UCE

Mathematics 4

(For S.4)

BY

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Second edition 2008

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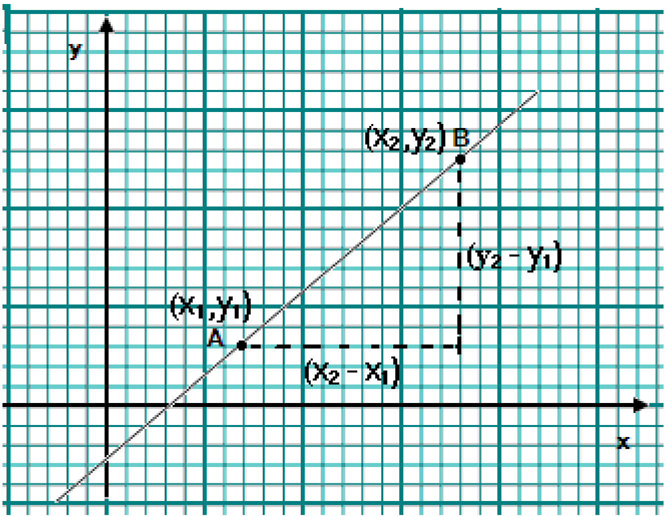
Chapter 1
Gradients and rates of change

The gradient of a straight line

Every straight line has a slope with respect to the horizontal axis. The measure of the slope is called the **gradient**. In the Cartesian plane, the gradient of a line is the measure of its slope or inclination to the x-axis. The gradient of a line is defined as the ratio of **the change in y-coordinate (vertical) to the change in the x-coordinate (horizontal).**

Thus, gradient = $\frac{\text{change in y - coordinate}}{\textit{change in x} - \textit{coordinate}}$

Consider a line passing through the points A(x_1, y_1) and B(x_2, y_2).



From A to B, the change in the x-coordinate (horizontal change) is (x2 - x1) and the change in the y-coordinate (vertical change) is (y2 - y1). By definition,

Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$

Example 1.1

Find the gradient of a line passing through the points (2, 5) and (7, 9).

Solution

gradient = $\frac{\text{change in y - coordinate}}{\textit{change in x} - \textit{coordinate}} = \frac{9-5}{7-2} = \frac{4}{5}$

Note: The gradient does not depend on the direction of change, but care must be taken to subtract the corresponding coordinates.

Thus, also, gradient = $\frac{5-9}{2-7}$ = $\frac{4}{5}$

Example 1.2

Find the gradients of line segments:
(a) AB (b) CD, given that line AB passes through the points (-3, -4) and (-1, -1); while line CD passes through the points (-3, 5) and (8, 1).

Solution

(a) Gradient of AB is given by: $\frac{-4-(-1)}{-3-(-1)}=\frac{-3}{-2}=\frac{3}{2}$

(b) Gradient of CD is given by: $\frac{1-5}{8-(-3)}$

$= \frac{4}{11}$

Note:

(i) All horizontal line have zero gradients.

(ii) Gradients of vertical lines cannot be defined.

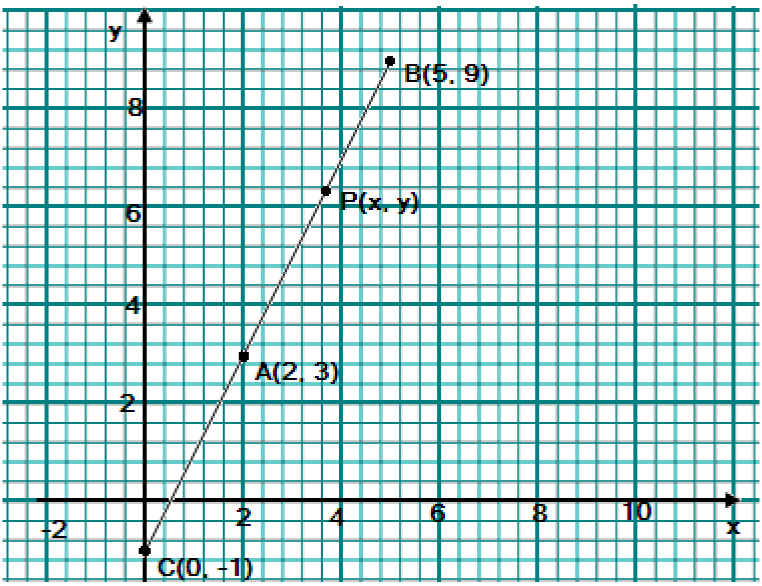
The equation of a straight line

The equation of a straight line gives the relationship between the x- and y-coordinates of the points that lie on it.
Consider a line passing through the points A(2, 3), B(5, 9) and C(0, -1). The gradient of line AB is given by:

gradient = $\frac{\text{change in y-coordinate}}{\textit{change in } x - \textit{coordinate}}$

$= \frac{9-3}{5-2}=\frac{6}{3} = 2$

The gradient of line AC = $\frac{3-(-1)}{2-0}=\frac{4}{2} = 2$



The gradients of lines AB and AC are the same. Since A, B and C lie on a straight line, then a straight line has a constant gradient.
Consider a point P, whose coordinates are (x, y) on line AB. The gradient of AP is the same as the gradient of AB. This is because a straight line has a constant gradient.

Thus, $\frac{y \quad 3}{x \quad 2} = 2$.

Therefore, $y - 3 = 2(x - 2) = 2x - 4$

$y = 2x - 1$.

This is the equation of the line passing through A, B, C and P.

In this case, coordinates of only two points are required to determine the gradient. The general point (x, y) enables us to find the equation of the line.

The equation of a line can be determined if:
(a) two points on the line are known,
(b) its gradient and a point on it are known.

Example 1.3

Find the equation of the line passing through the points (4, 5) and (8, 7).

Solution

Gradient = $\frac{7-5}{8-4} = \frac{2}{4} = \frac{1}{2}$.

Let (x, y) be a point on the same line. Using one of the points, say (4, 5),

gradient = $\frac{y-5}{x-4}$.

Therefore, $\frac{y-5}{x-4}. = \frac{1}{2}$

$$y - 5 = \frac{1}{2}(x - 4)$$

$$y - 5 = \frac{1}{2}x - 2$$

$$y = \frac{1}{2}x + 3$$

Example 1.4

Find the equation of a line whose gradient is 3 and passes through (-1, 4).

Solution

In this case the gradient is known. Let (x, y) be a point on the line.

Then, $\frac{y-4}{x-(-1)} = 3$

$$y - 4 = 3(x + 1)$$

$$y = 3x + 7$$

Exercise 1.1

1. Find the equations of the lines passing through the following pairs of points:

(a)	(1, 3); (4, 8)	(b)	(0, -2); (2, 5)
(c)	(8, 5); (-1, -6)	(d)	(7, -5); (-3, 1)
(e)	(-2, 3); (0, -3)	(f)	(15, 9); (-10, -4)
(g)	(6, -2); (3, -2)	(h)	(0, 3); (0.5, 9)

2. In each of the following, find the equation of a line whose gradient and a point through which it passes are:

(a)	$\frac{1}{2}$; (-1, 2)	(b)	2; (0, 4)
(c)	-3; (0, 0)	(d)	$\frac{2}{3}$; $\frac{1}{2}, \frac{1}{3}$
(e)	5; (3, 4)	(f)	$-\frac{5}{2}$; (4, 0)
(g)	0; (4, 3)	(h)	1; (-3, -2)

3. The gradients of two lines l_1 and l_2 are $-\frac{1}{2}$ and 3 respectively. Find their equations if they meet at the point (2, 3).

4. Line l_1 has a gradient of -1 and passes through the point (3, 0). Line l_2 has a gradient of $\frac{2}{3}$ and passes through the point (4, 4).Draw the two line on the same pair of axes and state their point of intersection.

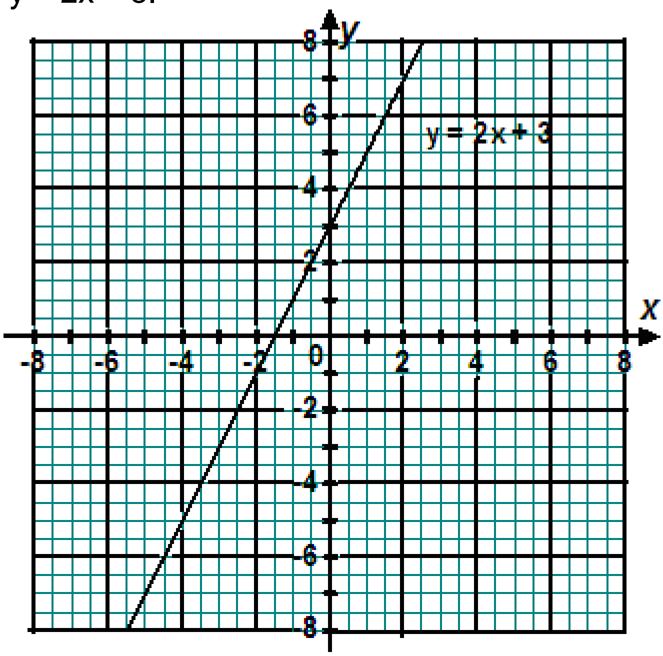
5. Line l_1 passes through the point (-1, 3) and has a gradient of 3. Line l_2 passes through the point (2, 3) and meets line l2 at the point (0, 6).
   (a) Find the equations of the two lines.
   (b) Draw the lines l_1 and l_2 on the same pair of axes.

The general equation of a straight line

Consider a line whose equation is y = 2x + 3.
We can obtain three points on this line by taking any values of x and substituting them in the equation to get y. Such as:

x	-1	1	2
y	1	5	7

Plotting points (-1, 1), (1, 5) and (2, 7) on a Cartesian plane gives the graph of the line y = 2x + 3.



The line cuts the y-axis at (0, 3). This point is called the **y-intercept** of the line. It is important to note that the y-intercept occurs when x = 0, which is the equation of the y-axis.
The gradient of the line y = 2x + 3 can be obtained by using any two points on the line.

Thus, using (1, 5) and (-1, 1), the gradient = $\frac{5-1}{1-(-1)} = \frac{4}{2} = 2$

For the line with equation y = 2x + 3 the gradient is 2 and the y-intercept is 3.

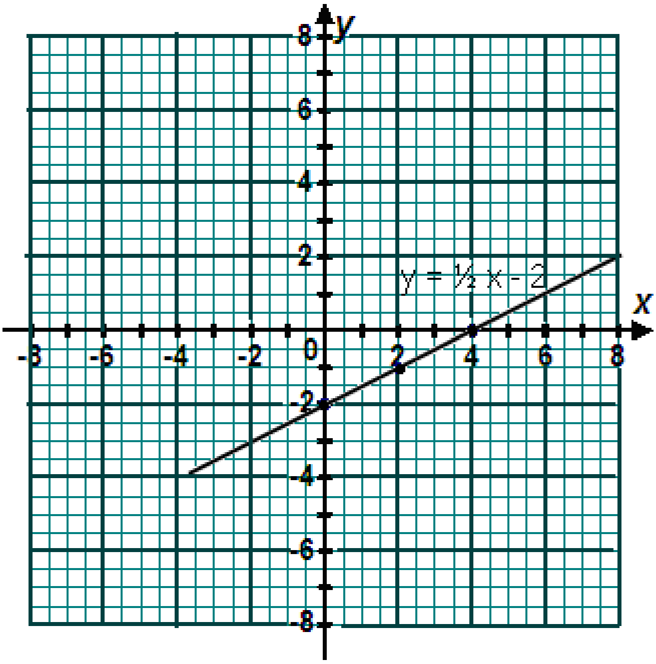
Consider a line whose equation is $y = \frac{1}{2}x - 2$.

Three points on this line are:

x	0	2	4
y	-2	-1	0

Plotting these points on the Cartesian plane gives the graph of the line

$y = \frac{1}{2}x - 2$.



Using the points (2, -1) and (4, 0), gradient is $\frac{-1-0}{2-4} = \frac{-1}{-2} = \frac{1}{2}$

The graph cuts the y-axis at (0, -2). Therefore, the line $y = \frac{1}{2}x - 2$ has a gradient of $\frac{1}{2}$ and the y-intercept is -2.

In general, a line whose equation is given by $y = mx + c$, has a gradient m and y-intercept c.

All equations of straight lines can be written in the form $y = mx + c$, where m and c are known

Example 1.5

Write down the gradient and the coordinates of the y-intercept of the following lines:

(a) $y = 5x + 4$ (b) $y = 2 - x$

(c) $2y = 6x - 3$ (d) $3y + 2x - 5 = 0$

Solution

We need to express each equation in the form $y = mx + c$.

(a) $y = 5x + 4$ is in the form $y = mx + c$.
Therefore, gradient $m = 5$ and y-intercept is at $(0, 4)$.

(b) $y = 2 - x$ can also be written as $y = -x + 2$.
So, gradient $m = -1$ and y-intercept is at $(0, 2)$

(c) $2y = 6x - 3$
In order to write it in the form $y = mx + c$, we divide both sides by 2.
Thus, $y = 3x - \frac{3}{2}$
Therefore, gradient = 3 and y-intercept is at $(0, -\frac{3}{2})$

(d) $3y + 2x - 5 = 0$
Writing in the form $y = mx + c$ gives,
$3y = -2x + 5$
$y = -\frac{2}{3}x + \frac{5}{3}$

Therefore, gradient = $-\frac{2}{3}$ and y-intercept is at $(0, \frac{5}{3})$

Example 1.6

Find the equation of a line whose gradient is 2 and passes through (4, 3).

Solution

The equation should be in the form $y = mx + c$, where $m = 2$ and $y = 2x + c$.
We need to find the value of c.
The line passes through (4, 3). Thus, when $x = 4$, $y = 3$, substituting 4 for x and 3 for y in the equation $y = 2x + c$ gives $3 = 2 \times 4 + c$. which means, $3 = 8 + c$.
So, $c = -5$
$y = 2x - 5$ is the required equation.

Exercise 1.2

1. Determine the gradients and the y-intercepts of the straight lines:

(a) $y = 8x + 1$ (b) $y = x$

(c) $y = 3 - 2x$ (d) $y + x = 0$

(e) $3y + x = 9$ (f) $2x + 5y + 10 = 0$

(g) $\frac{1}{2}y + \frac{1}{3}x = 2$ (h) $\frac{2}{5}y + \frac{1}{2}x + 5 = 0$

2. Show that the point (-1, -4) lies on the line $y = 3x - 1$
3. Show that the equation of the straight line passing through (0, k) and (k, 0) is $y + x = k$.
4. Given that the line $y = 3x + a$ passes through (1, 4), find the value of a.

The x-intercept of a line

It is important to remember that the equation of the y-axis is $x = 0$, and the equation of the x-axis is $y = 0$. We have already seen that the y-intercept occurs when $x = 0$. Similarly, the x-intercept occurs when $y = 0$. Therefore, given the equation of a line, its x-intercept can be obtained.

Example 1.7

Find the x and y intercepts of the line with equation $y = 5x + 6$.

Solution

The y-intercept occurs when $x = 0$. Therefore, $y = 5 \times 0 + 6$.
i.e. $y = 6$.
The y-intercept is 6.
The x-intercept occurs when $y = 0$. Therefore,
$0 = 5x + 6$
$-6 = 5x$
$\frac{6}{5} = x$

The x-intercept is $\frac{6}{5}$.

Exercise 1.3

Find the x and y intercepts of the lines given by each of the following equations:

1. $y = x + 3$
2. $y = 3x - 7$
3. $y = \frac{1}{2}x + \frac{4}{5}$
4. $x + y + 3 = 0$
5. $2y = 3x + 2$
6. $4x + 4y = 9$
7. $3x - 4y = 12$
8. $2x + 3y + 7 = 0$
9. $x + \frac{1}{2}y = 6$
10. $x = 7$
11. $y = -3$
12. $y = x$

The graph of a straight line

In order to draw the graph of a given equation, we need to obtain two or more points that lie on the line. Any value of x can be chosen and substituted in the equation to find the value of y.
Care must be taken to avoid values that give fractions.

