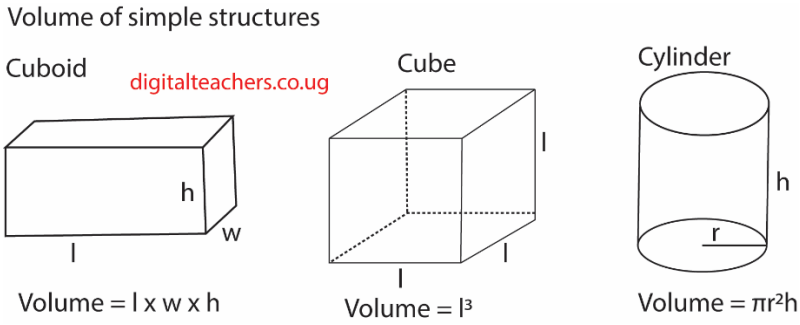
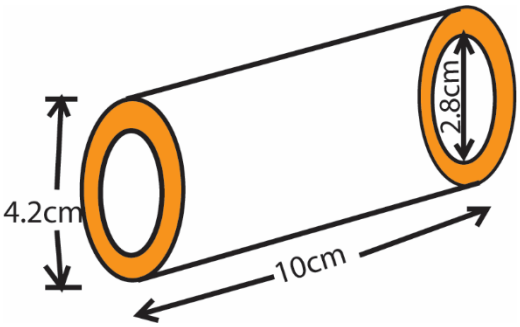
Volume

This is the amount of space taken up by an object, while capacity is the measure of an object's ability to hold a substance, like a solid, a liquid or a gas.



Example

The diagram below is of a piece of thick metallic pipe 10 cm long. The diameter of the pipe is 4.2 cm and the diameter of the bore (hole) is 2.8cm. Find the volume of the metal which makes up the pipe. (Take $\pi = {}^{22}/_{7}$)



Volume of the metal $= \pi h(R^2 - r^2)$

$$= \frac{22}{7}\ x\ 10\ x\ ((\frac{4.2}{2})^2 - (\frac{2.8}{2})^2)$$

$$= \frac{22}{7}\ x\ 10\ x\ (2.1^2 - 1.4^2)$$

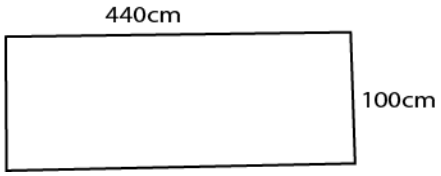
$= \frac{22}{7}\ x\ 10\ x\ (2.1 + 1.4)(2.1 - 1.4)$

$= \frac{22}{7}\ x\ 10\ x\ 3.5\ x\ 0.7$

$= 77\ cm^3$

Revision Questions

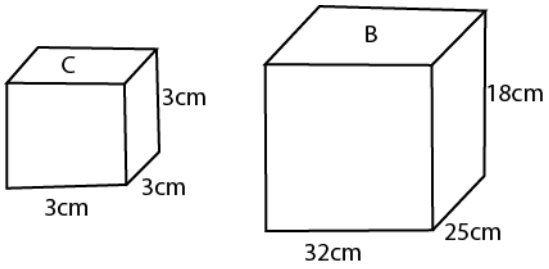
1. A welder was given a metal sheet with measurements as shown in the diagram below. He welded it into a hollow cylinder, making the height 100cm. (Take $\pi = \frac{22}{7}$)



(a) What is the surface area of the sheet of metal needed to cover the bottom of the cylinder?

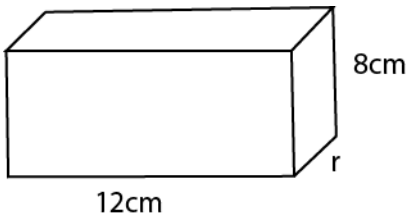
(b) What maximum volume of water will the cylinder hold?

