:

$=\quad \frac{-2 x}{-2} \geq \frac{4}{-2} \quad$ : the sign $\underline{\text { must }}$ be reversed at the step of dividing by a negative number.
$=\quad \mathrm{x} \geq-2$
2. Solve $-\mathrm{x}<10$

## Solution:

$=\quad-1 \mathrm{x}-\mathrm{x}>10 \mathrm{x}-1$ : note change of sign from $\langle\mathrm{to}\rangle$.
$=\quad \mathrm{x}>-10$
3. Rewrite the following inequalities:
(i) $-2 \leq x$
(ii) $0>x$
(iii) $3<x$

## Solution:

(i) $-2 \leq x$

$$
\Rightarrow \quad x \geq-2 \quad: \text { note the change of sign from } \leq \text { to } \geq .
$$

(ii) $0>x$
$\Rightarrow \quad x>0 \quad:$ note the change of sign from $>$ to $<$.
(iii) $3<x$
$\Rightarrow \quad x>3 \quad:$ note the change of sign from $\langle$ to $>$.

- Worked examples

1. Solve the inequality: $x+8 \leq 12$

## Solution:

$$
\begin{aligned}
& =\quad x+8-8 \leq 12-8: \text { by subtracting } 8 \text { from both sides } \\
& \therefore \quad x \leq 4
\end{aligned}
$$

2. Solve the inequality: $4 \mathrm{x}+4>16$

## Solution:

$$
\begin{aligned}
& =4 x+4-4>16-4: \text { by subtracting } 4 \text { from both sides } \\
& =\quad \frac{-4 x}{4} \geq \frac{12}{4} \quad: \text { by dividing } 4 \text { on both sides } \\
& \therefore \quad \mathbf{x}>3
\end{aligned}
$$

3. Solve the inequality: $7 \mathrm{y}-3 \leq 25$

## Solution:

$$
\begin{array}{ll}
=7 y-3+3 \leq 25+3 & : \text { by adding } 3 \text { on both sides } \\
= & \frac{7 y}{7} \leq \frac{28}{7} \\
= & \mathbf{y} \leq \mathbf{4}
\end{array}
$$

4. Solve the inequality: $1 / 2 x \geq 5$

## Solution:

$$
\begin{aligned}
& =\quad 2 \times 1 / 2 \times \geq 5 \times 2 \quad: \text { by multiplying by } 2 \text { on both sides } \\
& =\quad \mathbf{x} \geq \mathbf{1 0}
\end{aligned}
$$

5. Solve the inequality : $6(5-2 x)+8 \geq 0$

## Solution:

$$
\begin{array}{ll}
= & 6 \times 5-6 \times 2 \times+8 \geq 0 \\
= & 30-12 \times+8 \geq 0 \\
= & 38-12 \times \geq 0 \\
= & : \text { by the brackets. } \\
= & \frac{-12 x}{-12} \leq \frac{-38}{-12} \\
& \\
& \quad \text { by collecting like terms. } \\
& \text { the sign from } \geq \text { to } \leq .
\end{array}
$$

6. Solve the inequalities: $\frac{3}{x-1}>\frac{2}{x-5}$

## Solution:

$$
\begin{array}{cl}
3(x-5)>2(x-1) & : \text { by cross-multiplying. } \\
3 x-15>2 x-2 & : \text { by opening the brackets. } \\
3 x-2 x>15-2 & \text { : by collecting like terms. } \\
x>13 . &
\end{array}
$$

7. Solve the inequality : $\frac{3 x}{2}-\frac{2}{3}(1-2 \mathrm{x})<5$.

## Solution:

| $\frac{3 x}{2}$ |  |
| :---: | :---: |
| 1 | $-\begin{array}{c}\frac{2}{3}(1-2 \mathrm{x}) \\ 2\end{array}$ |
| $\begin{array}{l}5 . \\ \\ \text { : identify the number of terms in the inequality }\end{array}$ |  |
| (there are 3 terms). |  |