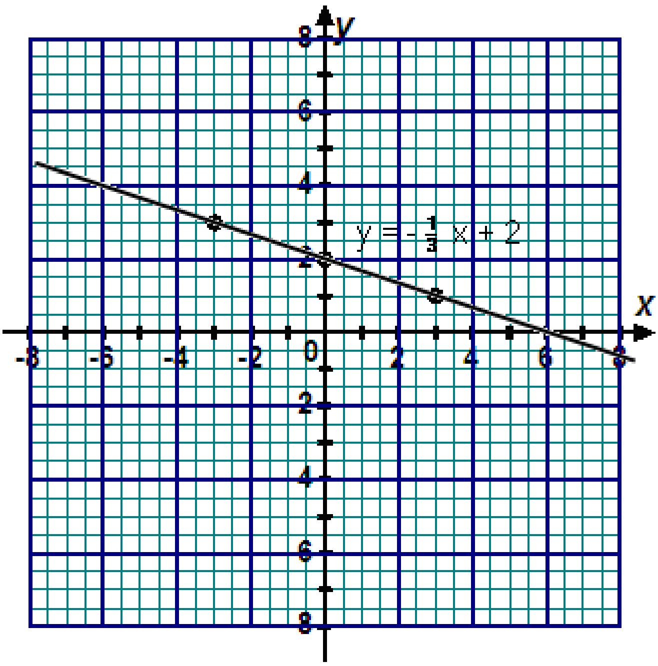
Example 1.8

Draw the graph of the line with an equation $y = -\frac{1}{3}x + 2$

Solution

Some points on the line are:

x	-3	0	3
y	3	2	1



Alternatively, the x- and y-intercepts can also be used to plot the points on the graph although you should avoid cases which give awkward fractions such as $\frac{1}{3}$, $\frac{1}{7}$ and so on.

Thus, in $y = -\frac{1}{3}x + 2$, when x = 0, y = 2 and when y = 0, x = 6.
The intercepts are at (0, 2) and (6, 0).

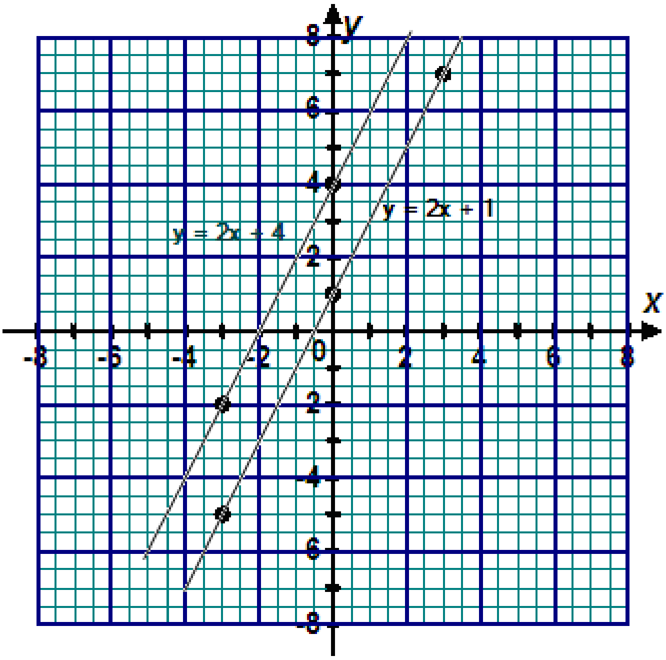
Exercise 1.4

Draw the graphs of the lines whose equations are given below:

1. $y = x + 3$
2. $y = x - 4$
3. $-2x + 1$
4. $y = \frac{1}{2}x - 3$
5. $y = \frac{2}{3}x + 4$
6. $y = -3x$
7. $y = \frac{3}{4}x - 1$
8. $y = 1$
9. $x = -2$
10. $2y = x + 1$

The gradients of parallel lines

Consider two lines l_1 and l_2 whose equations are $y = 2x + 4$ and $y = 2x + 1$ respectively.
When they are drawn, their graphs are as shown in the following figure.



The lines are parallel. Note that both lines have the same gradient, 2. Therefore, parallel lines have the same gradient. Conversely, lines that have the same gradient are parallel.

Example 1.9

Find the equation of a line which passes through the point (3,5) and is parallel to $y = -3x + 1$.

Solution

The equation of the required line is in the form $y = mx + c$. The gradient of the line $y = -3x + 1$ is -3.
Since the two lines are parallel, $m = -3$. Thus, $y = -3x + c$.
Therefore, $5 = -3 \times 3 + c$

$5 = -9 + c$

$c = 14$

The required equation is $y = -3x + 14$ or $y + 3x = 14$.

Exercise 1.5

1 Determine the gradients of the following pairs of equations and state whether their lines are parallel:

(a) $y = 2x - 7$
$3y = 6x + 2$

(b) $y = 4$
$y = -3$

(c) $y = 2x + 3$
$y = 4x + 6$

(d) $5y + 3x + 1 = 0$
$10y + 6x - 1 = 0$

(e) $\frac{1}{2}y + x = 2$
$3y + 2x = 0$

(f) $2x + y = 3$
$3x + y = 1$

(g) $x + 2y = 4$
$x + 3y = 6$

(h) $y = 2x + 3$
$2y = 4x - 7$

(i) $3y = 5x + 7$
$6y = 10x - 3$

(j) $5y = x + 2$
$4y = x + 3$

2. A line through the points (-2, 4) and (3, 5) is parallel to the line passing through the points (a, 6) and (-4, 1). Find a.

3. Line *l* is parallel to a line whose equation is $y = 4x - 7$ and passes through the point (1, -2). Find the equation of line *l*.

4. Find the equation of the line that is parallel to another line whose equation is $4y + 5x = 6$ and passes through the point (8, 5).

5. Find the equation of the line that is parallel to another line whose equation is $x + 2y + 8 = 0$ and passes through the point (-2, -3).

The gradients of perpendicular lines

If a line has a gradient *m*, then a line perpendicular to it has a gradient $-\frac{1}{m}$ so That the product of the gradients is

$$m \times -\frac{1}{m} = -1.$$

Example 1.10

Find the equation of a line perpendicular to another line whose equation is $2y + 3x = 1$ and passes through the point (-3, 1).

Solution

$2y + 3x = 1$ can be written as $2y = -3x + 1$.

Which means, $y = -\frac{3}{2}x + \frac{1}{2}$, and its gradient is $-\frac{3}{2}$.

The gradient of its perpendicular is the negative reciprocal of $-\frac{3}{2}$, which is $\frac{2}{3}$.

The equation of the perpendicular line is in the form $y = mx + c$, where $m = \frac{2}{3}$.

That is, $y = \frac{2}{3}x + c$

But the point (-3, 1) lies on the line. Therefore, $1 = \frac{2}{3}(-3) + c$.

Which means, $1 = -2 + c$

$c = 3$

The required equation is $y = \frac{2}{3}x + 3$ or $3y = 2x + 9$.

Exercise 1.6

1. The following are gradients of lines. In each case, state the gradient of the perpendicular line:

(a) 5 (b) 8

(c) -4 (d) -1

(e) $-\frac{1}{3}$ (f) $-\frac{2}{3}$

(g) $-4\frac{1}{3}$ (h) $1\frac{2}{5}$

(k) -0.7 (l) 0.5

2. State whether the following pairs of equations represent perpendicular lines

(a) $2y + 3x = 2$
$12y - 8x = 24$

(b) $y + x = 3$
$y = x + 4$

(c) $3y + 5x = 6$
$5y + 3x = 4$

(d) $y = 5 - 6x$
$\frac{1}{2}x = \frac{1}{12}x + 1$

3. Write down the equation of the line perpendicular to:

(a) $3x + 4y - 1 = 0$ and passes through (1, 2),

(b) $y = \frac{1}{2}x + \frac{1}{3}$ and passes through the origin,

(c) $3x - 2y + 7 = 0$ and passes through (-1, 0),

(d) $5y + x + 4 = 0$ and passes through (3, 5).

4. Find the equation of a line whose gradient is $-\frac{1}{2}$ and passes through the point (5, -2)

5. State which of the points (0, 3), (4, 6) and $(\frac{1}{2}, -2)$ lie on the line $y = 2x - 3$.

6. Determine whether each of the following points lie above or below the line $y = 4 - x$.

(a) (-2, 3) (b) (3, 3)

(c) (5, 0)

7. Find the equation of a line passing through the points (-1, -2) and (4, 6).

8. Determine the gradient and the y-intercept of the lines with equations:

(a) $3x + 2y + 5 = 0$ (b) $\frac{1}{2}y + 6 = x$

9. Find the equation of the line that passes through the point (2, -5) and is parallel to the line $y = 3 - 3x$.

10. Find the equation of the line perpendicular to $2x + 3y = 4$ that passes through the point (0, 3).

11. Determine the gradients and the coordinates of the x-intercept of the following equations:

(a) $3y = 2x + 9$

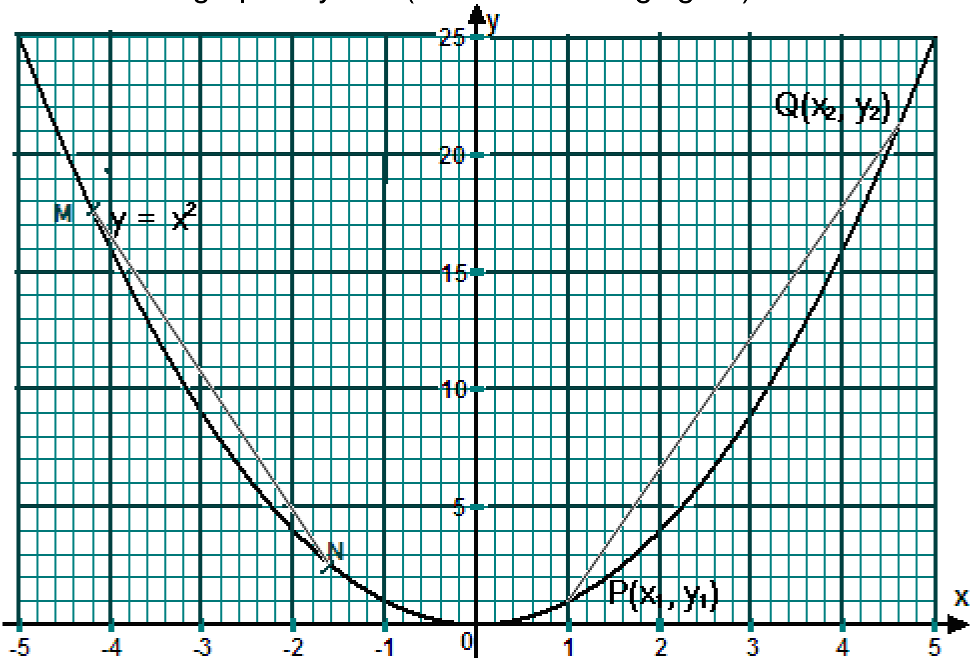
(b) $2y + 3x = 1$

(c) $y = 3 - 5x$

(d) $y + 3x = 14$

Rates of change

If a variable, y, is related to a variable, x, then as x changes, y also changes according to the rule relating them. We relate the change in y to the corresponding change in x by defining the average rate of change as: the change in y divided by the corresponding change in x.
Thus, if x_1 and x_2 are two values of x, and the corresponding values of y are y_1 and y_2, then the average rate of change of y as x changes from x_1 to x_2 is $\frac{y_2 \quad y_1}{x_2 \quad x_1}$.
Consider the graph of $y = x^2$ (see the following figure)



The average rate of change between P and Q is the gradient of chord PQ. This is given by $\frac{y_2 - y_1}{x_2 - x_1}$

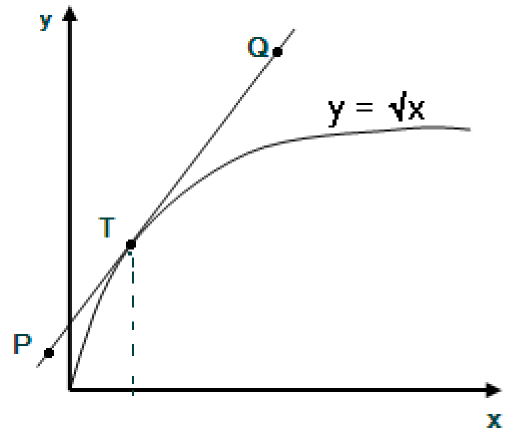
But $y_2 = x_2^2$ and $y_1 = x_1^2$. Therefore, the average rate of change is

$$\frac{x_2^2 - x_1^2}{x_2 - x_1} = \frac{(x_2 - x_1)(x_2 + x_1)}{x_2 - x_1} = x_2 + x_1.$$

The value of the result ($x_2 + x_1$) is different for different intervals of x. Thus the average rate of change from M to N has a different value of $x_2 + x_1$ from the one for P to Q.

The average rate of change occurs over a given **interval**. Suppose we want to find the rate of change at a particular point or, in the case of motion, at a particular time. This is said to be the rate of change at some instant and is called the **instantaneous rate of change**.

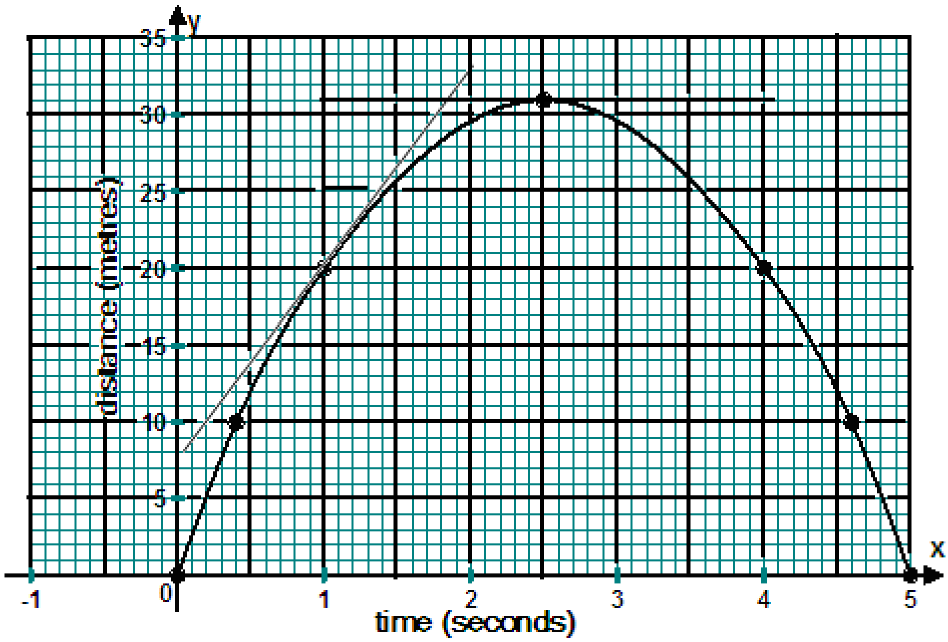
The following figure shows a sketch of the curve of $y = \sqrt{x}$. The instantaneous rate of change at point T is the gradient of tangent PQ.



The tangent to a curve is a straight line drawn such that it touches the curve only at one point, called the **point of contact**.

Example 1.11

The distance – time graph below shows the motion of a ball thrown upwards from the ground.



Use the graph to find:

(a) the average velocity of the ball from:
(i) t = 1 to t = 2 (ii) t = 2.5 to t = 4.
(b) the velocity of the ball when t = 1.
(c) the velocity of the ball when t = 2.5.

Solutions

(a) (i) when t = 1, s = 20 and when t = 2, s = 30. The average velocity is the gradient of the chord joining points (1, 20) and (2, 30). Therefore, the average velocity is

$$\frac{30-20}{2-1} = 10 \text{ m/s}$$

(ii) when t = 2.5, s = 31 and when t = 4, s = 20. The average velocity is the gradient of the chord joining points (2.5, 31) and (4, 20). Therefore, the average velocity is

$$\frac{20-31}{4-2.5} = -\frac{11}{1.5} = -7\tfrac{1}{3} \text{ m/s}$$

(b) The velocity of the ball when t = 1 is the gradient of the tangent to the Curve at t = 1.
From the graph, two points on the tangent are ((1, 20) and (2, 35).
Therefore, the velocity is $\frac{35-20}{2-1}$ = 15 m/s.

(c) The velocity of the ball when t = 2.5 is the gradient of the tangent to the curve at t = 2.5. This tangent is horizontal, thus, its gradient is zero. The velocity of the ball at t = 2.5 is zero. The height of the ball at this time is 31 m, which is the greatest height of the ball from the ground.