2. Below is a cube (C) and a box (B). Waiswa packs curry powder in cubes and packs the cubes in a box for export to Kigali.



Calculate space left empty after filling the box with cubes of curry powder.

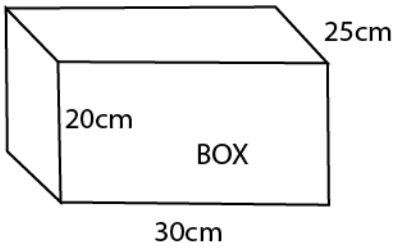
3. The volume of the box below is $480cm^3$

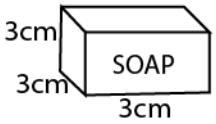


(a) Find the value of r

(b) Calculate its surface area

4. Pieces of soap each measuring 3cm by 3cm by 3cm are to be packed into a box measuring 30cm by 25cm by 20cm. (See the diagram below.)





a) How many pieces of soap can be packed into the box?

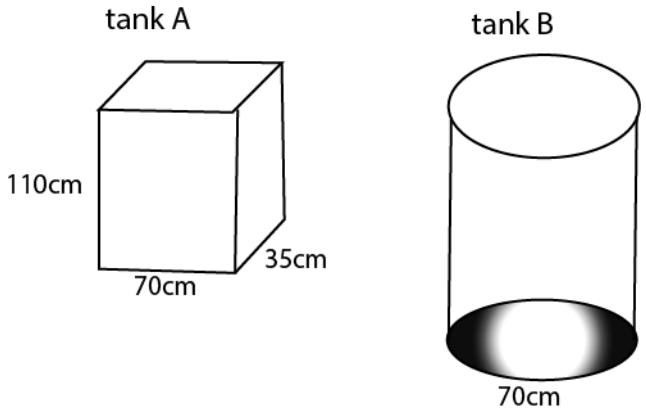
b) Calculate the space left empty after pieces of soap have been packed into the box.

5. (a) Kamya packs cylindrical beef tins whose height is 10 cm and diameter 5 cm in a rectangular carton measuring 60 cm long 10cm wide, 30cm high, How many tins will be packed in the carton?

(b) Find the space (in cm^3) that will remain after the tins have been packed in the carton.

(Take $\pi = 3.14$)

6. A cuboidal water tank (A), which is 70cm long by 35cm wide by 110cm, was filled with water. The water from tank (A) was all poured into the cylindrical tank (B) of diameter 70cm.



a) Find the volume of water in tank A when it is full

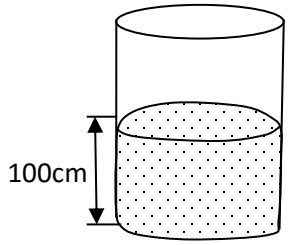
b) Find the new height of water after it has been poured into tank B

7. How many cubes of ½ cm ^{3} volumes are contained in a cube of 2cm^3 volume?

8. A milk seller has 36 litres of milk. He sells milks using a container measuring 6 cm by 10 cm by 6cm at shs. 150 per full container. How much money does he get selling all the milks?

(6 marks)

9. The figure below is a cylindrical tank containing 1540 litres of water

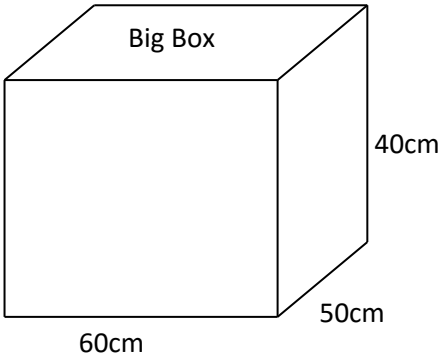


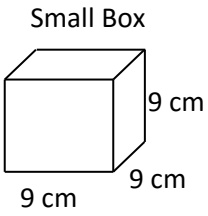
(a). Find the radius of tank. (use $\pi = \frac{22}{7}$) (4marks)

(b) If the tank is $\frac{4}{5}$ full, find its capacity (2mark)

10. The diagram below shows a big box 60cm long, 50cm wide, 40 cm high and a small box 9 cm long, 9cm wide, and 9 cm high.