$6 \times \frac{3 x}{2}-\frac{2}{3}(1-2 \mathrm{x}) \times 6<5 \times 6$ : by multiplying of the LCM of 2 and 3 by all the terms in inequality.
$3 \times 3 \mathbf{x}-2 \times 3(1-2 \mathbf{x})<30$
$9 \mathrm{x}-6(1-2 \mathrm{x})<30$
$9 x-6+12 x<30 \quad:$ by simplifying and opening the brackets.
$21 \mathrm{x}<36$ : by collecting like terms
$\frac{21 x}{21}<\frac{36}{21} \quad:$ by dividing by 21 on both sides.
$\therefore \quad \mathbf{x}<\frac{12}{7} \quad:$ by canceling by 3 .
8. Solve $2 \leq \sqrt{x} \leq 4$ and show the solution on a number line.

## Solution:

$2 \leq \sqrt{x} \leq 4$
$(2)^{2} \leq(\sqrt{x})^{2} \leq(4)^{2} \quad:$ by squaring all the terms
$4 \leq \mathrm{x} \leq 16$.

9. Solve $2 x-1 \leq 3(x+3)$ and show the solution set on a number line.

## Solution:

$$
\begin{aligned}
2 \mathrm{x}-1 \leq 3 \mathrm{x}+9 & : \text { by opening the brackets } \\
2 \mathrm{x}-3 \leq 9+1 & : \text { by collecting like terms } \\
-\mathrm{x} & \leq 10 \\
\frac{-x}{-1} & \geq \frac{10}{-1}
\end{aligned} \quad \begin{array}{ll} 
& \text { by dividing by }-1 \text { on both sides and changing the } \\
& \\
& \text { sign from } \leq \text { to } \geq .
\end{array}
$$

$$
\therefore \mathrm{x} \geq-10
$$


10. Solve $-7<x+1 \leq 9$ and show the solution set on a number line.

## Solution:

$-7-1<\mathrm{x}+1-1 \leq 9-1$ : by subtracting 1 from all the terms.
$\underline{-8<x<8}$ Ans

11. Solve $16<x^{2}<25$ and show the solution set on a number line.

## Solution:

$\sqrt{16}<\sqrt{x^{2}}<\sqrt{25} \quad:$ by finding the square root of all the terms.
$4<x<5$ Ans


## Miscellaneous Exercise:

Solve the following inequalities and show the solution sets on a number line where possible.
1.
(a) $5 y+2<17$
(b) $\frac{3 x}{4}+\frac{3}{5}>0$
(c) $\frac{2 x+3}{5}-\frac{3 x-2}{10} \leq 0$
2.
(a) $7(2 y-3) \geq 5(4 y-7)$
(b) $x+5>9$
(c) $\sqrt{x} \leq 5$
3.
(a) $-2<4 \mathrm{x}<12$
(b) $2 x+6 \geq 18$
(c) $5(\mathrm{x}+2)-3(\mathrm{x}-5)>29$
4.
(a) $\frac{3 x}{4}-\frac{2}{3}(1-2 \mathrm{x})<5$
(b) $2+\frac{3}{x}<4$
(c) $4 x+\frac{3}{2}<\frac{5}{2}$
5.
(a) $5(3 x-1)>4(5 x+1)-4$
(b) $2 x>4$
(c) $\quad \frac{1}{2}(x-2)<4$
6.
(a) $11 \leq 3 y+5 \leq 17$
(b) $\frac{x}{2}-\frac{x}{5}>\frac{2+x}{3}$
(c) $2+x \leq 5 \frac{1}{2} x$
7.
(a) $\frac{1-x}{3} \leq \frac{x}{12}-\frac{x}{4}$
(b) $9 \leq 3(x+5)$
(c) $\frac{1}{4}(\mathrm{x}-3)<\frac{1}{3} \mathrm{x}$
8.
(a) $\frac{4}{x-2} \leq \frac{2}{x+1}$
(b) $-2 x+1<x-5<5-x$
(c) $8-x<12 \leq 16-2 x$
9.
(a) $\frac{2}{3} \mathrm{x}-\frac{1}{2} \leq \frac{1}{2} \mathrm{x}+\frac{1}{3}$
(b) $\frac{x}{2}-\frac{x}{4}>-2$
(c) $x-\frac{1}{2}(x-6)<2 x$
10.
(a) $\frac{x+2}{3}>2+\frac{x}{2}$
(b) $\frac{x}{5}-2<\frac{x}{3}$
(c) $4 x+1>7 x-5$