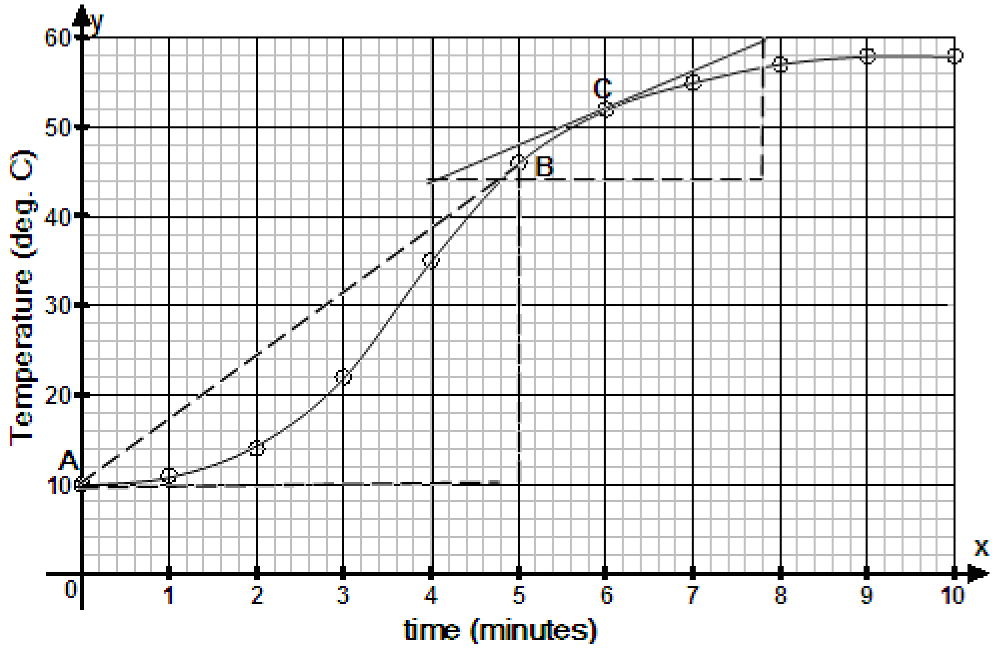
Example 1.12

A beaker is filled with liquid and heated slowly. The temperature is taken at intervals of one minute, the results being:

Time (t min)	0	1	2	3	4	5	6	7	8	9	10
Temp. (^{0}C)	10	11	14	22	35	46	52	55	57	58	58

Draw the graph of these readings on a scale 1 cm to 1 min horizontally and 1 cm = 5^0C vertically.
(a) Calculate the average rate of heating, in deg/min., during the first 5 min of the experiment and in the whole 10 minutes.
(b) Find the rate of increase of the temperature at the instant when t = 6 minutes.
(c) At what time is the rate of heating the greatest?

Solution



(a) The average rate of heating during the first 5 minutes is given by the gradient of chord AB, i.e. $\frac{46-10}{5-0}=\frac{36}{5}=7.2^{0}C/\min$.

(b) The rate of increase at the instant when t = 6 min. is given by the gradient of the tangent at point C, i.e. $\frac{60-44}{7.8-4}=4.211^{0}C/\min$.

(c) The rate of heating is greatest when t = 3 min. i.e. the point on the curve with greatest slope.

Distance - Time graphs

In a distance-time graph the distance is represented on the vertical axis and the time on the horizontal axis. However, before plotting any point, we must choose suitable scales for both axes. A suitable scale is the scale that will enable you to fit in all your points adequately. Also make sure that the intervals are uniform on each axis.

Example 1.13

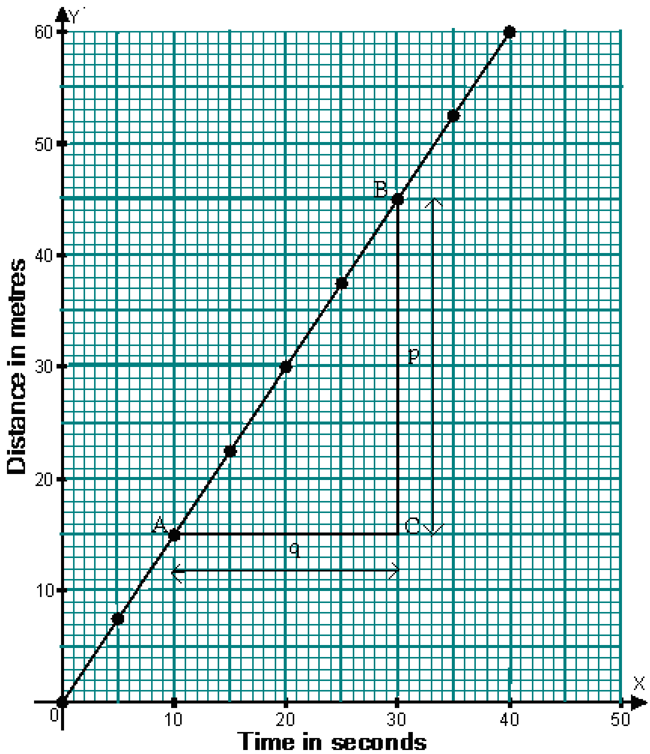
The table below shows the distance, in metres, walked along a road and the time taken, in seconds

.

Distance (m)	Time (s)
0	0
7.5	5
15.0	10
22.5	15
30.0	20
37.5	25
45.0	30
52.5	35
60.0	40

(a) Draw a distance-time graph to represent this information.
(b) Determine the gradient of the graph.

Solution



(b) Choose two suitable points on the line. For example, A and B. Draw a line from point B parallel to the vertical axis and another line from point A parallel to the horizontal axis both to meet at point C. Label length BC and AC as p and q respectively.

Gradient = $\frac{p}{q}$

$= \frac{45-15}{30-10} = \frac{30}{20}$ = 1.5 m/s. (The gradient represents speed).

No matter which two points we take, the gradient is the same. This means that the speed is constant (uniform). Also if the vertical axis represents displacement, then the gradient would represent velocity.

Example 1.14

In a bicycle race, a cyclist covered 70 km as follows: 30 km in 30 minutes, 10 km in $1\frac{1}{2}$ hours and 30 km in 30 minutes.

(a) Draw a distance-time graph for the journey.

(b) From the graph, determine the average speed for the whole journey.

Solution

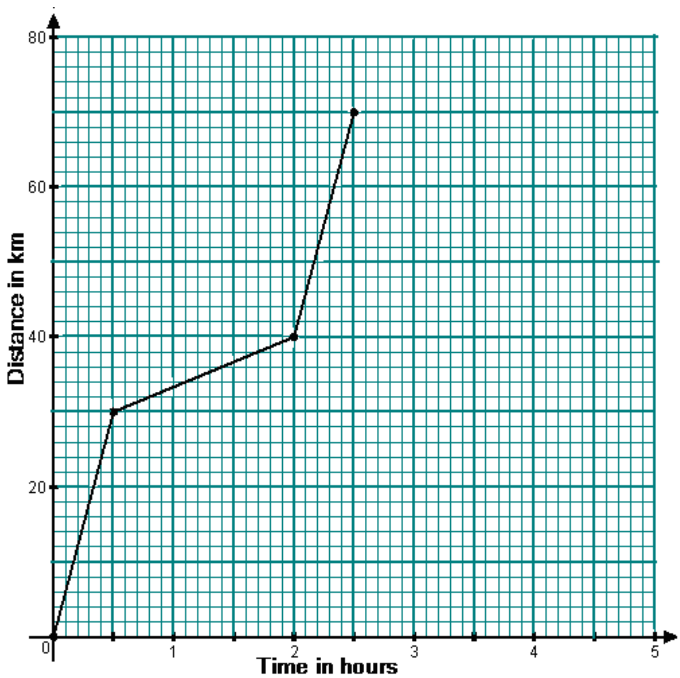
(a) In each stage of the journey, we need two points. That is the start and the end of each stage.

Stage I: (0, 0), $\left(\frac{1}{2}, 30\right)$

Stage II: $\left(\frac{1}{2}, 30\right)$, (2, 40)

Stage III: (2, 40) , ($2\frac{1}{2}$, 70)

Choose a suitable scale for each axis. In this case, in the vertical scale, 1 cm represents 20 km and in the horizontal scale, 1 cm represents 30 minutes. Draw the axes and plot the points.
Join the points in each stage with a straight line as shown in the figure below.



(b) The average speed is found by joining the first point of stage I to the last point of stage III. Then find the gradient of this line by choosing suitable points such as (0, 0) and $(1\frac{1}{4}, 35)$.

Gradient = (35 - 0) ÷ $(1\frac{1}{4} - 0)$ = 28 km/h.

Checking by calculation,

Average speed = $\frac{total\ distance}{total\ time}$ = 70 ÷ $\frac{5}{2}$ = 28 km/h.

Example 1.15

Two towns, X and Y, are 110 km apart. A cyclist, C_1, leaves Y at 9.00 a.m. and travels towards X at an average speed of 44 km/h. At the same time, another cyclist, C_2, leaves X and travels towards Y at an average speed of 25 km/h.

(a) On the same axes draw, distance-time graphs for each cyclist.

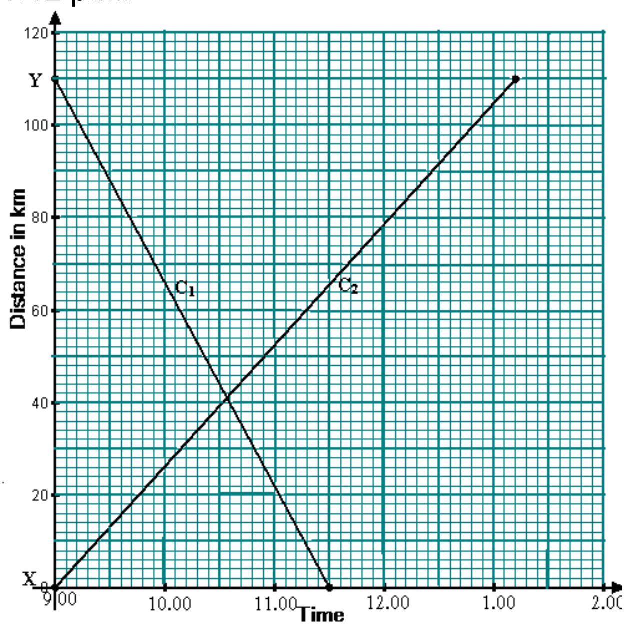
(b) From the graph determine:

  (i) when the two cyclists met,

  (ii) the distance cyclist C_1 had traveled before meeting cyclist C_2.

Solution

(a) Using suitable scales, mark on the vertical axis points X and Y at the correct distance. On the horizontal axis, mark the time starting from 9.00 a.m. then 10.00 a.m. etc. Since the speed for each motion is constant the graphs are straight lines. C_1 takes 2 hours and 30 minutes and arrives at X at 11.30 a.m. C_2 takes 4 hours and 12 minutes. She arrives at Y at 1.12 p.m.



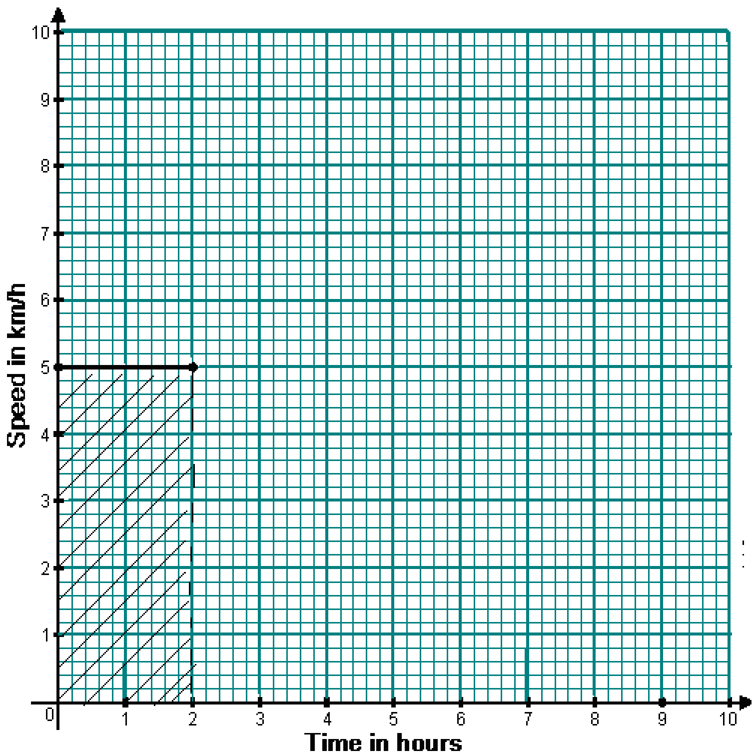
(b) (i) They meet at 10.36 a.m.
(ii) C_1 had covered 70 km from Y which is the intersection of graphs C_1 and C_2.

Speed-time graphs

In speed-time graphs, speed is represented on the vertical axis and time on the horizontal axis. If the motion is in a specific direction then velocity should be represented on the vertical axis.

Example 1.16

A man walking at a steady speed covers a distance of 10 km in 2 hours. We therefore say that his speed is 5 km/h.
Note: steady speed means that the speed stayed the same or constant.



The graph shows that speed remained constant. The graph is called a **speed - Time graph**.
Since the speed is constant, the distance traveled = speed × time

$= 5 \times 2 = 10$ km.

If we shade the graph as in the figure above, we notice that the shaded area is equal to the distance traveled by the man in 2 hours.

Area of a rectangle = length × width

$= 2\text{ h} \times 5\text{ km/h}$
$= 10\text{ km/h}.$

In general, in speed-time graphs, the distance traveled is obtained by finding the area enclosed by the graph and the axes.

Exercise 1.7

1. An athlete sets off in a long distance race at a steady speed of 4 m/s. Draw a graph to show the distance *d* meters he has covered after *t* seconds, taking values of *t* from 0 to 25. Use scales of 2 cm to 5 seconds and 2 cm to 20 meters. From your graph find:
   (a) the distance covered after: (i) 5 sec. (ii) 22 sec. (iii) 10.5 sec.
   (b) the time taken to run: (i) 40 m (ii) 70 m (iii) 36 m
   (c) Give an equation connecting *d* and *t.*

2. Dauda left school at 4.00 pm and walked home at 6 km/h.
   (a) How far from the school was he after
   (i) 30 minutes (ii) 20 minutes (iii) 10 minutes?
   (b) Draw a distance – time graph extending from 4.00 pm to 5.00 pm. Take 2 cm to 10 minutes and 2 cm to 1 km.
   (c) Use the graph to find:
   (i) the distance Dauda had walked by 4.40 pm.
   (ii) the time he reached home if he lived 4.5 km from the school.

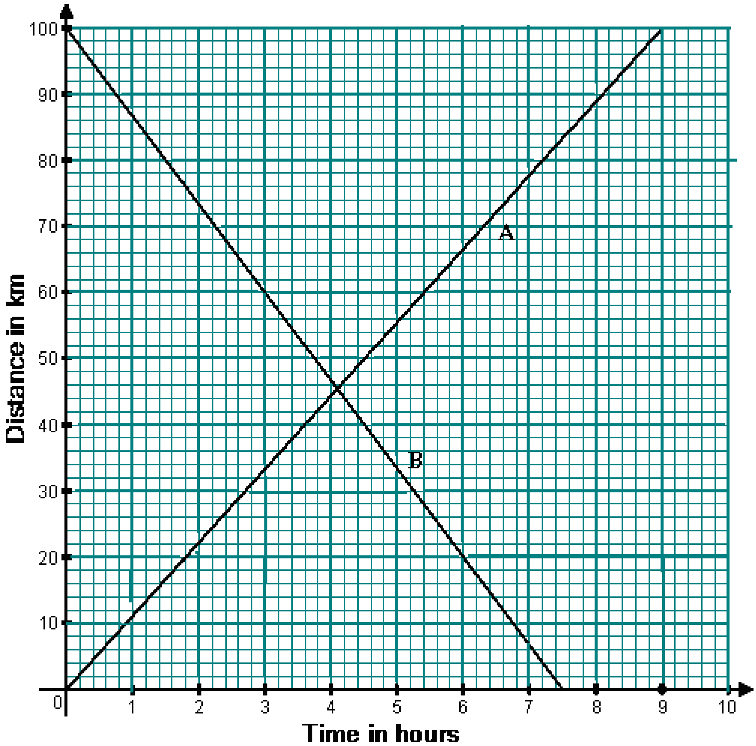
3. From Kampala to Jinja is 80 km. At 08:15 hours a lorry leaves Kampala for Jinja at 60 km/h. Draw a graph showing the distance traveled with respect to time. Use scales of 1 cm to 5 minutes and 1 cm to 10 km. From your graph find:
   (a) how far from Kampala the lorry is when the time is:
   (i) 08:20 hours (ii) 08:50 hours (iii) 09:15 hours.
   (b) the time when the lorry reaches Jinja.

4. In a 100-m race, an athlete took 9.8 seconds.
   (a) Draw a distance-time graph to represent this information.
   (b) From the graph determine:
   (i) the average velocity correct to 1 decimal place,
   (ii) the distance traveled in three seconds,
   (ii) the time taken to cover 80 m.

5. A safari rally car travels at an average speed of 180 km/h for 5 hours between two towns.
   (a) Draw a distance-time graph to illustrate its motion.