Study it carefully and answer the questions that follow.





If such small boxes are to be packed into the big box.

(a) Find the number of small boxes that will be packed in the first layer of the big box.

(1marks)

(b) How many layers will fill the big box? (2 marks)

(c) How many small boxes will fill the big box? (2 marks)

11. A cylindrical tin of radius 7cm contains 3080cm^3 of cooking oil.

(a) Joan used 2156cm^3 of the cooking oil. What is the height of the cooking oil remaining in the tin? (Take $\pi = \frac{22}{7}$) (03marks)

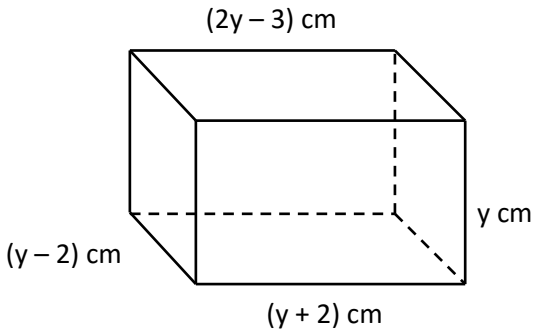
(b). Joan poured the remaining cooking oil into a rectangular tin with base area 77cm^2. What was the height of the oil in the tin?

12. A tank was $\frac{2}{3}$ full of water. When $^1/_4$ of the water in the tank was drawn, 2,500 litres remained. Find the capacity of the tank when full.

(04 marks)

Let the volume the total volume be V

13. The figure below is a cuboid. Study and use it to answer the questions that follow.

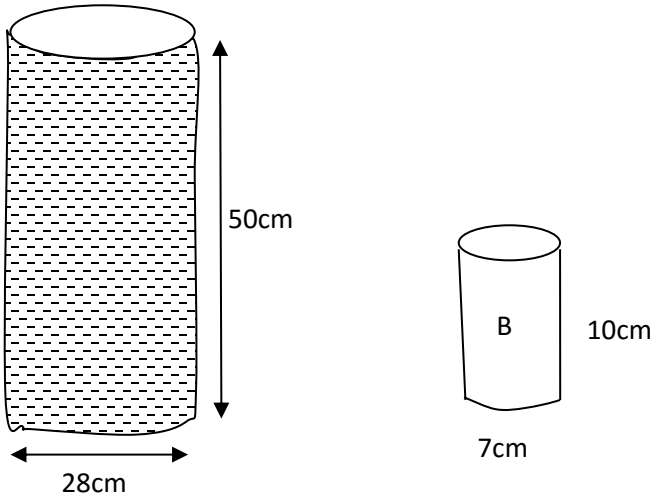


(a) Find the value o f y (02 marks)

(b) Find the volume of the cuboid (03 marks)

14. A bucket was $\frac{3}{4}$ full of water. When 4 liters were removed, it become $\frac{1}{2}$. What is the capacity of the bucket?

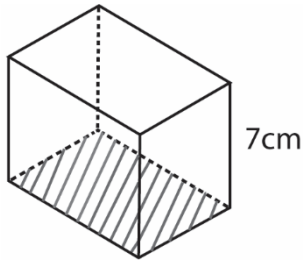
15. Bbosa filled container A below with drinking water, she served visitor with the water using cups of size B shown in the diagram



Find the total number of full cups of water she served the visit

(Use $\pi\frac{22}{7}$) (06marks)

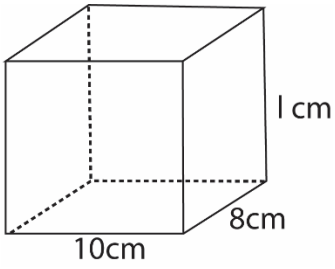
16. The area of the shaded part of the cuboid below is $12cm^2$. Calculate the volume of the cuboid.



