We are accustomed to writing numbers in base ten, using the symbols for 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 probably because we have 10 fingers. For example, 75 means 7 tens and five units. However numbers can be written in any number base.

Example 1

Change 75 base ten to base eight

$75_{ten} = 113_{eight}$

8	75	R
8	9	3
	1	1

Example 2

Change 113eight to base 10

$113_{eight} = [(1 \text{ x } 8^2) + (1 \text{ x } 8^1) + (3 \text{ x } 8^0)]_{\text{ ten}}$

$= (1 \text{ x } 64) + (1 \text{ x } 8) + (3 \text{ x } 1)$

$= 64 + 8 + 3$

$= 75$

Therefore, if we use base 8 instead of base ten, then 75 is written as 113 which denotes one sixty-four (8^2), one eight (8^1) and 3 units (instead of hundreds, tens and units).

Base 2 is particularly useful as it only requires two symbols, for zero and one, and it is the way numbers are represented in computers.

Example 3

Change 75ten to base 2

2	75	R
2	37	1
2	18	1
2	9	0
2	4	1
2	2	0
	1	0

Thus, $75_{ten} = 1001011_{two}$

Just as, in base ten, the columns represent powers of 10 and have 'place value' 1, 10, 10^2 , 10^3 etc. (reading from right to left), so in base 2, the columns represent powers of 2. Hence the number 1001011 denotes (reading from right to left):

1 unit (2^0), 1 two (2^1), no fours (2^2), 1 eight (2^3), no sixteens (2^4), no thirty-twos (2^5), 1 sixty-four (2^6).

The number 1001011 in base 2 is the same as the number 75 in base ten.

Example 4

Change 75ten to base five

5	75	R
5	15	0
	3	0

Thus, $75_{ten} = 300_{five}$

We use the symbols 0, 1, 2, 3 and 4 to represent numbers in base 5. The columns in base 5 have 'place value' 1, 5, 25, 125, 625 etc. reading from right to left. The number 75 in base ten is the same as the number 300 in base five, that is 3 twenty-fives, no fives and no units.

Example 5

Change 203_{six} to base ten

$203six = [(2 \text{ x } 6^2) + (0 \text{ x } 6^1) + (3 \text{ x } 6^0)]_{ten}$

$= [2 \text{ x } 36 + 0 + 3 \text{ x } 1]_{ten}$

$= 72 + 3 = 75_{ten}$

Writing the number 75 in base six we get 203, which represents 2 thirty-sixes, no sixes and 3 units.

We have seen that 75 (base10), 1001011 (base 2), 300 (base 5), 113 (base 8), and 203 (base 6) all represent the same number.

Similarly, we can write 75 in any base we choose and we can write all numbers in any base.

Revision questions

1. Subtract $1101_{two} - 110_{two}$.

2. Change 72_{ten} to binary

3. Express 45_{ten} to binary

4. Write: 21_{ten} in base two

5. Change 1010_{two} to base ten

6. Add: $101_{two} + 11_{two}$

8. Change 110_{two} to base ten.

9. Change 3 to binary system.

10. Work out:

11. Work out: $110_{two} \times 11_{two}$

12. Change $11010_{\text{ two}}$ to base ten.

13. Change 1011_{two} to base ten (2marks)

14. Change 11011_{two} to base ten.

15. Work out: $1101_{two} + 111_{two}$

16. Work out: $110_{two} \times 11_{two}$

17. (a) Write the place value of 2 and 1 in 201three (02 mark)

(b) Work out: 42five x 21 five (02 marks)

18. (a) work out $333_{five} + 123_{five}$ (02 marks)

(b) Given that $34_t = 112_{four}$, find the value of t (03marks)

Suggested answers

1. Subtract 1101 $_{two}$-110$_{two}$.

 1101 two

 –ll0two

 111 two

Or

First change the numbers to base ten, subtract the numbers and then change the answer to base two.

1101two = $(1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$ = 13

-110two = $(1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$ = - 06

= 7ten

Converting 7to base two

2	7	R
2	3	1
2	1	1
	0	1

Hence, 1101two-110two=111two

2. Change 72 $_{ten}$ to binary

2	72	r
2	36	0
2	18	0
2	9	1
2	4	0
2	2	0
2	1	

thus 72 $_{ten}$ = 100100$_{two}$

3. Express 45$_{ten}$ to binary

2	45	R
2	22	1
2	11	0
2	5	1
2	2	1
2	1	0

45$_{ten}$ = 101101$_{two}$

4. Write: 21_{ten} in base two

2	21	R
2	10	1
2	5	0
2	2	1
	1	0

21ten = 10101two

5. Change 1010_{two} to base ten

$1010_{two} = (1x2^3) + (0 \times 2^2) + (1 \; x2^1) + (0 \times 2^0)$

$= 8 + 0 + 2 + 0$

$= 10$

6. Add: $101_{two} + 11_{two}$

101_{two}

$+ \; 11_{two}$

1000_{two}

7. Change 110_{two} to base ten.

$110_{two} = (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

$= 4 + 2 + 0$

$= 6$

8. Change 3 to binary system.

2	3	r
	1	1

$\therefore \quad 3_{ten} = 11_{two}$

9. Work out:

1010two

\+ 111two

11two

10. Work out: $110_{two} \times 11_{two}$

110

x 11

110

110

10010

11. Change 11010_{two} to base ten.

$1^4 1^3 0^2 1^1 0^0{}_{two} = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 16 + 8 + 0 + 2 + 0$

$= 26_{ten}$

12. Change 1011_{two} to base ten (2marks)

$(1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0) = (8 + 0 + 2 + 1) = 11$

13. Change 11011_{two} to base ten.

$1^4 1^3 0^2 1^1 1^0{}_{two} = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 16 + 8 + 0 + 2 + 1$

$= 27$

14. Work out: $1101_{two} + 111_{two}$

1101_{two}

$+ \ 111_{two}$

10100

15. Work out: 110_{two} x 11_{two}

$$\begin{array}{r} 110 \\ \times\ 11 \\ \hline 110 \\ 110 \\ \hline 10010 \\ \hline \end{array}$$

16. (a) Write the place value of 2 and 1 in 201three (02 mark)

Place value of 2 = 2 x 3^2 = or 2nines

Place value of 1 = 1 x 3^0 or 1 unit

(b) Work out: 42five x 21 five (02 marks)

$$\begin{array}{r} 42 \\ \times\ 21 \\ 42 \\ 134 \\ \hline 1432_{five} \\ \hline \end{array}$$

17. (a) work out (02 marks)

$$\begin{array}{r} 333_{\ five} \\ +\ 123_{\ five} \\ \hline 1011_{five} \\ \hline \end{array}$$

(b) Given that $34_t = 112_{four}$, find the value of t (03marks)

34t m = $112_{\ four}$

8. It implies that $3t^1 + 4t^0$ = $1 x4^2 + 1x4^1 + 2x4^0$

3t +4 = 16+4+2

3t = 18

t = 6