(b) From the graph determine:
    (i) the distance traveled in 36 minutes,
    (ii) the time it took to cover 290 km.

6. From the distance-time graph drawn below, calculate
   (a) the velocity, in km/h, of A and B respectively,
   (b) how
       (i) long they took before meeting
       (ii) far they had traveled before meeting.



7. Rashidah left her house at 8.00 am for Kampala city. After traveling at a constant speed for 1 hour, she arrived at Mukono town, 15 km from her home. She rested for 15 minutes and then proceeded at a constant speed towards Kampala 30 km from Mukono, she arrived in Kampala at 12.15 p.m.
   (a) Using a scale of 2 cm represents 1 hour and 1 cm represents 5 km draw a distance-time graph for the whole journey.
   (b) From the graph, determine Rashidah's average speed from:
       (i) her house to Mukono town,
       (ii) Mukono town to Kampala city.
   (c) Calculate her average speed for the whole journey.

8. Three towns, R, P and Q are such that Q is between P and R. From P to Q is 105 km and from Q to R is 15 km. A car leaves Q at 9.00 a.m. and travels towards P at an average speed of 60 km/h. At the same instant, a cyclist leaves P and travels towards Q at an average speed of 24 km/h. An ambulance leaves R at 9.30 a.m. and travels towards P via Q at an average speed of 160 km/h.
   (a) On the same axes, draw a distance-time graph for each vehicle.
   (b) From the graph determine:
      (i) when the ambulance caught up with the car,
      (ii) when the ambulance and the cyclist met,
      (iii)when the car and the cyclist met,
      (iii) the distance the car had travelled before the ambulance caught up with it.
9. The distance from Masaka to Mbarara is 140 km. At 08 00 hours a motorist leaves Masaka for Mbarara at 80 km/h. At 08 40 hours another motorist leaves Mbarara for Masaka at 70 km/h. On the same pair of axes draw the two graphs for the motorists.
   Take 3 cm to 1 hour on the horizontal axis and 1 cm to 10 km on the vertical axis.
   From your graphs find:
   (i) the distance from Masaka to the place where the two motorist met;
   (ii) the distance between the two motorists at 09 00 hours;
   (iii) the tme when they met.

Exercise 1.8

1. The speed v m/s of a car after t s is given by the following table:

t	0	2	4	6	8	10	12	14	16	18
v	0	15	26	35	39	40	36	28	16	0

   Represent this information on a graph using 1 cm to represent 2 s horizontally and 1 cm to 5 m/s vertically.
   Estimate the rate at which the speed is changing when t = 14 and find the average rate of change in the speed between 2 and 7 s.
2. The temperature of water in an electric kettle is T^0C after the heating element has been switched off for t min. The following table shows the relationship between T^0C and t min.

t (min)	0	1	2	3	4	5	6	7	8	9	10
T^0C	100	76	60	48	38	32	28	24	22	21	20

   (a) Calculate, in deg./min, the average rate of cooling during the first ten minutes.
   (b) Estimate the rate of cooling, in degrees per min, when t = 2.

3. The stopping distance d metres of a car traveling at s km/h is given by the formula $d = \frac{s^2}{40}$

Copy and complete the following table:

s	10	20	30	40	50	60
d	2.5	--	22.5	--	--	--

Draw a graph to show the relationship between d and s. Use your graph to find s when the stopping distance is 25 m. Find also the gradient of the graph at the point when s is 20.

4. A stone is thrown vertically upwards into the air and its height h metres above the ground is given by h = 16(5t – t2) where t is the time in seconds for which the stone has been in flight. Calculate the values of h for t = 0, 1, 2, 3, 4, 5.

Draw a graph to show the relationship between h and t. Use your graph to find:

(a) the greatest height of the stone,
(b) the times at which the stone is 50 m above the ground,
(c) the length of time for which the stone is more than 70 m above the ground and
(d) the speed of the stone at 4 seconds.

5. A train traveled between two stations A and B, 7 km apart, and the following table shows the time (in min) since leaving A and the distance (km) from A.

Time (min)	0	2	4	6	8	10	12
Distance (km)	0	0.25	1.15	2.83	5.40	6.65	7

Draw the graph of these readings using a scale of 1 cm to represent 1 min. on the horizontal axis and 2 cm to 1 km on the vertical axis.

From your graph, estimate (by drawing a tangent) the speed of the train in km/min when the train has traveled 6 km.

Two minutes after this train left A, another train passed through B and traveled towards A at a steady speed of 60 km/h. Using the same axes and scale, draw the graph to represent its journey between B and A and use it to find the distance from A when the trains pass each other.

6. The table below shows the distance, s metres, of a particle from point P after t seconds.

t	0	1	2	3	4	5
s	0	2	8	18	32	50

(a) Draw the distance-time graph using 1 cm to represent 0.5 seconds and 1 cm to represent 5 m.

(b) From your graph find:

(i) the average speed of the particle during the third second.

(ii) the average speed of the particle between the first and the fifth seconds

(c) Using a suitable tangent, determine the rate of change when t = 3 seconds.

7. Draw a graph to represent the following:
A man starts at 7 a.m. from point P and walks at a speed of 6 km/h for 4 hours. He then rests for 1 hour before proceeding at 5 km/h untilhe reaches his destination, Q, at 1 p.m. His return journey is made at a constant speed of 4 km/h without stops. From the graph find

(a) his distance from the starting point at

(i) 10 a.m. (ii) 12 noon

(b) the time taken to travel

(i) the first 14 km,

(ii) from P to Q

(iii)…the whole trip.

8. Man A begins at point P and travels a distance of 360 km to point Q at a speed of 60 km/h. Man B, beginning at the same time travels at a constant speed from P to Q in 2 hours more. Draw graphs t represent this information.
From the graphs find:

(i) when A and B are 50 km apart,

(ii) when and where B overtakes A,

(iii) the distance apart of A and B when they have been traveling 3.5 hours

Chapter 2

Equations

Simultaneous equations

To find the value of two unknowns in a problem, two different equations must be given that relate the unknowns to each other. These two equations are called **simultaneous** equations.

Substitution method

This method is used when one equation contains a unity quantity of one of the unknowns, as in equation (2) of the example below.

Example 2.1

Solve the following pair of simultaneous equations.

$3x - 2y = 0$ ....(1)

$2x + y = 7$ ....(2)

Solution

Procedure:

- label the equations so that the working is made clear.
- In this case, write y in terms of x from equation (2).
- Substitute this expression for y in equation (1) and solve to find x.
- Find y from equation (2) using this value of x.

$2x + y = 7$ ....(2)

$y = 7 - 2x$

substituting in (1)

$3x - 2(7 - 2x) = 0$

$3x - 14 + 4x = 0$

$7x = 14$

$x = 2$

substituting in (1)

$2 \times 2 + y = 7$

$y = 3$

The solutions are x = 2, y = 3.

These values of x are the only pair which simultaneously satisfy *both* equations.

Exercise 2.1

Use the substitution method to solve the following:

1.	$2x + y = 5$ $x + 3y = 5$	2.	$x + 2y = 8$ $2x + 3y = 14$
3.	$3x + y = 10$ $x - y = 2$	4.	$2x + y = 5$ $x - y = -3$
5.	$4x + y = 14$ $X + 5y = 13$	6.	$x + 2y = 1$ $2x + 3y = 4$
7.	$2x + y = 5$ $3x - 2y = 4$	8.	$2x + y = 13$ $5x - 4y = 13$
9.	$7x + 2y = 19$ $x - y = 4$	10.	$b - a = -5$ $a + b = -1$
11.	$a + 4b = 6$ $8b - a = -3$	12.	$a + b = 4$ $2a + b = 5$
13.	$3m = 2n - 6\frac{1}{2}$ $4m + n = 6$	14.	$2w + 3x - 13 = 0$ $x + 5w - 13 = 0$
15.	$x + 2(y - 6) = 0$ $3x + 4y = 30$	16.	$2x = 4 + z$ $6x - 5z = 18$
17.	$3m - n = 5$ $2m + 5n = 7$	18.	$5c - d - 11 = 0$ $4d + 3c = -5$

Elimination method

Use this method when the first method is unsuitable (some prefer to use it for every question)

procedure

- Label the equations so that the working is made clear.
- Choose an unknown in one of the equations and multiply the equations by a factor or factors so that this unknown has the same coefficient in both equations.

- Eliminate this unknown from the two equations by subtracting (or adding) the two equations , depending on whether the equal coefficients have like or opposite signs. Then solve for the remaining unknown.
- Substitute in the first equation and solve for the eliminated unknown.

Example 2.2

Solve the simultaneous equations:
$x + 2y = 8$; $2x + 3y = 14$

solution

Let: $x + 2y = 8$ ....(i)   and $2x + 3y = 14$ ....(ii)
Multiplying equation (i) by 2 gives,

$2x + 4y = 16$ ....(iii)
$2x + 3y = 14$ ....(ii)

Since the equal coefficients (of x) are both positive, we subtract (ii) from (iii), to get

$4y - 3y = 16 - 14$
$y = 2$

substituting for y in (i) gives

$x + 2(2) = 8$
$x + 4 = 8$
$x = 8 - 4 = 4$

The solutions are $x = 4$ and $y = 2$.

Example 2.3

Solve: $2x + 3y = 5$ ....(i)
$5x - 2y = -16$ .... (ii)

Solution

(i) × 2 $4x + 6y = 10$ ....(iii)
(ii) × 3 $15x - 6y = -48$ .... (iv)
Since the equal coefficients (of y) have opposite signs, we add the equations.
That is, (iii) + (iv) gives

$19x = -38$
$x = -2$

substituting for y in either equation (i) or (ii),

$2(-2) + 3y = 5$, using equation (i)
$-4 + 3y = 5$
$3y = 9$
$y = 3$

The solutions are $x = -2$, $y = 3$.

Exercise 2.2

Use the elimination method to solve the following:

1. $2x + 5y = 24$
   $42 + 3y = 20$

2. $5x + 2y = 13$
   $2x + 6y = 26$

3. $3x + y = 11$
   $9x + 2y = 28$

4. $x + 2y = 17$
   $8x + 3y = 45$

5. $3x + 2y = 19$
   $x + 8y = 21$

6. $2a + 3b = 9$
   $4a + b = 13$

7. $2x + 3y = 23$
   $3x + 4y = 15$

8. $3x + 8y = 27$
   $4x + 3y = 13$

9. $2x + 7y = 17$
   $5x + 3y = -1$

10. $5x + 3y = 23$
    $2x + 4y = 12$

11. $7x + 5y = 32$
    $3x + 4y = 23$

12. $3x + 2y = 4$
    $4x + 5y = 10$

13. $3x + 2y = 11$
    $2x - 2 = -3$

14. $3x + 2y = 7$
    $2x - 3y = -4$

15. $x + 2y = -4$
    $3x - y = 9$

16. $5x - 7y = 27$
    $3x - 4y = 16$

17. $3x - 2y = 7$
    $4x + y = 13$

18. $x - y = -1$
    $2x - y = 0$

19. $y - x = -1$
    $3x - y = 5$

20. $x - 3y = -5$
    $2y + 3x + 4 = 0$

21. $x + 3y - 7 = 0$
    $2y - x - 3 = 0$

22. $3a - b = 9$
    $2a + 2b = 14$

23. $x + 2y = 4$
    $3x + y = 9\frac{1}{2}$

24. $2x - y = 5$
    $\frac{x}{4} + \frac{y}{3} = 2$

25. $3x - y = 17$
    $\frac{x}{5} + \frac{y}{2} = 0$

26. $3x - 2y = 5$
    $\frac{2x}{3} + \frac{y}{2} = \frac{7}{9}$

27. $2x = 11 - y$
    $\frac{x}{5} \quad \frac{y}{4} = 1$

28. $4x - 0.5y = 12.5$
    $3x + 0.8y = 8.2$

29. $0.4x + 3y = 2.6$
    $x - 2y = 4.6$

30. $-3c + 4d = 4$
    $9c - 2d = 3$

Graphical solution

This method involves drawing the graphs of the given equations on the same coordinate axes. The coordinates of the point of intersection of these lines satisfy both equations and hence will be the solution to the simultaneous equations. Since two lines can only intersect in at most one point, there will be at most one pair of solutions to two simultaneous linear equations

Example 2.4

Solve the following pair of simultaneous equations using graphical method.

$3x + y = 6$, $x - y = 2$

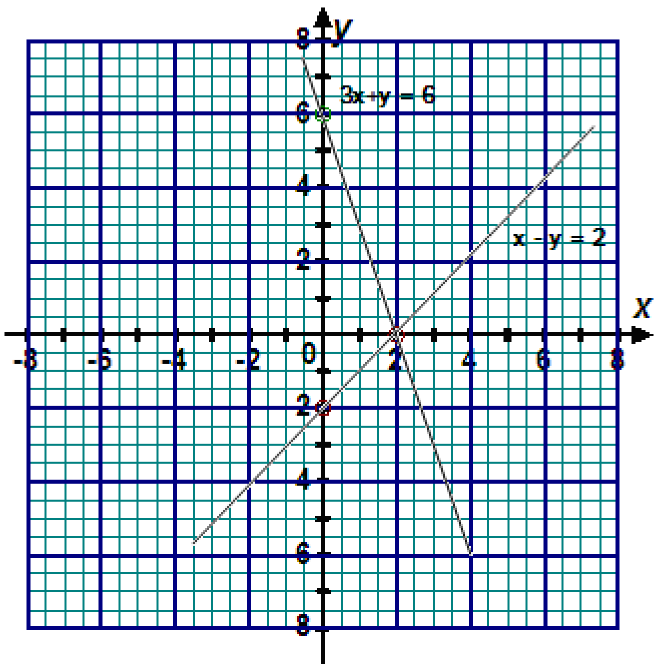
Solution:

$3x + y = 6$

x	0	1	3
y	6	3	-3

$x - y = 2$

x	0	4	-2
y	-2	2	-4



Point of intersection (2, 0) is the only ordered pair which satisfies both equations, i.e. (2, 0) lies on both lines.
Therefore, the solutions are $x = 2$, $y = 0$

Problem solving involving simultaneous equations.

Many problems in everyday life involve more than one unknown quantity and simultaneous equations can often be used to find the required values of these quantities.
Firstly, the unknown quantities must be given appropriate pronumerals. Then the information is translated into equations. If there are two unknowns, there must be two pieces of information leading to two equations before the problem can be solved. Since the problem is originally in words, the algebraic solution must be translated back into words to give the final answer.

Example 2.5

The sum of two numbers is 20 and their difference is 2. Find the numbers.

Solution

Let the larger number be x and the smaller number be y. Then

$x + y = 20$ . . . . . .(i)

$x - y = 2$ . . . . . . (ii)

To eliminate y, (i) + (ii)

$2x = 22$

$x = 11$

Substitute x= 11 into equation (i) to find y

$11 + y = 20$

$y = 9$

So the numbers are 11 and 9.

Example 2.6

The total cost of tickets to a show for 2 adults and 3 children is sh. 16,000 whilst the cost for 3 adults and 2 children is sh. 19,000. Find the cost of an adult ticket and of a child's ticket.

Solution

Let the cost of an adult's ticket be sh. A
Let the cost of a child's ticket be sh.c

$2a + 3c = 16,000$ ………….(i)

$3a + 2c = 19,000$ …………..(ii)

To eliminate c:

$2 \times \text{(i)} - 3 \times \text{(ii)} \Rightarrow -5a = -25,000$

$a = 5,000$

substitute a = 5,000 in (i)

$10,000 + 3c = 16,000$

$3c = 6,000$

$c = 2,000$

So an adult ticket costs sh. 5,000 and a child's ticket costs sh. 2,000.

Exercise 2.3

Solve each problem by forming a pair of simultaneous equations:

1. Find two numbers with a sum of 15 and a difference of 4.
2. Twice one number added to three times another number gives 21. Find the numbers, if the difference between them is 3
3. The line, with equation $y + ax = c$, passes through the points (1, 5) and (3, 1). Find a and c
4. A cyclist completes a journey of 500 m in 22 seconds, part of the way at 10 m/s and the remainder at 50 m/s. How far does she travel at each speed
5. A bag contains forty coins, all of them either Sh. 200 or sh. 500 coins. If the value of the money in the bag is sh. 10,500, find the number of each kind.
6. Thirty tickets were sold for a concert, some at sh. 10,000 and the rest at sh. 5,000. If the total raised was sh. 800,000, how many had the cheaper tickets?
7. The wage bill for five men and six women workers is sh. 6,700,000, while the bill for eight men and three women is sh. 6,100,000. Find the wage for a man and for a woman.
8. The denominator of a fraction is 2 more than the numerator. If both denominator and numerator are increased by 1 the fraction becomes $\frac{2}{3}$. Find the original fraction.
9. Find two numbers where three times the smaller number exceeds the larger number by 5 and the sum of the numbers is 11.
10. A straight line passes through the points (2, 4) and (-1, -5). Find its equation.
11. A wallet containing sh. 40,000 has three times as many sh.1,000 notes as sh. 5,000 notes. Find the number of each kind.
12. At the present time a man is four times as old as his son. Six years ago he was 10 times as old. Find their present ages.