17. A cylindrical tank of diameter 70cm contains water to a height of 100cm.
    Find in litres the amount of water the tank contains (Use $\pi = \frac{22}{7}$) (04 marks)

18. Bottles of 300 millilitres (ml) were used to fill a nine litre bucket with water. Find the number of full 300ml bottles that were used.

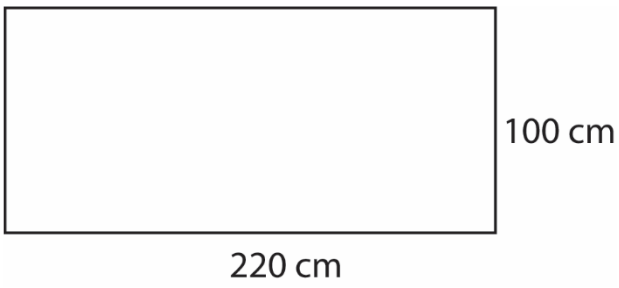
19. The sum of the lengths of the edges of the prism below is 92cm



    (i) Find the length l. (03cm)

(ii) Find the volume of the prism.(02marks)

20. The figure below shows a rectangular sheet of metal. The sheet is curved to form the wall of cylindrical tank whose height is 100cm.



(a) Find the diameter of the tank formed. (use $\pi = \frac{22}{7}$) (02marks)

(b) Calculate

(i) the area of the sheet required to cover the base of the tank. (02marks)

(ii) Capacity of the tank (01mark)

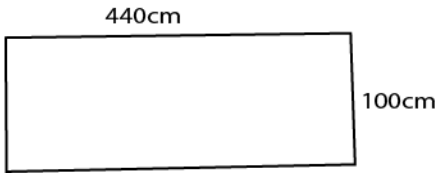
21. A Cylinder with radius 7cm contains 2,926cm^3 of liquid soap (Take $\pi=22/7$)

(i) If 616 cm^3 of the liquid soap is used, what is the height of the liquid soap remaining?

(ii) If the 616cm^3 was allowed to pour into a rectangular tin with a base area of 88cm^2, to what height will the liquid soap rise?

Suggested answers

1. A welder was given a metal sheet with measurements as shown in the diagram below. He welded it into a hollow cylinder, making the height 100cm. (Take $\pi = \frac{22}{7}$)



(a) What is the surface area of the sheet of metal needed to cover the bottom of the cylinder?

Radius of the bottom = $2\pi r = 440$

$$= 2 \times \frac{22}{7} \times r = 440$$

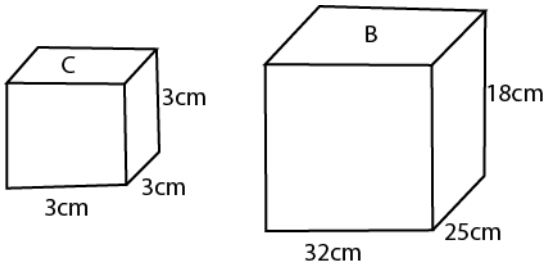
$$r = 70$$

Area = $2\pi r^2 = 2 \times \frac{22}{7} \times 70 \times 70 = 30{,}800$

(b) What maximum volume of water will the cylinder hold?

Volume = Area x h = 30800 x 100 = 3080000cm^3

2. Below is a cube (C) and a box (B). Waiswa packs curry powder in cubes and packs the cubes in a box for export to Kigali.



Calculate space left empty after filling the box with cubes of curry powder.