- Solution sets:

We can write solution sets of given inequalities such as:
(i) $\quad \mathrm{x}<2$
(ii) $\quad \mathrm{x}<5$
(iii) $\quad \mathrm{x} \geq-4$

- Inequality

1. $\mathrm{x}<2$
2. $\mathrm{x} \leq 5$
3. $x \geq-4 \quad\{x: x \geq-4\}$

## Solution set

$\{x: x<2\}$
$\{x: x \leq 5\}$

## How the set solution set is read.

x such that x is less than 2 .
x such that x is less than or equal to 5 .
$x$ such that $x$ is greater than or equal to ${ }^{-4}$

NB: $\quad \mathrm{x}$ is identified than we can list the member otherwise we cannot.

- Do not write the solution set of $\mathrm{x}<5$ as $\{\mathrm{x} ; \mathrm{x}=\ldots 2,3,4\}$. This is wrong if x is not defined.
- However, if $x$ is defined such as " $x$ is a positive integer". Then the solution set of $x<5$ will be $\{x: x=1,2,3,4\}$.
- Worked examples:

1. Write down the solution of $\mathrm{x}<10$, if a positive even number.

## Solution:

$\{x: x \in E, x<10\}=\{2,4,6,8\}$.
2. List the numbers of the following sets
(i) $\quad\{x: x \in E, 1<x<12\}$.
(ii) $\quad\{x: x$ is a negative integer, $3 \leq x<5\}$.

## Solution:

(i) $\{x: x \in E, 1<x<12\}=\{2,4,6,8,10\}$.
(iii) $\{\mathrm{x}: \mathrm{x}$ is a negative integer, $3 \leq \mathrm{x}<5\}=\{-3,-2,-1\}$.
3. List the members of the following set.
$\{\mathrm{x}: \mathrm{x}$ is a positive integer divisible by $3,-2<\mathrm{x} \leq 12\}$

## Solution:

$$
\{3,6,9,12\}
$$

- $\quad$ Showing wanted and un wanted regions of inequalities.

Note the following:

Symbol
0 <

- >
- $\leq$
- $\geq$
the nature of the boundary line.
-------- (broken line)
-------- (broken line)
$\qquad$ (continuous line)
- Normally we shade the un wanted region.
- Worked examples

1. Show the region $x>3$ on a graph by shading out the unwanted region.

Step I: $\quad$ Replace the inequality sign with equal signs i.e let $\mathrm{x}=3$.
Step II: Draw your xy - plane.
Identify the nature of the boundary line by considering the inequality sign.
Since sign is >, then the boundary line must be broken ---- -


- $\quad$ Remember x - lines are vertical lines
- Draw a broken vertical line through 3 on the $\mathrm{x}-$ axis.
- Name the line, $\mathrm{x}=3$.


## Step III:

- Determine the wanted region.
- $\quad$ Since we have x as the only unknown in the inequality, then we consider only the x - coordinates.
- $\quad$ Pick on any $x$ - coordinate from any side of the boundary line (broken line)
- test this x - coordinate picked by substituting it in the inequality.
i.e let $\mathrm{x}=2$

But $x>3$
Is $2>3$ ? No.