Quadratic equations

Introduction

When an equation of the form y = $ax + b$ ($a \neq 0$ and a, b are constants) is plotted on a Cartesian plane, the graph is a straight line. Hence we call an expression of the form $ax + b$ a **linear expression** and an equation of the form $ax + b = 0$ is called a linear equation.
Thus 3x + 5 is a linear expression,
y = 3x + 5 represents a straight line graph.
When an equation of the form $y = ax^2 + bx + c$ ($a \neq 0$; a, b and c are constants) is plotted on a Cartesian plane, the graph is a curve known as a parabola.

An expression of the form $ax^2 + bx + c$ is called a **quadratic expression**.
An equation of the form $ax^2 + bx + c = 0$ is called a quadratic equation.
Thus, $2x^2 + 3x + 1$, $3x^2 + 5$ and $x^2 + 3x$ are quadratic expressions, whereas, $2x^2 + 3x + 1 = 0$; $3x^2 + 5 = 0$ and $x^2 + 3x = 0$ are quadratic equations.

Solving quadratic equations by factors method

We solve an equation by finding the value of the unknown. For any two numbers, p and q, if pq = 0, then, either:

(a) p = 0 which means 0 × q = 0
(b) q = 0 which means p × 0 = 0
(c) p = q = 0 which means 0 × 0 = 0

Similarly, if (x + 2)(x+ 3) = 0, then either x + 2 = 0 or x + 3 = 0.
Solving for x, gives either x = -2 or x = -3.
Thus, in order to solve a quadratic equation, the quadratic expression is factorized so that the equation is in the form (x + a)(x + b) = 0.

Example 2.7

Factorize $x^2 + 7x + 6 = 0$. Hence solve the equation.

Solution

$x^2 + 7x + 6 = 0$
Factorizing, $x^2 +7x + 6$, gives (x + 6)(x + 1) = 0.
Therefore, x + 6 = 0 or x + 1 = 0; which means x = -6 or x = -1

These are the only two values of x which satisfy the equation $x^2 + 7x + 6 = 0$.
We can check these solutions by substituting each of them in the equation.
Thus, when x = -6,
$(-6)^2 + 7(-6) + 6 = 36 - 42 + 6 = 0$

And when x = -1
$(-1)^2 + 7(-1) + 6 = 1 - 7 + 6 = 0$

Note: *Every quadratic equation has two solutions*.

Example 2.8

Solve: $x^2 + x - 72 = 0$

Solution

$x^2 + x - 72 = 0$
$\Leftrightarrow (x - 8)(x + 9) = 0. \quad \Rightarrow x - 8 = 0$ or $x + 9 = 0$
$\therefore \quad x = 8$ or $x = -9$.

Example 2.9

Solve: $x^2 - x - 29 = 1$

Solution

Always ensure that the quadratic expression is equated to zero. This is the only time the method used in the examples above can apply.
Thus, $x^2 - x - 29 = 1$ should be rewritten as $x^2 - x - 29 - 1 = 0$. That is, $x^2 - x - 30 = 0$.

The factors of 30, whose sum is 1, are -6 and 5.
Therefore, $(x - 6)(x + 5) = 0$.
Either $x - 6 = 0$ or $x + 5 = 0$
$\therefore x = 6$ or $x = -5$
The roots are -5 and 6.

Example 2.10

Solve:
(a) $x^2 - 49 = 0$ (b) $x^2 - 6x = 0$
(c) $x^2 - 16x + 64 = 0$ (d) $6x^2 + 5x - 4 = 0$

Solutions

(a) $x^2 - 49 = 0$ can be written as $x^2 - 7^2 = 0$
$(x - 7)(x + 7) = 0$
$x - 7 = 0$ or $x + 7 = 0 \Rightarrow x = 7$ or $x = -7$.
The roots are -7 and 7.

(b) Factorizing $x^2 - 6x = 0$ gives $x(x - 6) = 0$
Either $x = 0$ or $x - 6 = 0$
$\Rightarrow x = 0$ or $x = 6$
The roots are 0 and 6.

(c) $x^2 - 16x + 64 = 0$
Factorizing $x^2 - 16x + 64 = 0$ gives $(x - 8)(x - 8) = 0$
Either $x - 8 = 0$ or $x - 8 = 0$
$\Rightarrow x = 8$ or $x = 8$
The roots are 8 and 8.

Note: *$x^2 - 16x + 64$ is a perfect square and therefore it has identical factors. The equation $x^2 - 16x + 64 = 0$ has two equal roots.*

(d) $6x^2 + 5x - 4 = 0$

When the coefficient of x^2,(in this case it is 6), in the quadratic expression is numerically greater than 1, we proceed as follows when factorizing:

- Multiply the coefficient of x^2 by the constant term, i.e. $6 \times -4 = -24$.
- Find the factors of -24 whose sum is 5, (the coefficient of x), i.e. -3 and 8.
- Rewrite the equation as:

$6x^2 + (-3 + 8)x - 4 = 0$

$\Rightarrow 6x^2 - 3x + 8x - 4 = 0$

$\Rightarrow 3x(2x - 1) + 4(2x - 1) = 0$

$\Rightarrow (2x - 1)(3x + 4) = 0$

Then, either $2x - 1 = 0$ or $3x + 4 = 0$

$\Rightarrow \quad 2x = 1$ or $3x = -4$

$\therefore \quad x = \frac{1}{2}$ or $x = -\frac{4}{3}$.

Example 2.11

Factorize $3x^2 - 22x + 7$. Hence solve $3x^2 - 22x + 7 = 0$

Solution

$3x^2 - 22x + 7$

Multiplying 3 by 7, gives 21. The factors of 21 whose sum is -22, are -1 and -21.

Then, $3x^2 - 22x + 7 \Leftrightarrow 3x^2 + [-1 + (-21)]x + 7$

$\Leftrightarrow 3x^2 - 1x - 21x + 7$

$\Leftrightarrow x(3x - 1) - 7(3x - 1)$

$\Leftrightarrow (3x - 1)(x - 7)$

Hence, $3x^2 - 22x + 7 = (3x - 1)(x - 7)$.

The equation $3x^2 - 22x + 7 = 0$ can be written as $(3x - 1)(x - 7) = 0$

Either $3x - 1 = 0$ or $x - 7 = 0$

$3x = 1$ or $x = 7$

$\therefore \quad x = \frac{1}{3}$ or $x = 7$

Exercise 2.4: Solve.

1. $(x + 4)(x + 2) = 0$
2. $(x - 5)(x - 3) = 0$
3. $(x - 5)(x + 7) = 0$
4. $(x - 9)^2 = 0$
5. $(x + 13)^2 = 0$
6. $x^2 + x - 12 = 0$
7. $x^2 + 9x + 14 = 0$
8. $x^2 - 11x - 12 = 0$
9. $u^2 + 6u + 9 = 0$
10. $t^2 - t - 42 = 0$

11. $a^2 - 2a + 1 = 0$
12. $y^2 + 8y = 0$
13. $x^2 + 10x = 24$
14. $x^2 = 4x - 3$
15. $v^2 - 36 = 0$
16. $4x^2 - 9 = 0$
17. $1 - y^2 = 0$
18. $81 - 16q^2 = 0$
19. $x^2 = 6x$
20. $5x^2 = 45$
21. $6x(2x + 3) = 0$
22. $-4x(3x - 5) = 0$
23. $(3x - 2)(2x + 1) = 0$
24. $6x^2 - 29x + 35 = 0$
25. $6x^2 - x + 1 = 0$
26. $(4x - 3)(3x - 4) = 0$
27. $4y^2 - 48y = 16$
28. $3y^2 - 24y = 3$

Solutions of quadratic equations by formula

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are given by the formula

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Use this formula only after trying (and failing) to factorise.

Example 2.12

Solve the equation $2x^2 - 3x - 4 = 0$.

Solution

In this case a = 2, b = -3, c = -4.

$$x = \frac{-(-3) \pm \sqrt{((-3)^2 - (4 \times 2 \times -4))}}{2 \times 2} =$$

$$= \frac{3 \pm \sqrt{(9 + 32)}}{4} = \frac{3 \pm \sqrt{41}}{4} = \frac{3 \pm 6.403}{4}$$

Either $x = \frac{3 + 6.403}{4} = 2.35\,(2\text{ d.p})$ or $\frac{3 - 6.403}{3} =$ -0.85 (2 d.p)

Example 2.13

Solve the equation $2x^2 + 7x = 2$

Solution

Writing the equation in the general form $ax^2 + bx + c = 0$ we get
$2x^2 + 7x - 2 = 0$
Hence a= 2, b = 7, c = -2
Substitute these values in

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$= \frac{-7 \pm \sqrt{47 - (4 \times 2 \times -2)}}{2 \times 2} = \frac{7 \pm \sqrt{65}}{4}$$

Hence x = 0.27 or x = -3.77 (2 d.p)

Example 2.14

Solve: $2x(x - 1) = (x + 1)^2 - 5$

Solution

First we re-arrange the terms in the equation as follows.

$2x(x - 1) = (x + 1)^2 - 5$

$\Leftrightarrow 2x^2 - 2x = x^2 + 2x + 1 - 5$

$2x^2 - 2x - x^2 - 2x - 1 + 5 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

$x = 2$

In this example the quadratic equation has a repeated root

Exercise 2.5

Solve the following, giving answers to two decimal places where necessary:

1. $2x^2 + 11x + 5 = 0$
2. $3x^2 + 11x + 6 = 0$
3. $6x^2 + 7x + 2 = 0$
4. $3x^2 - 10x + 3 = 0$
5. $5x^2 - 7x + 2 = 0$
6. $6x^2 - 11x + 3 = 0$
7. $2x^2 + 6x + 3 = 0$
8. $x^2 + 4x + 1 = 0$
9. $5x^2 - 5x + 1 = 0$
10. $x^2 - 7x + 2 = 0$
11 $2x^2 + 5x - 1 = 0$
12. $3x^2 + x - 3 = 0$
13. $3x^2 + 8x - 6 = 0$
14. $3x^2 - 7x - 20 = 0$
15. $2x^2 - 7x - 15 = 0$
16. $x^2 - 3x - 2 = 0$
17. $2x^2 + 6x - 1 = 0$
18. $6x^2 - 11x - 7 = 0$
19. $3x^2 + 25x + 8 = 0$
20. $3y^2 - 2y - 5 = 0$
21. $2y^2 - 5y + 1 = 0$
22. $\frac{1}{2}y^2 + 3y + 1 = 0$
23. $2 - x - 6x^2 = 0$
24. $3 + 4x - 2x^2 = 0$
25. $1 - 5x - 2x^2 = 0$
26. $3x^2 - 1 + 4x = 0$
27. $5x - x^2 + 2 = 0$
28. $24x^2 - 22x - 35 = 0$

29. $36x^2 - 17x - 35 = 0$

30. $20x^2 + 17x - 63 = 0$

31. $x^2 + 2.5x - 6 = 0$

32. $0.3y^2 + 0.4y - 1.5 = 0$

33. $10 - x - 3x^2 = 0$

34. $x^2 + 3.3x - 0.7 = 0$

35. $x^2 = 6 - x$

36. $x(x + 10) = -21$

37. $3x + 2 = 2x^2$

38. $x^2 + 4 = 5x$

39. $6x(x + 1) = 5 - x$

40. $(2x - 1)^2 = (x - 1)^2 + 8$

41. $2x + 2 = \frac{7}{x} - 1$

42. $\frac{2}{x} + \frac{2}{x+1} = 3$

43. $3x(x+2) - x(x-2) + 6 = 0$

44. $(x - 3)^2 = 10$

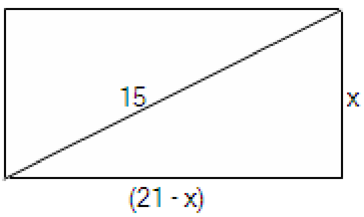
Problems solved by quadratic equations

Example 2.15

The perimeter of a rectangle is 42 cm. If the diagonal is 15 cm, find the width of the rectangle.

Solution

Let the width of the rectangle be x cm.
Since the perimeter is 42 cm, the sum of the length and the width is 21 cm.
Therefore, length of rectangle = (21 - x) cm.



By Pythagoras' theorem

$x^2 + (21 - x)2 = 152$
$x^2 + 441 - 42x + x^2 = 225$
$2x^2 - 42x + 216 = 0$
$x^2 - 21x + 108 = 0$
$(x - 12)(x - 9) = 0$
$x = 12$ or $x = 9$

note that the dimensions of the rectangle are 9 cm by 12 cm, whichever value of x is taken.
Therefore, the width of the rectangle is 9 cm.

Example 2.16

A man bought a certain number of golf balls for sh. 2,000. If each ball had cost

Sh. 200 less, he could have bought five more for the same money. How many golf balls did he buy?

Solution

Let the number of balls be x.

Cost of each ball = sh. $\frac{2000}{x}$

If five more balls had been bought, cost of each ball now = $\frac{2000}{(x+5)}$

Therefore, . $\frac{2000}{x}$ - $\frac{2000}{(x+5)}$ = 200

Multiplying by x,

x. . $\frac{2000}{x}$ - x. $\frac{2000}{(x+5)}$ = 200x

multiply by (x+5)

2000(x+5) – x. $\frac{2000}{(x+5)}$ (x+5) = 200x(x+5)

$2000x + 10,000 - 2000x = 200x^2 + 1000x$

$200x^2 + 1000x - 10,000 = 0$

$x^2 + 5x - 500 = 0$

$(x - 20)(x + 25) = 0$

x = 20 or x = -25

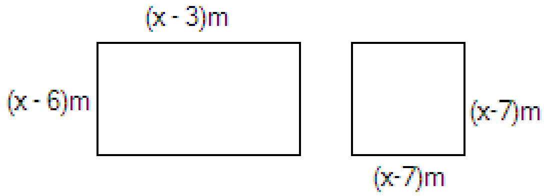
We discard x = -25 as meaningless.

The number of balls bought = 20.

Exercise 2.6

Solve by forming a quadratic equation:

1. Two numbers which differ by 3, have a product of 88. Find them.
2. The product of two consecutive odd numbers is 143. Find the numbers.
3. The length of a rectangle exceeds the width by 7 cm. If the area is 60 cm^2, find the length of the rectangle.
4. The length of a rectangle exceeds the width by 2 cm. If the diagonal is 10 cm long, find the width of the rectangle.
5. The area of the rectangle exceeds the area of the square by 24 m^2. Find x.



6. The perimeter of a rectangle is 68 cm. If the diagonal is 26 cm, find the dimensions of the rectangle.

7. A man walks a certain distance due North and then the same distance plus a further 7 km due East. If the final distance from the starting point is 17 km, find the distances he walks North and East.

8. A farmer makes a profit of sh. X on each of the (x + 5) eggs her hen lays. If her total profit was sh. 84,000, find the number of eggs the hen lays.

9. A number exceeds four times its reciprocal by 3. Find the number.

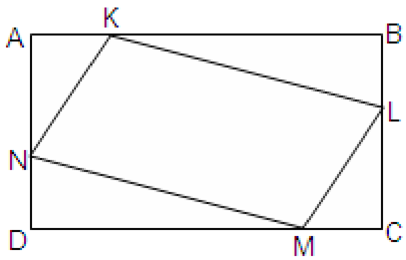
10. Two numbers differ by 3. The sum of their reciprocals is $\frac{7}{10}$; find the numbers.

11. A cyclist travels 40 km at a speed of x km/h. Find the time taken in terms of x. Find the time taken when his speed is reduced by 2 km/h. If the difference between the time is 1 hour, find the value of x.

12. A train normally travels 240 km at a certain speed. One day, due to bad weather, the train's speed is reduced by 20 km/h so that the journey takes two hours longer. Find the normal speed.

13. An aircraft flies a certain distance on a bearing of 135^0 and then twice the distance o a bearing of 225^0. Its distance from the starting point is then 350 km. find the length of the first part of the journey.

14. In the following figure, ABCD is a rectangle with AB = 12 cm and BC = 7 cm. AK = BL = CM = DN = x cm. If the area of KLMN is 54 cm^2 find x.



15. The numerator of a fraction is 1 less than the denominator. When both numerator and denominator are increased by 2, the fraction is increased

$\frac{1}{12}$. Find the original fraction.