Number of boxes along the length = $\frac{32}{3} = 10\frac{2}{3}$

Number of boxes along the width = $\frac{25}{3} = 8\frac{1}{3}$

Number of layer = $\frac{18}{3} = 6$

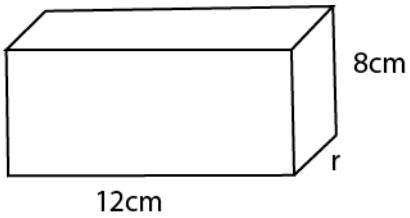
Number of cubes that are packed in the box = 10 x 8 x 6 = 480 cubes

Volume occupied by the cube = 480 x 3 x 3 x3 = 12960 cm^3

Volume of the box = 32 x 25 x 18 = 14400cm^3

Unoccupied space = 14400 – 12960 = 1440cm^3

3. The volume of the box below is 480cm^3



(a) Find the value of r

Volume = l x w x h

480 = 12 x r x 8

480 = 96r

r = 5cm

(b) Calculate its surface area

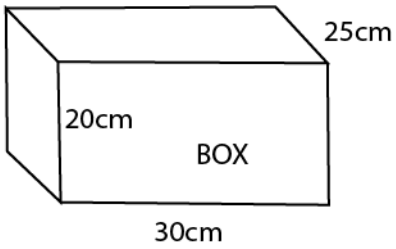
Surface area = 2(12 x 5) + 2(12 x 8) + 2(5 x 8)

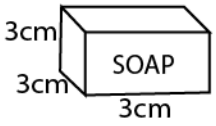
=2(60 +96 +40)

= 2 x 196

= 392

4. Pieces of soap each measuring 3cm by 3cm by 3cm are to be packed into a box measuring 30cm by 25cm by 20cm. (See the diagram below.)





a) How many pieces of soap can be packed into the box?

Bar

Bar long the length = 30 /3 = 10

Bars across = 25/3 = 8

Bars along the height = 20/3 = 6

Total bars = 10 x 8 x6 = 480

b) Calculate the space left empty after pieces of soap have been packed into the box.

Volume of the box = 30 x 20 x 25 = 15000 cm^3

Volume of soap = 480 x (3 x 3 x 3) = 12960 cm^3

Empty space = 15000 – 12960 = 2040cm^3

5. (a) Kamya packs cylindrical beef tins whose height is 10 cm and diameter 5 cm in a rectangular carton measuring 60 cm long 10cm wide, 30cm high, How many tins will be packed in the carton?

Number of tins = $\frac{60}{5} \; x \; \frac{10}{5} \; x \; \frac{30}{10}$

= 12 x 2 x 3 = 72 tins

(a) Find the space (in cm^3) that will remain after the tins have been packed in the carton.

(Takeπ = 3.14)

Volume of 1 tin = $\pi r^2 h$

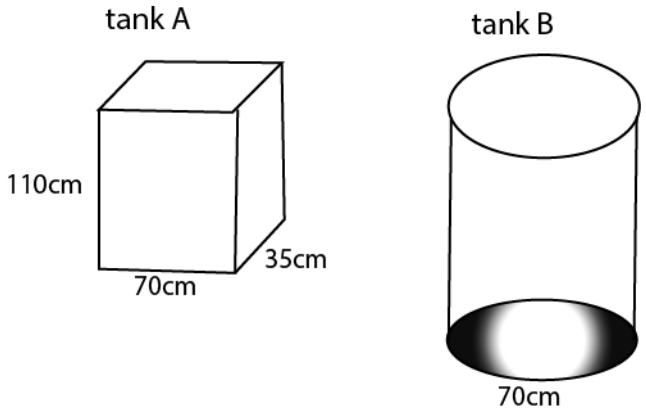
$= \frac{22}{7}\ x\ \frac{5}{2}\ x\ \frac{5}{2}\ x\ 10 = 14140.8cm^3$

Volume of 48tins = 196.4 x 72 = 9427.2 cm^3

Volume of the box = 60 x 10 x 30 =18000

Unoccupied volume = 18000 – 14140.8 = 3859.2cm^3

6. A cuboidal water tank (A), which is 70cm long by 35cm wide by 110cm, was filled with water. The water from tank (A) was all poured into the cylindrical tank (B) of diameter 70cm.