- The value 2 gives a No answer implying that it has been picked from the un wanted region. Hence shade this region.
- If the value picked, gives a yes answer, Implying that it has been picked from the wanted region, then automatically the opposite side becomes the unwanted region and therefore it is shaded.

2. Show by shading the unwanted represented by the inequality $\mathrm{y} \leq 3$.

Step I: $\quad$ Replace the inequality $\operatorname{sign}(\leq)$ with equal signs, i.e let $y=3$.
Step II: - Draw your coordinate axes.

- Identify the nature of the boundary line by considering the inequality sign.
- $\quad$ Since sign is $\leq$, then the boundary line must be continuous $\qquad$

- $\quad$ Remember y - lines are horizontal lines.
- Draw a continuous horizontal line through 3 on the $y$-axis.
- Name this line y $=3$.


## Step III

- Determine the wanted region.
- $\quad$ Since we have $y$ as the only unknown in the inequality then we consider only the y - coordinates.
- $\quad$ Pick on any y - coordinate from above or below the boundary line (continuous line)
- Test the y - coordinate picked by substituting it in the inequality. i.e let $\mathrm{y}={ }^{-1}$

But $\mathrm{y} \leq 3$
Is ${ }^{-1}<3$ ? Yes, at this step, you can ignore the lower part of the sign $\leq$

- $\quad$ The value ${ }^{-}$, gives us a Yes answer implying that it has been picked from the wanted region.
- $\quad$ Therefore we shade the opposite side (above the boundary line).

3. Show by shading the unwanted region of the inequality $\mathrm{x} \leq 0$.

## Solution:

Step I: - $\quad$ Replace the inequality sign $\leq$ with equal signs i.e. let $\mathrm{x}=0$
Step II: - Draw the coordinate axes.

- Identify the nature of the boundary line by considering the inequality sign.
- $\quad$ Since the sign is $\leq$, then the boundary line must be a continuous line $\qquad$

- $\quad$ Remember the x - lines are vertical lines.
- Draw a continuous vertical line through 0 on the x - axis (this is the y - axis).
- Name this line $\mathrm{x}=0$.

Step III: - Determine the wanted region.
Since we have x as the only unknown in the inequality, then we consider only x coordinates.

- Pick on any x - coordinate from any side of the boundary line.
- Test the x - coordinate picked by substituting it in the inequality.
i.e. let $x=5$

But $\mathrm{x} \leq 0$.
Is $5<0$ ? No. at this step, you can ignore the lower part of the inequality sign $\leq$.

- The value 5 gives us a No answer implying that it has been picked from the unwanted region and therefore shade this region.
- Inequalities involving two variables (unknowns).
- $\quad$ Solutions of inequalities in one variable (unknown) can be shown on a number line but those of inequalities in two variables can only be shown on the coordinate axes.
- Worked examples.

4. Show by shading the unwanted region of the inequality $\mathrm{x}+\mathrm{y}<2$.

Step I: - $\quad$ Replace the inequality sign < with equal signs, i.e. let $\mathrm{x}+\mathrm{y}=2$.
Step II: - Draw table and use it to obtain three points i.e.

| X | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- |
| Y | 2 | 1 | 0 |

## Step III: - Draw the coordinate axes.

- $\quad$ Plot the points $(0,2),(1,1)$ and $(2,0)$ from the table above.
- Identify the nature of the boundary line by considering the inequality sign.
- $\quad$ Since the sign is < , then the boundary line must be broken $\qquad$
- Draw a broken line through the points plotted.
- Name this line $\mathrm{x}+\mathrm{y}=2$.


Step IV: - Determine the wanted region.

- $\quad$ Since we have x and y as the variables, then we consider both x and y coordinates.
- Pick on any point on either side of the boundary line.
- If the boundary line does not pass through the origin $(0,0),(0,0)$ is the simplest to take.
- $\quad$ Let $(x, y)=(0,0)$.
- Test this point by substituting the coordinates in the inequality.
i.e. $x=0$ and $y=0$

But $x+y<2$.
Is $0+0<2$ ? Yes.