16. The perimeters of a square and a rectangle are equal. One side of the rectangle is 11 cm and the area of the square is 4 cm^2 more than the area of the rectangle. Find the side of the square.

17. In a right angled triangle PQR, $\angle Q = 90^0$, QR = x cm, PQ = (2x – 2) cm and PR = 30 cm. Find x.

Table of values for a given quadratic relation

To draw a curve $y = x^2 - 2x - 6$ for the values of x between -3 and 5, we prepare a table of values in the following way:

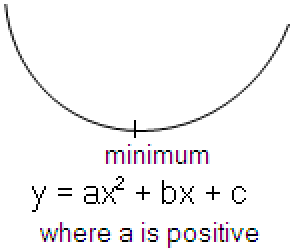
x	-3	-2	-1	0	1	2	3	4	5
x^2	9	4	1	0	1	4	9	16	25
-2x	6	4	2	0	-2	-4	-6	-8	-10
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
$y= x^2 - 2x - 6$	9	2	-3	-6	-7	-6	-3	2	9

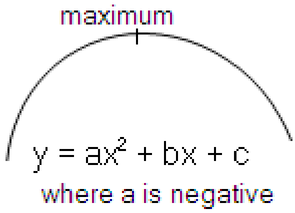
The points (-3, 9), (-2, 2), (-1, -3), (0, -6), …, (5, 9) are then plotted on a graph paper and joined by a smooth curve.
Alternatively, to find the y-coordinates we can substitute the values of x in the equation $y = x^2 - 2x - 6$ in the following way:
when x = -3, $y = (-3)^2 - 2(-3) - 6 = 9 + 6 - 6 = 9$,
when x = -2, $y = (-2)^2 - 2(-2) - 6 = 4 + 4 - 6 = 2$
when x = -1, $y = (-1)^2 - 2(-1) - 6 = 1 + 2 - 6 = -3$, e.t.c.

Graphs of quadratic functions

Quadratic function

'y is a function of x', i.e. for every value assigned to x, there is always a corresponding value of y. Then these pairs of values of x and y can be plotted and a distinctive graph, a *curve* or *a straight line*, will be obtained. This is, in fact, the graph of the function.
If the function is of the first degree, of which the general form is y = mx + c, then it is called a **linear function** and the graph is a straight line.
The expression $ax^2 + bx + c$, where a, b and c are constants, is called a **quadratic function** of x or a function of the second degree (highest power of x is 2). If such an expression is plotted against x, the resulting curve will have one of the shapes shown below.





The coefficient of x^2 determines which way up the curve is.

Example 2.17

Draw the graphs of $y = x^2$ and $y = -x^2$ for values of x between -2 and +2.

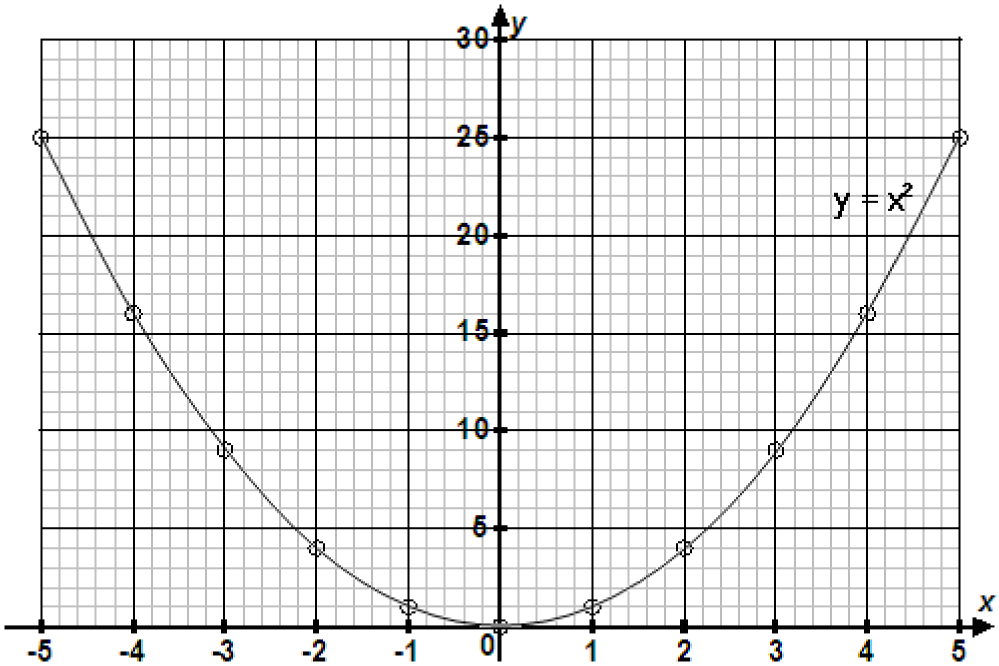
Solution

Graph of $y = x^2$:

Make a table of values for (x, y)

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	25	16	9	4	1	0	1	4	9	16	25

Plot these points on a graph paper and label the axes x, y. Since the values of y are all positive, x-axis should be drawn near the bottom of the paper.
As the values of x are between -2 and +2, y-axis should be taken in the middle of the paper. The scale chosen must be convenient but not necessarily the same o both axes. The resulting diagram should cover an area which is more than half the page on which it is drawn (but the whole diagram should fit on the same page).



Note:

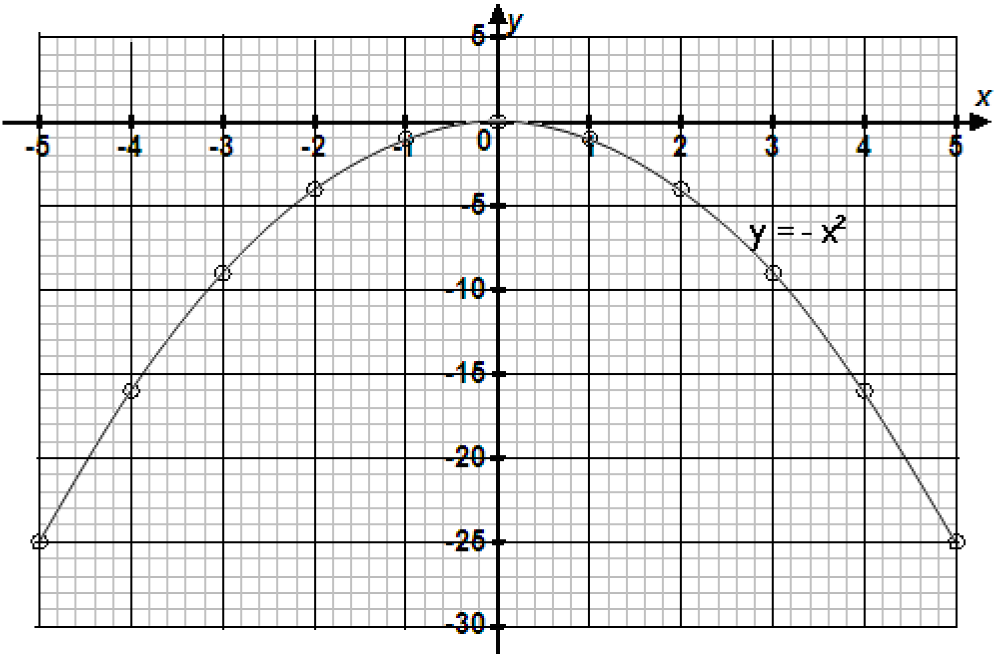
(a) The minimum value of the curve is 0 at the origin, i.e. the curve turns at the origin

(b) The curve is symmetrical about the y – axis.

The graph of $y = -x^2$

All values of y are numerically the same as the corresponding values of y in $y = x^2$ but are **negative**. The shape of the curve will be the same but inverted.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	-25	-16	-9	-4	-1	0	-1	-4	-9	-16	-25



Note:

(a) The maximum value of the curve is 0 at the origin.

(b) The curve is symmetrical about the y-axis.

Example 2.18

Draw the graph of $y = x^2 - 3x + 2$, for values of x between -1 and +4.

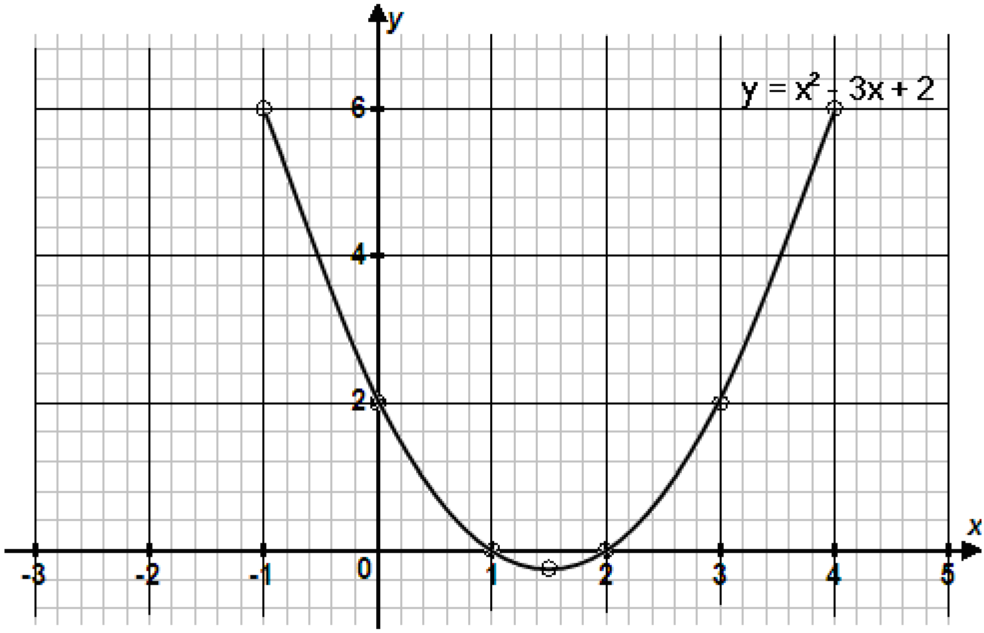
Solution

Make a table of values of x and y:

x	-1	0	1	1.5	2	3	4
x^2	1	0	1	2.25	4	9	16
-3x	3	0	-3	-4.5	-6	-9	-12
+2	+2	+2	+2	+2	+2	+2	+2
y	6	2	0	-0.25	0	2	6

Since both x = 1 and x = 2 give y = 0, it is advisable to take a value of x between 1 and 2, i.e. 1.5.

Note that the maximum value of the function is -0.25.



Graphical solution of equations

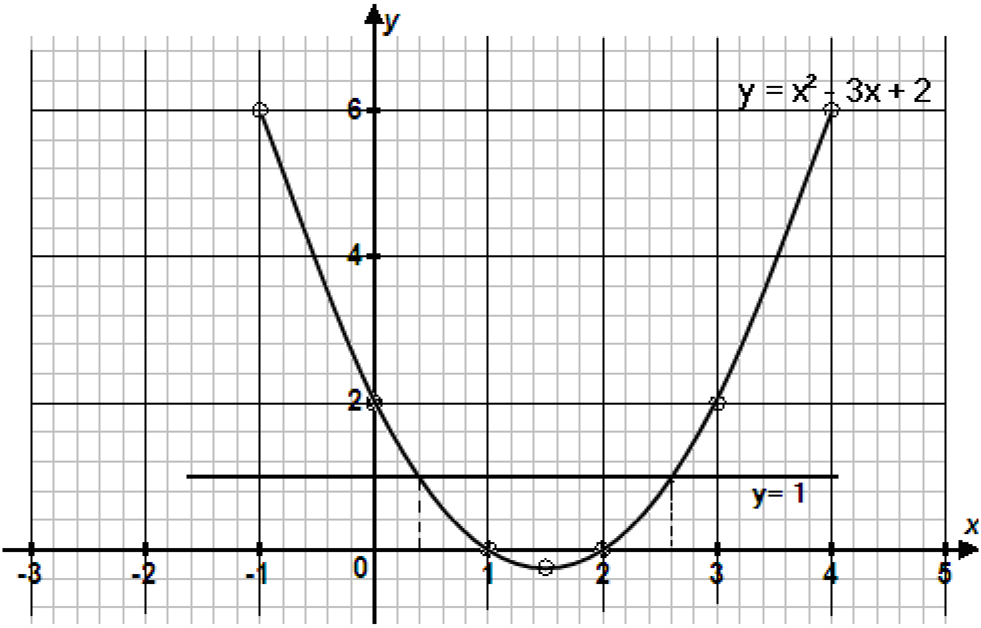
(Use the graph of example 2.18)

(a) The curve is the graph of function $x^2 - 3x + 2$. It crosses the x-axis at points where x = 1 and x = 2, i.e. these are the points on the curve for which y = 0.
In short, y = 0 when x = 1 and x = 2.
i.e. $x^2 - 3x + 2 = 0$ when x = 1 and also when x = 2.
Therefore **x = 1** and **x = 2** are the solutions of the equation $x^2 - 3x + 2 = 0$.

In general, if $y = ax^2 + bx + c$, the solutions of $ax^2 + bx + c = 0$ are the values of x at the points where the curve crosses the x-axis.

(b) Use the above graph to solve $x^2 - 3x + 1 = 0$
Equation to be solved is $x^2 - 3x + 1 = 0$
i.e. $x^2 - 3x = -1$
substituting $x^2 - 3x = -1$ in the equation of the curve $y = x^2 - 3x + 2$,
we get $y = x^2 - 3x + 2$
$= -1 + 2$
$= 1.$

We need the points on the curve when y = 1. Draw the line y = 1 on the same axes. The values of x at the points of intersection of the line and the curve are the solution of the equation.
From the graph, values of x are **0.4** and **2.6**.



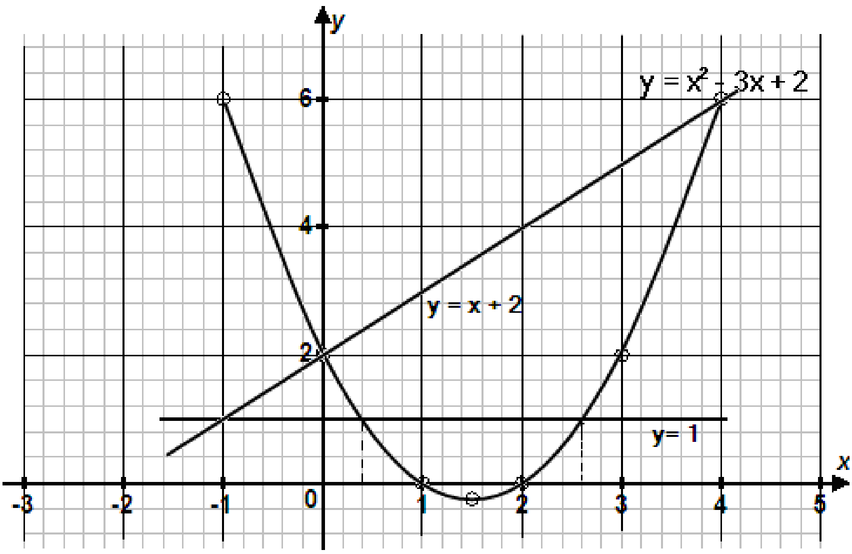
Alternative method:

Equation of graph: $y = x^2 - 3x + 2$
Equation to be solved: $\underline{0 = x^2 - 3x + 1}$
Subtracting: $y = 1$ is the equation of the line to be drawn.

(c) Use the above graph to solve $x^2 - 4x = 0$

$y = x^2 - 3x + 2$
$\underline{0 = x^2 - 4x}$ (subtracting)
$y = x + 2$

Draw a straight line graph of the function $y = x + 2$ on the same axes as shown below:



The line crosses the curve at the points where x = 0 and x = 4.
Hence the solutions of $x^2 - 4x = 0$ are **x = 0** and **x = 4**.

Range of values (Use graph of example 2.18)

Example 2.20

Find the range of values of x for which the function $x^2 - 3x + 2$
(a) is negative,
(b) is less than (x + 2),
(c) is greater than 1.

Solution

(a) $x^2 - 3x + 2$ is negative (i.e. below the x-axis) for values of x from 1 to 2. The range is **$1 < x < 2$**.
(b) $x^2 - 3x + 2$ is less than x + 2 (i.e. below the line y = x + 2) fro x = 0 to x = 4, i.e. **$0 < x < 4$**.
(c) In the given domain $-1 \leq x \leq 4$, the function $x^2 - 3x + 2$ is greater than 1 in two ranges:
From -1 to 0.4 and from 2.6 to 4,
i.e. **$-1 < x < 0.4$** and **$2.6 < x < 4$**.