a) Find the volume of water in tank A when it is full

Volume = L x W x H

= 70 x 35 x 110

= 269 500 cm^2

b) Find the new height of water after it has been poured into tank B

Volume of cylinder = πr^2h

Radius r = $\frac{diameter}{2} = \frac{70}{2} = 35cm$

⇨ 269500 = $\frac{22}{7}\ x\ 35\ x\ 35\ xh$

⇨ h = 70cm

7. How many cubes of ½ cm ^{3} volumes are contained in a cube of 2cm^3 volume?

Number or cube $2 \div \frac{1}{2}$ = 2 x $\frac{2}{1} = 4cubes$

8. A milk seller has 36 litres of milk. He sells milks using a container measuring 6 cm by 10 cm by 6cm at shs. 150 per full container. How much money does he get selling all the milks?

(6 marks)

Volume of milk in cm^3 = 36 x 1000cm^3

= 36000cm^3

Volume of the unit = L x W x H

= 10 x 6 x 6

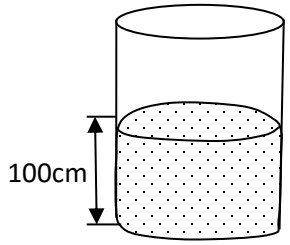
= 360cm^3

Number of units = $\frac{36000}{360} = 100units$

But 1 unit costs =150/=

$\therefore he\ would\ earn\ 100\ x\ 150 = 15,000/=$

9. The figure below is a cylindrical tank containing 1540 litres of water



(a) Find the radius of tank. (use $\pi = \frac{22}{7}$) (4marks)

Volume of a cylinder = $\pi r^2 h$

1L = 1000cm^3

1540 l = 1540 x 1000 = 1540000cm^3

$\therefore$ 1540000 = $\pi r^2 \, x100$

$r^2 = \frac{1540000}{(\pi \, x \, 100)}$

r = $\sqrt{\left(\frac{1540000}{(\pi \, x \, 100)}\right)}$ = 70cm

(b) If the tank is $\frac{4}{5}$ full, find its capacity (2mark)

Let the total volume be V

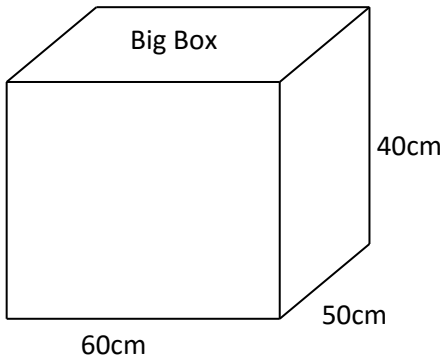
$\frac{4}{5}V = 1540$

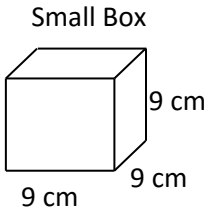
$4V = 1540 \, x \, 5$

V = 1925litres

10. The diagram below shows a big box 60cm long, 50cm wide, 40 cm high and a small box 9 cm long, 9cm wide, and 9 cm high.

Study it carefully and answer the questions that follow.





If such small boxes are to be packed into the big box.

(a) Find the number of small boxes that will be packed in the first layer of the big box.

(1marks)

Number of small boxes = $\frac{L}{l}\ x\ \frac{W}{w} = \frac{60}{9}\ x\ \frac{50}{9} =\ 30\ small\ boxes$

(b) How many layers will fill the big box? (2 marks)

Number of layers = $\frac{H}{h} =\ \frac{40}{9} = 4$

(c) How many small boxes will fill the big box? (2 marks)

Number of small boxes = 30 x 4 = 120 small boxes

11. A cylindrical tin of radius 7cm contains 3080cm^3 of cooking oil.

(a) Joan used 2156cm^3 of the cooking oil. What is the height of the cooking oil remaining in the tin? (Take $\pi = \frac{22}{7}$) (03marks)

Volume of remaining oil = 3080cm^3

= - 2156cm^3

924 cm^3

Volume = base area x height (h)

= (πr^2)h

924 = $(\frac{22}{7}\ x\ 7\ x\ 7)\ xh$

h = 6cm^3

(b) Joan poured the remaining cooking oil into a rectangular tin with base area 77cm^2. What was the height of the oil in the tin?