- The point $(0,0)$ gives a Yes answer implying that it has been picked from the wanted region and therefore shade the opposite side of the boundary.

5. Show by shading the unwanted region of the inequality $\mathrm{y} \leq \mathrm{x}-1$.

## Solution:

Step I: $\quad$ Replace the inequality $\leq$ with equal signs $=$ i.e. let $\mathrm{y}=\mathrm{x}-1$.
Step II: Draw a table and use it to obtain three points

| X | 0 | 1 | 4 |
| :--- | :--- | :--- | :--- |
| Y | -1 | 0 | 3 |

Step III: - Draw the coordinate axes.

- Plot the points $(0,-1),(1,0)$ and $(4,3)$ from the table above.
- Identify the nature of the boundary line by considering the inequality sign.
- $\quad$ Since the sign is $\leq$, then the boundary line must be continuous $\qquad$
- Draw a continuous line through the points plotted.
- $\quad$ Name this line $\mathrm{y}=\mathrm{x}-1$.


Step IV: - Determine the wanted region.

- $\quad$ Since we have x and y as the unknown in the inequality, then we consider both x and y coordinates.
- $\quad$ Pick on any point from either side of the boundary line preferably $(0,0)$.
- Test this point by substituting in the inequality.
i.e $\mathrm{x}=0$ and $\mathrm{y}=0$

But $\mathrm{y} \leq \mathrm{x}-2$.
Is $0<0-2$ ? Ignore the lower part of the sign.
$0<-2$ ? No at this step.

- The values $\mathrm{x}=0$ and $\mathrm{y}=0$ give us a No answer implying that it has been picked from the wanted region and therefore shade this region.

6. Show by shading the unwanted region of the inequality $\mathrm{y}>\frac{1}{3} \mathrm{x}+2$.

## Solution:

Step I: $\quad$ Replace the inequality $>$ with equal signs $=$ i.e. let $\mathrm{y}=\frac{1}{3} \mathrm{x}+2$.
Step II: Draw a table and use it to obtain three points
NB: - This particular equation, it is better to pick only values $x$ that are divisible by three. (this will help you to avoid coordinates with decimal points).

| X | 0 | 3 | -3 |
| :--- | :--- | :--- | :--- |
| Y | 2 | 3 | 1 |

Step III: - Draw the coordinate axes.

- $\quad$ Plot the points $(0,2),(3,3)$ and $(3,1)$ from the table above.
- Identify the nature of the boundary line by considering the inequality sign.
- $\quad$ Since the sign is > , then the boundary line must be broken $\qquad$
- Draw a broken line through the points plotted.
- Name this line $\mathrm{y}=\frac{1}{3} \mathrm{x}+2$.


Step IV: - Determine the wanted region.

- $\quad$ Since we have x and y as the unknown in the inequality, then we consider both x and y coordinates.
- $\quad$ Pick on any point from either side of the boundary line.
- $\quad$ let $(x, y)=(0,0)$.
- $\quad$ Test the point $(0,0)$ by substituting it in the inequality.
i.e $\mathrm{x}=0$ and $\mathrm{y}=0$

But $\mathrm{y}>\frac{1}{3} \mathrm{x}+2$.
Is $0<\frac{1}{3} \times 0+2$ ?
Is $0>0+2$ ? No.

- $\quad$ The point $(0,0)$ gives us a No answer implying that it has been picked from the wanted region and therefore shade this region.
- Intersecting regions:

Two are more wanted regions can be shown on one graph.
7. on the same graph show by shading the unwanted regions of the following inequalities $\mathrm{y}>\mathrm{x}$ and $\mathrm{x}+\mathrm{y} \leq 5$.