Intersecting graphs

Example 2.21

Draw the graph of $y = 1 + x - 2x^2$, taking values of x in the domain $-3 \leq x \leq 3$.
Using the same scale and axes, draw the graph of y = 2x - 5.
Use your graphs to answer the following questions:
(a) What is the maximum value of the function $1 + x - 2x^2$?
(b) Write down the x-coordinates of the points of intersection of the functions $y = 1 + x - 2x^2$ and y = 2x - 5. Show that these values of x satisfy the equation $2x^2 + x - 6 = 0$.

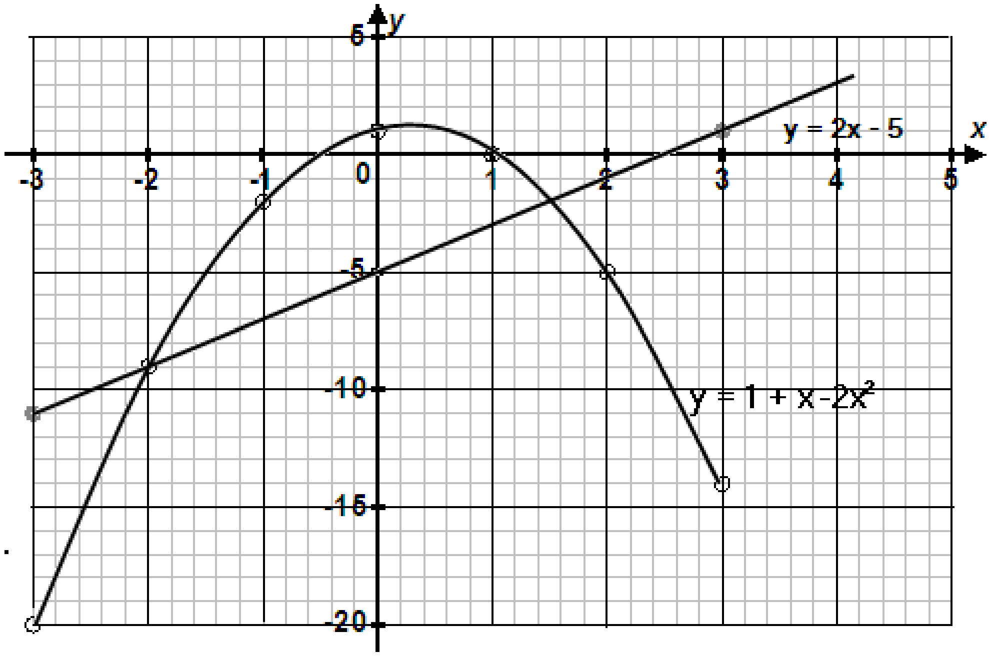
Table of values for $y = 1 + x - 2x^2$

x	-3	-2	-1	0	1	2	3
1	1	1	1	1	1	1	1
x	-3	-2	-1	0	1	2	3
$-2x^2$	-18	-8	-2	0	-2	-8	-18
y	-20	-9	-2	1	0	-5	-14

Table for y = 2x - 5

x	-3	0	3
y	-11	-5	1

When plotted, a curve and a straight line as shown in the figure below are obtained.



From the graphs,

(a) Maximum value of $1 + x - 2x^2 \approx$ **1.1**

(b) Values of x where the straight line intersects the curve are x = -2 and x = 1.5. At these points both $1 + x - 2x^2$ and $2x - 5$ are equal.

Hence $1 + x - 2x^2 = 2x - 5$

$1 + 5 + x - 2x - 2x^2 = 0$

$6 - x - 2x^2 = 0$

Or $2x^2 + x - 6 = 0$

Solutions of $2x^2 + x - 6 = 0$ are x = **-2** and x = **1.5**

Exercise 2.7

1. Draw the graph of $y = x^2 - 4x + 4$ for values of x from -1 to +5. Solve from your graph the equations:
   (a) $x^2 - 4x + 4 = 0$ (b) $x^2 - 4x + 1 = 0$,
   (c) $x^2 - 4x - 1 = 0$.

2. Draw the graph of the function $x^2 - 6x + 5$ for $-1 \leq x \leq 7$. Find the least value of this function and the corresponding value of x. Use your graph to solve the equations:
   (a) $x^2 - 6x + 5 = 0$ (b) $x^2 - 6x = 1$.

3. Draw the graph of $y = 2x^2 - 7x - 2$ for values of x from -3 to +3. State the least value of y and the corresponding value of x. Use your graph to solve the equations:

(a) $2x^2 - x = 4$, (b) $2x^2 - x + 6 = 0$,
(c) $2x^2 - x - 4 = 2x$.

4. Draw the curve of $y = \frac{1}{4}x^2$ for domain $-4 \le x \le 4$. Using the same scale and axes, draw the curve of $y = \frac{1}{4}x^2 - 3$.
What transformation will map the first curve onto the second curve? Use the first curve to find values of x such that $x^2 = 8$.

5. Draw the graph of the function $2 - x - x^2$ fro domain $-3 \le x \le 3$. Find the maximum value of the function and the corresponding value of x. Use your graph to solve the equation $x^2 + x = 2$.

6. Draw the of the function y = 2x2 - 7x - 2 for values of x from -1 to 5. Find the minimum value of the function and the corresponding values of x. By drawing suitable lines on the same axes, solve, where possible, the following equations:
(a) $2x^2 - 7x = 2$, (b) $2x^2 - 8x + 4 = 0$
(c) $2x^2 - 7x + 7 = 0$.

7. Copy and complete the following table of values for $y = 6 + 3x - 2x^2$.

x	-2	-1	0	0.5	1	2	3
y	-8	--	--	--	7	4	--

Draw the graph of $y = 6 + 3x - 2x^2$ for domain $-2 \le x \le 3$, taking 2 cm as one unit on the y-axis. Use the graph to obtain solutions of the equations:
(a) $6 + 3x - 2x^2 = 0$, (b) $2 + 3x - 2x^2 = 0$,
(c) $3 + x - x^2 = 0$.

8. Draw the graph of $y = x^2 - x - 2$ after completing the following table for values of x and y.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y	--	1.75	0	-1.25	--	--	--	-1.25	--	1.75	--

By drawing a suitable straight line on your graph, solve the equation
$x^2 - 2x - 2 = 0$

9. Draw the graph of $y = \frac{1}{2}(x - 3)(x + 1)$ for domain $-3 \le x \le 4$, using 2 cm to represent one unit on both axes. From your graph find the values of x for which $(x - 3)(x + 1) = 2$.

10. Draw the graph of the function $x^2 - 3x$ for domain $-3 \le x \le 4$, using scales of 2 cm to one unit on both axes.

Use your graph to find:
(a) the least value of the function and the corresponding value of x,
(b) the range of values of x for which the function is negative,
(c) the solutions of the equations $x^2 - 3x = 1$ and $x^2 - 2x - 1 = 0$.

11. Given that y = (3x + 1)(2x - 5), copy and complete the following table for values of x and y.

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5	4
y	14	6	-	-	-12	-11	-	-	-	23	-

Draw the graph of y = (3x + 1)(2x - 5) from x = -1 to x = 4, using a scale of 2cm to one unit on the x-axis and 1 cm to 5 units on the y-axis.
Draw on the same axes the graph of y = 8x - 7.
By considering the points of intersection of the two graphs, a certain quadratic equation in x can be solved. Write down and simplify the equation and obtain its roots from the graphs.

12. Some of the values of the function $3 - 3x - x^2$ for domain $-5 \leq x \leq 2$ are given in the following table. Complete the table and use it to draw the graph of the function.

x	-5	-4	-3	-2.5	-2	-1.5	-1	0	1	2
$3 - 3x - x^2$	-7	….	3	4.25	5	…	5	3	-1	…

From your graph find:
(a) The greatest value of the function and the corresponding value of x,
(b) The range of values of x for which the function has values greater than 2.

13. Copy and complete the following table for value of: x(5 - x).

x	0	0.5	1	2	2.5	3	4	4.5	5
x (5-x)		2.25	4				4	2.25	0

Draw the graph of y = x(5 - x) from x = 0 to x = 5,using a scale of 2 cm to 1 unit on each axis.

With the same axes, draw the graph of $y = \frac{6}{x+1}$ from x = 0 to x = 5.

Use your graphs to obtain:
(a) an equation whose solutions are the points of intersection of the two graphs.;
(b) the range of values of x for which $x(5 - x) > \frac{6}{x+1}$.

14. Assuming the graph of $y = x^2 + x + 1$ has been drawn; find the equation of the line which should be drawn to solve the equations:

(a) $x^2 + x + 1 = 6$ (b) $x^2 + x + 1 = 0$
(c) $x^2 + x - 3 = 0$ (d) $x^2 - x + 1 = 0$
(e) $x^2 - x - 3 = 0$

15. Assuming the graph of $y = x^2 - 8x - 7$ has been drawn; find the equation of the line which should be drawn to solve the equations:

(a) $x = 8 + \frac{7}{x}$ (b) $2x^2 = 16x + 9$

(c) $x^2 = 7$ (d) $x = \frac{4}{x-8}$

(e) $2x - 5 = \frac{14}{x}$.

16. Draw the graph of $y = x^2 + 4x + 5$ for $-6 \le x \le 1$. Draw suitable straight lines to find approximate solutions of the equations:
(a) $x^2 + 3x - 1 = 0$ (b) $x^2 + 5x + 2 = 0$

17. Draw the graph of $y = 2 + 3x - 2x^2$ for $-2 \le x \le 4$.
(a) Draw suitable straight lines to find approximate solutions of the equations:
(i) $2 + 4x - 2x^2 = 0$
(ii) $2x^2 - 3x - 2 = 0$
(c) Find the range of values of x for which $2 + 3x - 2x^2 \ge -5$.

18. Draw the graph of $y = \frac{18}{x}$ for $1 \le x \le 10$, using scales of 1 cm to 1 unit on both axes. Use the graph to solve approximately:

(a) $\frac{18}{x} = x + 2$ (b) $\frac{18}{x} + x = 10$
(c) $x^2 = 18$

19. Draw the graph of $y = x^2 - 2x + 2$ for $-2 \le x \le 4$, using scales of 2 cm to 1 unit for x and 1 cm to 1 unit for y.
By drawing other graphs, solve the equations:
(a) $x^2 - 2x + 2 = 8$
(b) $x^2 - 2x + 2 = 5 - x$
(c) $x^2 - 2x - 5 = 0$

20. Draw the graph of $y = x^2 - 7x$ for $0 \le x \le 7$, using scales of 2 cm to 1 unit on the x-axis and 1 cm to 1 unit on the y-axis. Use your graph to solve the equations:
(a) $x^2 - 7x + 9 = 0$
(b) $x^2 - 5x + 1 = 0$

21. Complete the table below, for the function $y = x^2 - x - 3$; for $-3 \le x \le 3$
(a) Draw the curve $y = x^2 - x - 3$

(b) Write down the coordinates of the minimum point on your graph.
(c) For what range of values of x is $y \le -1$?
(d) Use your graph to solve the equation $x^2 - x - 3 = 0$.

x	-3	-2	-1	0	1	2	3
x^2		4			1		
-x		2			-1		
-3		-3			-3		
$y= x^2 - x - 3$		3			-3		

Simultaneous equations – one linear and one quadratic

These are equations which involve two unknowns. One equation includes terms of the second degree such as x^2, xy and y^2 and the other equation is of the first degree, i.e. linear.
The usual method of solving such simultaneous equations is to use the linear equation to eliminate one of the unknowns from the quadratic equation.

Example 2.22

Solve the simultaneous equations:

$x + y = 0$ ....(i)

$y^2 - xy = 8$ ....(ii)

From $x + y = 0$, we get $x = -y$
Substituting $x = -y$ in (ii), we get

$$y^2 - (-y)y = 8$$
$$y^2 + y^2 = 8$$
$$2y^2 = 8$$
$$y^2 = 4$$
$$y = +2 \text{ or } -2$$

To find x substitute $y = 2$ and $y = -2$ in $x + y = 0$, or $x = -y$
When $y = 2$, $x = -y = -2$
When $x = -2$, $x = -y = +2$
The solutions are:
When **$x = +2$, $y = -2$** and when **$x = -2$, $y = +2$**
The solutions should be arranged in corresponding pairs.

Note: Before starting to solve the equations, look at them carefully and see which is the easier unknown to eliminate. For example, in
$3x - y = 8$ and $3x^2 - xy + 9 = y^2$, it is easier to eliminate y from
$3x - y = 8$, giving $y = y = 3x - 8$

For x, we have $x = \frac{8+y}{3}$ which involves a fraction.

Example 2.23

Solve the equations: x + y = 19 , xy = 84.

(You can eliminate either x or y. It makes no difference).
From x + y= 19, we have y = 19 - x
Substituting in equation xy = 84, we get

$x(19 - x) = 84$
$19x - x^2 = 84$
$x^2 - 19x + 84 = 0$
$(x - 12)(x - 7) = 0$
Either x = 12 or x = 7.

Substitute values of x in x + y = 19, to get

When x = 12, 12 + y = 19 or y = 7
When x = 7, 7 + y = 19 or y = 12.

Solutions are: when **x = 12, y = 7** and when **x = 7, y = 12**.

Exercise 2.8

Solve the following simultaneous equations:

1. $2x - y = 0$
$x^2 + xy = 75$

2. $x + y = 8$
$xy = 15$

3. $x + y = 0$
$x^2 - xy + y^2 = 12$

4. $y^2 = 8x$
$y = 3x - 16$

5. $x - 2y = 2$
$Xy = 12$

6. $3x + y = 6$
$2x^2 - x = y + 6$

7. $x + y = 7$
$x^2 + y^2 = 25$

8. $p + 2q = 15$
$pq = 25$

9. $x + y = 3$
$2x + 2y = 3xy$

10. $2x - y = 2$
$x^2 + y^2 = 89$

11. A rectangle is (x+ 3) cm long and y cm wide. The perimeter of the rectangle is 25 cm. The area of the rectangle is 22.5 cm^2.
(a) Form and solve the simultaneous equations in x and y.
(b) Write down to 2 d.p., the length and width of the rectangle.

12. The longest side of a right angled triangle is 25 cm and the two sides containing the right angle are x cm and y cm. If one of the shorter sides exceeds the other by 17 cm. obtain two equations in x and y and solve them.

Chapter 3

Circles

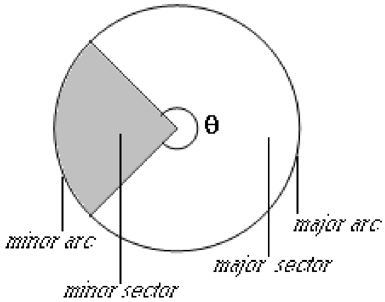
Area of a part of a circle

Given a circle of radius, r, its area is found using the formula, area = πr^2.

Where $\pi = \frac{22}{7}$ or 3.142 (to 3 d.p)

Area of a sector

A sector is a part of a circle enclosed by two radii and an arc (minor or major). The figure below shows the major and minor sectors of a circle.



If the angle subtended by the major arc at the centre is θ, then the area of the major sector $= \frac{\theta}{360}\pi r^2$.

Example 3.1

A circle has a radius of 18 cm. Find the area of a sector of the circle whose arc subtends an angle of 70^0 at the centre. (Take $\pi = \frac{22}{7}$).

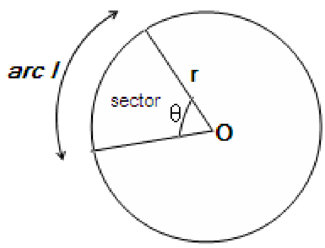
Solution

Area of a sector $= \dfrac{\theta}{360}\pi r^2 = \left(\dfrac{70}{360}\times\dfrac{22}{7}\times 18\times 18\right)cm^2 = 198\ cm^2$.

Circumference $= \pi d = 2\pi r$

Arc length An arc is any part of the circumference of a circle. The length of an arc of a circle is proportional to the angle it subtends at the centre.

Arc length, $l = \dfrac{\theta}{360}\times 2\pi r$

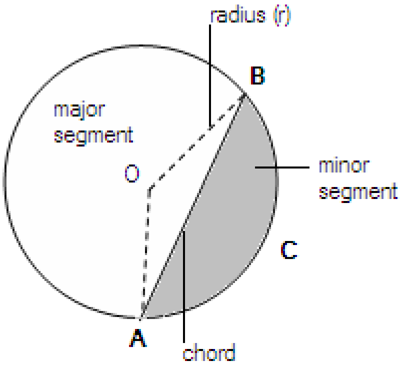


Example 3.2

The length of an arc which subtends an angle of 140^0 at the centre of a circle of radius 12 cm is given by

Arc length $= \dfrac{140}{360}\times 2\times\dfrac{22}{7}\times 12 = \dfrac{88}{3} = 29\frac{1}{3}$ cm.