Volume = base area x height (h)

924 = 77 h

h = 12cm

12. A tank was $\frac{2}{3}$ full of water. When $^1/_4$ of the water in the tank was drawn, 2,500 litres remained. Find the capacity of the tank when full.

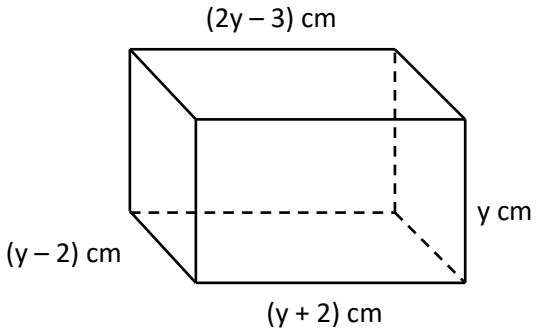
(04 marks)

Let the volume the total volume be V

$(\frac{2}{3}\left(1-\frac{1}{4}\right) x\ V = 2{,}500 litres$

V = 5000litres

13. The figure below is a cuboid. Study and use it to answer the questions that follow.



(a) Find the value o f y (02 marks)

2y – 3 = y + 2

y = 3

(b) Find the volume of the cuboid (03 marks)

Length = y + 2 = 5 + 2 = 7

Width = y -2 = 3

Height =y = 5

Volume = length x width x height= 7 x 3 x 5 =105cm^3.

14. A bucket was $\frac{3}{4}$ full of water. When 4 liters were removed, it become $\frac{1}{2}$. What is the capacity of the bucket?

Let the capacity of bucket be X

$\frac{3}{4}X - 4 = \frac{1}{2}X$

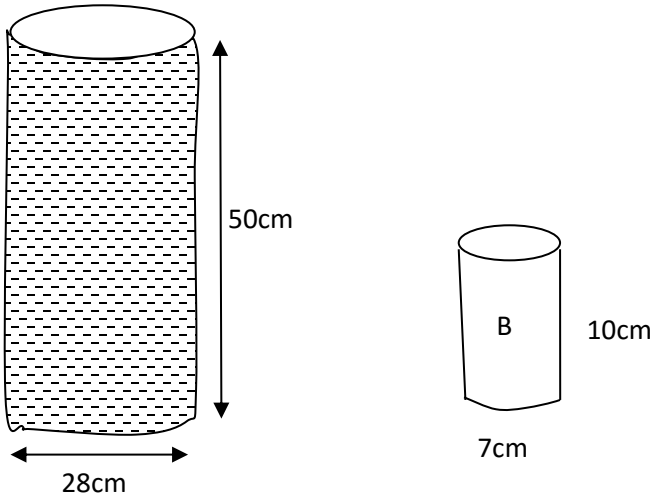
Multiply be 4 throughout

3X -16 = 2X

Collect like terms

X = 16

15. Bbosa filled container A below with drinking water, she served visitor with the water using cups of size B shown in the diagram



Find the total number of full cups of water she served the visit

(Use $\pi\frac{22}{7}$) (06marks)

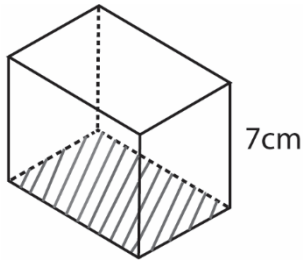
Volume of a cylinder = $\pi r^2 h$

Volume of container A = $\frac{22\ x\ 14^2\ x\ 50}{7} = 30800 cm^3$

Volume of each cup = $\frac{22\ x\ 3.5\ x\ 3.5\ x\ 10}{7} = 385\ cm^3$

Number of filled cups = $\frac{volume\ of\ A}{volume\ of\ the\ cup} = \frac{30800}{385} = 80\ cups$