## Solution:

Step I: Replace the inequality signs $>$ and $\leq$ with equal signs $=$ i.e. let $\mathrm{y}=\mathrm{x}$. and $\mathrm{x}+\mathrm{y}=5$.

Step II: - for $\mathrm{y}=\mathrm{x}$, you do not need a table.

- for $\mathrm{x}+\mathrm{y}=5$, draw a table and use it to obtain three points.

| X | 0 | 3 | 5 |
| :--- | :--- | :--- | :--- |
| Y | 5 | 2 | 0 |

Step III: - Draw the coordinate axes.

- Plot the points $(0,5),(3,2)$ and $(5,0)$ from the table above and $(2,2),(-3,-3)$ and $(4,4)$ for $\mathrm{y}=\mathrm{x}$.
- Identify the nature of the boundary line by considering the inequalities sign.
- $\quad$ For $\mathrm{x}+\mathrm{y} \leq 5$, the sign is $\leq$, therefore the boundary line must be a continuous line $\qquad$ .
- $\quad$ For $y>x$, the sign is >, therefore the boundary line must be broken
- Name the continuous line $\mathrm{x}+\mathrm{y}=5$ and the broken line $\mathrm{y}=\mathrm{x}$.


Step IV: - Determine the wanted regions.

- $\quad$ Since x and y are the unknowns in the two inequalities, then we consider both x and y coordinates.
- $\quad$ Consider one boundary line at a time and pick one point from any side.
- $\quad$ For $\mathrm{x}+\mathrm{y}=5$, take $(\mathrm{x}, \mathrm{y})=(0,0)$.
- Test this point by substituting in the inequality.
i.e $\mathrm{x}=0$ and $\mathrm{y}=0$

But $x+y \leq 5$.
Is $0+0<5$ ? Yes, ignore the lover part $\leq$.

- $\quad$ The point $(0,0)$ gives us a Yes answer implying that it has been picked from the wanted region and therefore shade the opposite side (region).
- $\quad$ For $\mathrm{y}>\mathrm{x}$, you cannot pick $(0,0)$ because the line $\mathrm{y}=\mathrm{x}$ passes through this point. Therefore pick some other point.
- $\quad$ i.e( $-2,5$ ), $x=-2$ and $y=5$.

But y > x Is $5>-2$ ? Yes.

- The point $(-2,5)$ gives us axes answer implying that it has been picked from the wanted region and therefore shade the opposite side.

8. On the same graph show by shading the unwanted regions of the following inequalities, $\mathrm{x}<5, \mathrm{x}+\mathrm{y} \geq 2$ and $\mathrm{y}<1 / 2 \mathrm{x}+2$.

## Solution:

Step I: Replace the inequality signs $<, \geq$ and $<$ with equal signs $=$ i.e. let $\mathrm{x}=5$.
$x+y=2$.
and $\mathrm{y}=1 / 2 \mathrm{x}+2$.
Step II: - for $\mathrm{x}=5$, you do not need a table to get the values, this is simply a vertical line through 5 .

- for the other two, draw tables to obtain three points in each case.
$\mathrm{x}+\mathrm{y}=2$

| X | 0 | 1 | 2 |
| :--- | :--- | :--- | :--- |
| Y | 2 | 1 | 0 |
|  |  |  |  |
| X | 0 | -4 | 2 |
| Y | 2 | 0 | 3 |

$$
y=1 / 2 x+2
$$

Step III: - Draw the coordinate axes.
Plot the points $(0,2),(1,1)$ and $(2,0)$ from the first.

- Also plot points $(0,2),(-4,0)$ and $(2,3)$ from the second table
- Identify the nature of the boundary lines in each by considering the inequalities signs.
- For $\mathrm{x}+\mathrm{y} \geq 2$, the sign is $\geq$, therefore the boundary line must be a continuous line
$\qquad$
For $\mathrm{y}<1 / 2 \mathrm{x}+2$, the sign is $<$, therefore the boundary line must be a broken line ----
$\qquad$


Step IV: - Determine the wanted regions.