16. The area of the shaded part of the cuboid below is $12cm^2$. Calculate the volume of the cuboid.



Volume = Area x height
= 12 x 7 = $84cm^3$

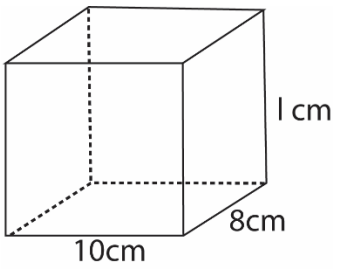
17. A cylindrical tank of diameter 70cm contains water to a height of 100cm.
Find in litres the amount of water the tank contains (Use $\pi = \frac{22}{7}$) (04 marks)

Volume = $\pi r^2 h = \frac{22}{7} \; x \; \left(\frac{70}{2}\right)^2 \; x \; 100 = 385000cm^3 = \frac{385000}{1000} = 385L$

18. Bottles of 300 millilitres (ml) were used to fill a nine litre bucket with water. Find the number of full 300ml bottles that were used.

9l = 9 x 1000 = 9000ml
Number of bottles = $\frac{9000}{300}$ = 30 bottles

19. The sum of the lengths of the edges of the prism below is 92cm



(i) Find the length l. (03cm)

The sum of the length of edges of prism = 4(l + W + h)

$92 = 4(10 + 8 + l)$

$\frac{92}{4} = 18 + l$

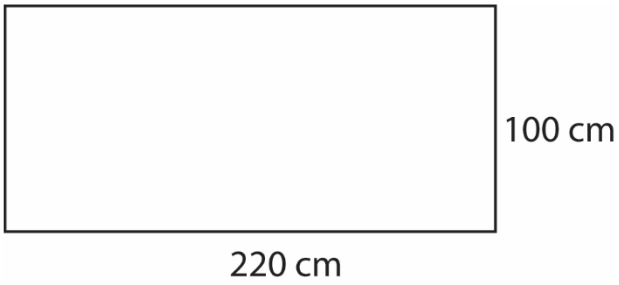
$23 = 18 + l$

$l = 5$

(ii) Find the volume of the prism.(02marks)

Volume = l x w x h
= 10 x 8 x 5
= 400cm^3

20. The figure below shows a rectangular sheet of metal. The sheet is curved to form the wall of cylindrical tank whose height is 100cm.



(a) Find the diameter of the tank formed. (use $\pi = \frac{22}{7}$) (02marks)

Circumference = πd

$\frac{22}{7} \; x \; d = 220$

$d = \frac{220 \; x \; 7}{22} = 70cm$

(b) Calculate

(iii) the area of the sheet required to cover the base of the tank. (02marks)

radius = $\frac{d}{2} = \frac{70}{2} = 35cm$

area = πr^2 = $\frac{22}{7} \; x \; 35 \; x \; 35$ = 3850cm^2

(iv) Capacity of the tank (01mark)

Volume = area x height = 3850 x 100 = 385000cm^3

21. A Cylinder with radius 7cm contains 2,926cm^3 of liquid soap (Take π=22/7)

(i) If 616 cm^3 of the liquid soap is used, what is the height of the liquid soap remaining?

Volume of liquid soap remaining = 2926 – 616 = 2310cm^3

Let the height of remaining liquid soap be h

Volume = $\pi r^2 h$ = 2310

$\frac{22}{7}$ x7 x7xh = 2310

h = 15cm

(i) If the 616cm^3 was allowed to pour into a rectangular tin with a base area of 88cm^2, to what height will the liquid soap rise?

Volume = area of the base x height

616cm^3 = 88 x h

h = 7cm

the height of the liquid will rise to 7cm