- $\quad$ Pick one point from any of the sides of each boundary line.
- For $\mathrm{x}=5$, consider only the $\mathrm{x}=$ coordinates i.e. $\mathrm{x}=4$.
- Test this value by substituting in the inequality.
i.e $\mathrm{x}<5$

Is $4<5$ ? Yes.

- $\quad$ The value 4 gives us a yes answer, implying that it has been picked from the unwanted region and therefore shade the opposite side (region).
- For y $<1 / 2 \mathrm{x}+2$, take $(0,0)$ and test it by substituting it in the inequality.
- i.e $x=0$ and $y=0$.

But $y<1 / 2 x+2$,
Is $0<1 / 2 \times 0+2$ ?
Is $0<2$ ? Yes.

- $\quad$ The point $(0,0)$ gives us a yes answer implying that it has been picked from the wanted region and therefore shade the opposite region.
- $\quad$ For $x+y \geq 2$, take $(x, y)=(0,0)$ and test it.
i.e. $\mathrm{x}=0$ and $\mathrm{y}=0$

But $\mathrm{x}+\mathrm{y} \geq 2$.
Is $0+0>2$ ?

Is $0>2$ ? No. Ignore the lower part of $\leq$.
The point $(0,0)$ gives us a No answer implying that it has been picked from the unwanted region and therefore shade this region.

- Miscellaneous Exercise

Show by shading the unwanted regions of the following inequalities.
1.
(a) $\mathrm{x}=\leq 2$
(b) $x<-5$
(c) $\mathrm{y} \leq-3$
2.
(a) $x=\leq 0$
(b) $\mathrm{y} \leq 0$
(c) $\mathrm{x}+\mathrm{y}<0$
3.
(a) $2 x+1<y$
(b) $x-y \geq 3$
(c) $y \geq 2-x$
4.
(a) $\quad 1 / 2 y \leq x-4$
(b) $2 y+2>x$
(c) $x+y>2$
5.
(a) $3 x-y \geq 1$
(b) $x-y \leq 0$
(c) $y<1 / 3 x-3$

- On the same coordinate axes (graph) show by the unwanted regions of the following inequalities.

6. 

(a) $\mathrm{x}+\mathrm{y} \leq 3$ and $\mathrm{y}>\mathrm{x}-2$
(b) $\mathrm{y}<5-1 / 2 \mathrm{x}$ and $2 \mathrm{x}<10-\mathrm{y}$
7.
(a) $\mathrm{x} \leq 4$ and $\mathrm{y}<3$
(b) $3 x+2 y>16$ and $x+y \leq 10$
8. (a) $2 x+3 y \geq 18$ and $4 x+3 y \geq 24$ (b) $2 x+3 y \leq 12$ and $6 x+5 x>30$
9.
(a) $\mathrm{x}+\mathrm{y} \leq 3$ and $\mathrm{y}>\mathrm{x}-2$
(b) $\mathrm{y}<5-1 / 2 \mathrm{x}$ and $2 \mathrm{x}<10-\mathrm{y}$
10.
(a) $\mathrm{x} \geq 0, \mathrm{y}>0$ and $\mathrm{x}+\mathrm{y}<3$
(b) $y \leq x+1, x<4$ and $y \geq 1$
11.
(a) $\mathrm{x} \geq 0, \mathrm{y} \geq 0$ and $\mathrm{x}+\mathrm{y} \leq 1$
(b) $\mathrm{x} \leq 2, \mathrm{y} \leq 2, \mathrm{y} \leq \mathrm{x}+1$
12.
(a) $y \leq x+4, \mathrm{x} \leq 3$ and $\mathrm{y} \geq 1$
(b) $y \leq x, y \geq 1 / 2 x$ and $2 x+y \geq 2$