Chord of a circle

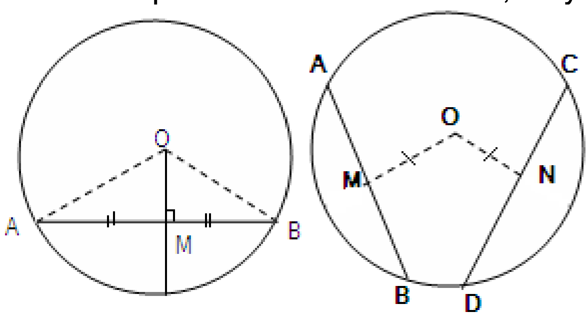


The line AB is a chord. It is a line segment joining any two points on the circumference.
OB = OA = radius of the circle.
ΔAOB is an isosceles triangle.

The area of a circle cut off by a chord is called a **segment**. In the diagram the **minor** segment is shaded and the **major** segment is unshaded.
Area of shaded part (segment) = Area of sector AOBC – the area of triangle AOB.

Properties of chords

(a) Perpendicular bisector of any chord of a circle passes through the centre of the circle.
(b) A perpendicular drawn from the centre of a circle to a chord bisects the chord. Note in both cases the angle at the centre is bisected by the perpendicular bisector of the chord.
(c) Equal chords are equidistant from the centre. Conversely, if chords of a circle are equidistant from the centre, they are equal in length.

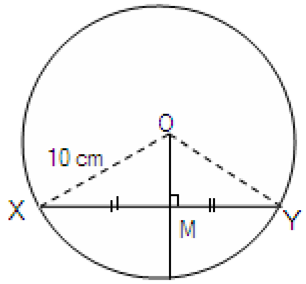


AM = MB OM = ON since AB = CD.

Example 3.3

XY is a chord of length 12 cm of a circle of radius 10 cm, centre O. Calculate:
(a) the angle XOY

(b) the area of the minor segment cut off by the chord XY.



Let the mid-point of XY be M.
Therefore MY = 6 cm

$\text{Sin } M\hat{O}Y = \frac{6}{10}$

Therefore, $M\hat{O}Y = 36.87^0$

So, $X\hat{O}Y = 2 \times 36.87 = 73.74^0$

Area of minor segment = area of sector XOY - area of ΔXOY

Area of sector XOY = $\frac{73.74}{360} \times \pi \times 10^2 = 64.32 \text{ cm}^2$

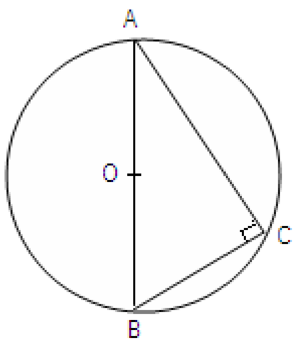
Area of ΔXOY = $\frac{1}{2} \times 10 \times 10 \times \sin 73.74^0 = 48 \text{ cm}^2$

Therefore area of minor segment = 64.32 - 48 = 16.3 cm^2 (3 s.f.)

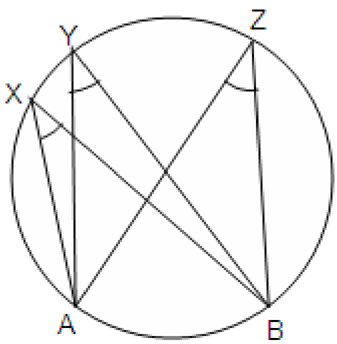
Angle properties of a circle

1. *Angle in a semi-circle is a right angle.*

   In the diagram, AB is a diameter.
   $A\hat{C}B = 90^0$

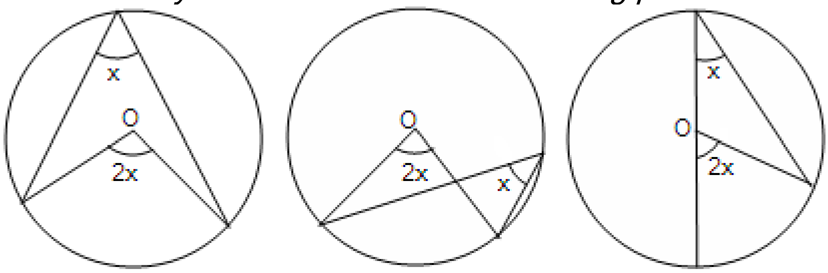


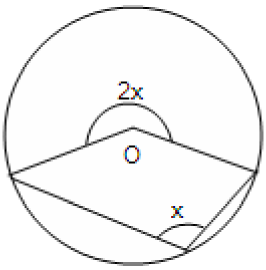
2. *Angles in the same segment of a circle are equal.*
   (angles subtended by an arc in the same segment of a circle are equal).



$A\hat{X}B = A\hat{Y}B = A\hat{Z}B$

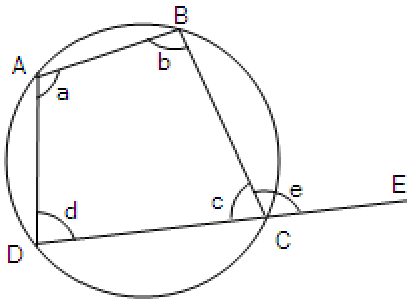
3. *The angle subtended at the centre by an arc of a circle is twice the angle subtended by the same arc on the remaining part of the circumference.*



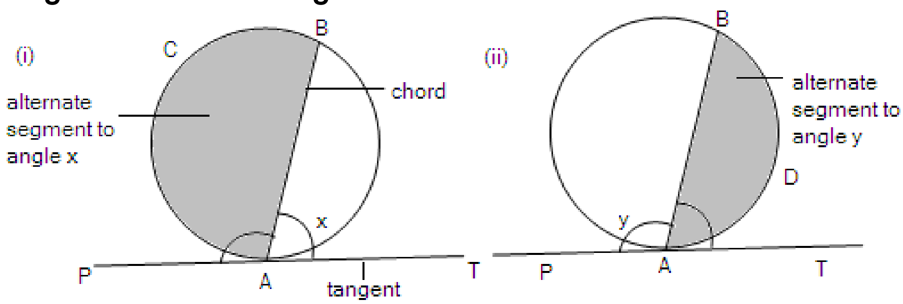


4. In a cyclic quadrilateral,
   (i) the *opposite angles add up to 180^0* (the angles are supplementary)
   $a + c = 180^0$; $b + d = 180^0$
   (ii) its *exterior angle is equal to the interior opposite angle*.
   e = a.

In the following figure, ABCD is a cyclic quadrilateral. The vertices lie on the circumference of the circle.

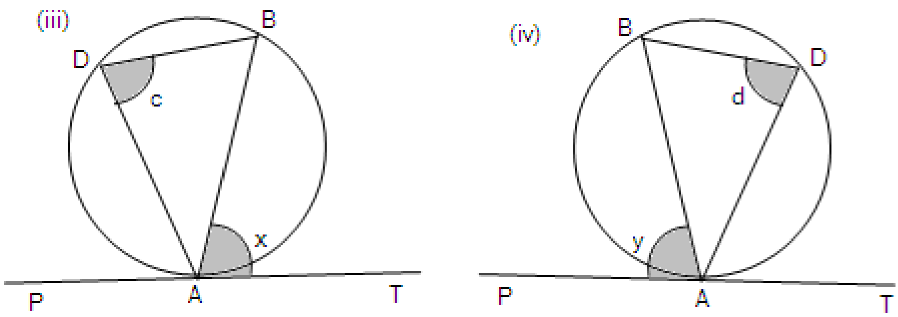


5. **Angles in alternate segments**



A **tangent** to a circle is a straight line drawn from a point outside the circle such that it touches the circle at only one point, the point of contact.
In fig. (i), the segment ACB of the circle on opposite side of the chord AB to angle x is called the alternate segment corresponding to angle BAT.

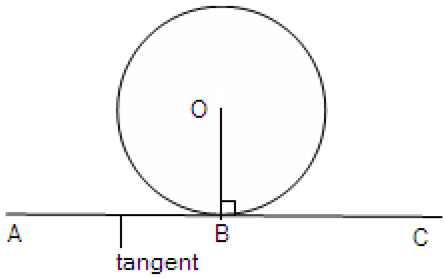
In fig. (ii), ADB is an alternate segment to angle BAP, i.e. the angle y.



In fig. (iii), x and c are angles in alternate segments.
In fig. (iv), y and d are angles in alternate segments.

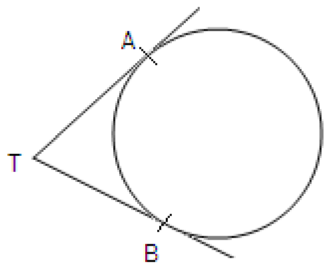
The angle between a tangent to a circle and a chord drawn from the point of contact is equal to any angle which the chord subtends in the alternate segment.

6. *The angle between a tangent and the radius drawn to the point of contact is 90^0.*



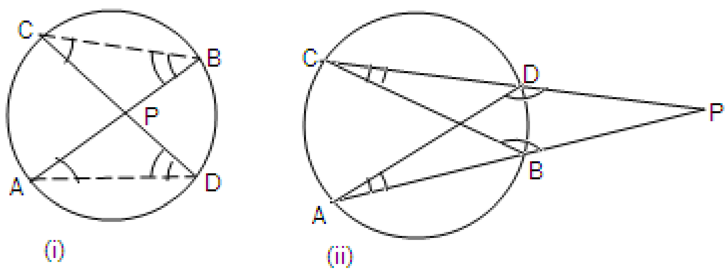
$C\hat{B}O = 90^0$

7. *From any point outside a circle just two tangents to the circle may be drawn and they are equal in length.*



TA = TB

8. Intersecting chords



In fig. (i), the chords AB and CD intersect internally at a point P. ΔCPB and ΔAPD are similar, as $\angle C = \angle A$, $\angle B = \angle D$ (angles in the same segment)

$C\hat{P}B = A\hat{P}D$ (third angles or vertically opposite angles)

$\therefore \frac{CP}{AP} = \frac{PB}{PD} \quad (= \frac{CB}{AD}).$

Therefore, **AP × PB = CP × PD**

Similarly, in fig. (ii), chords AB and CD intersect externally at a point P. ΔCPB and ΔAPD are similar.

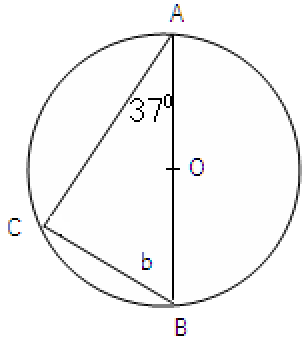
$\hat{C} = \hat{A}$, (angles in the same segment); $C\hat{B}P = A\hat{D}P$ (third angles)

Therefore, $\therefore \frac{CP}{AP} = \frac{PB}{PD} \quad (= \frac{CB}{AD})$

Therefore, **AP × PB = CP × PD**

Example 3.4

In the following diagram, AB is a diameter of the circle, centre O. Find angle b.



Solution

$A\hat{C}B = 90^0$ (Angle in a semi-circle)
Therefore, b = $180^0 - (80 + 37)^0 = 53^0$

Example 3.5

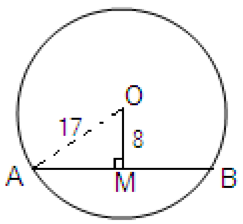
A chord is 8 cm away from the centre of a circle of radius 17 cm. Find the length of the chord

Solution

Let OM be the perpendicular bisector of AB.
Then AM = MB and in ΔAOM, $AM^2 = AO^2 - OM^2$

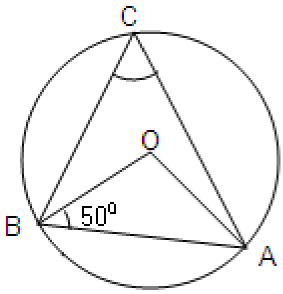
$$= 17^2 - 8^2$$
$$= 225$$
$$AM = \sqrt{225} = 15 \text{ cm}$$

Therefore, the chord AB = 15 × 2 = 30 cm.



Example 3.6

Given $A\hat{B}O = 50^0$, and O is the centre of the circle, find $B\hat{C}A$.



Solution

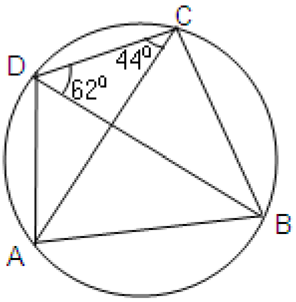
ΔOBA is isosceles (OA = OB). Therefore, $O\hat{A}B = 50^0$

$B\hat{O}A = 80^0$ (angle sum of a triangle)

Therefore, $B\hat{C}A = \frac{1}{2} \times 80^0 = 40^0$ (angle at the circumference)

Example 3.7

Given $B\hat{D}C = 62^0$ and $D\hat{C}A = 44^0$, find $B\hat{A}C$ and $A\hat{B}D$.



Solution

$B\hat{D}C = B\hat{A}C$ (both subtended by arc BC)

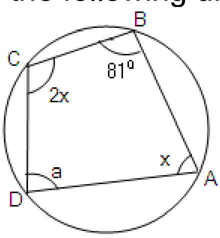
Therefore, $B\hat{A}C = 62^0$

$D\hat{C}A = A\hat{B}D$ (both subtended by arc DC)

Therefore, $A\hat{B}D = 44^0$

Example 3.8

Find a and x in the following diagram.



Solution

$x + 2x = 180^0$ (opposite angles of a cyclic quadrilateral)

$3x = 180^0$

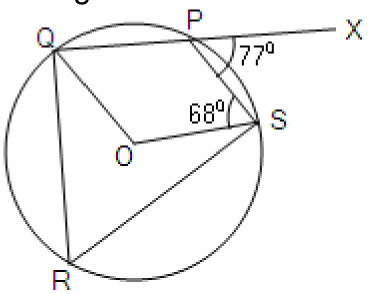
$x = 60^0$

$a = 180^0 - 81^0$ (opposite angles of a cyclic quadrilateral)

Therefore, $a = 99^0$

Example 3.9

In the following figure, P, Q, R, S are points on a circle centre O. QP is produced to X. If angle XPS = 77^0 and angle PSO = 68^0, find $P\hat{Q}O$.



Solution

$Q\hat{R}S = 77^0$ (= *ext. angle of cyclic quadrilateral*)

$Q\hat{O}S = 2 \times 77^0 = 154^0$ (*angle at centre = 2 × angle at circumference*)

$Q\hat{P}S = 180^0 \quad 77^0 = 103^0$ (*angles on a straight line*)

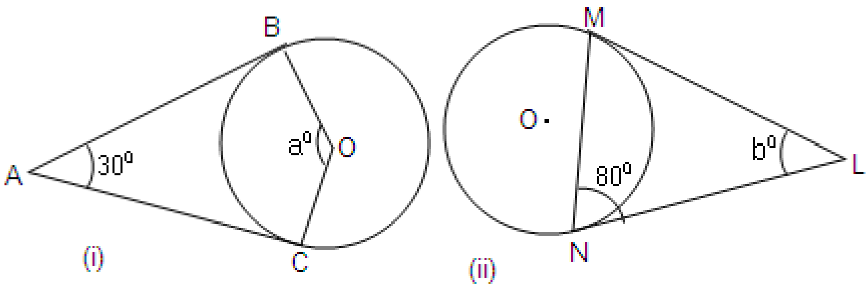
In quadrilateral PQOS,

$P\hat{Q}O = 360^0 - 154^0 - 103^0 - 68^0$ (*angle sum of a quadrilateral*)

Therefore, $P\hat{Q}O = 35^0$.

Example 3.10

Find the values of the angles represented by letters in the diagrams below. In each diagram, O is the centre of the circle.



Solutions

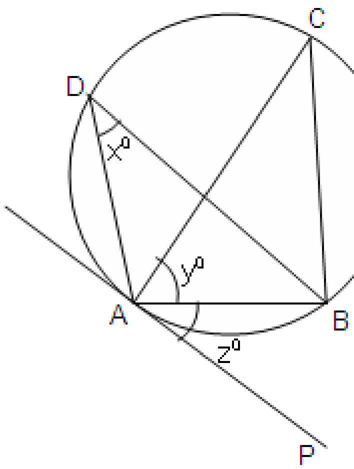
$A\hat{B}O = 90^0$ (AB is a tangent and OB is a radius)

$A\hat{C}O = 90^0$ (AC is a tangent and OC is a radius)
So a = $360^0 - (30^0 + 90^0 + 90^0)$
$= 360^0 - 210^0$
$= 150^0$

(ii) LM = LN (tangents from an external point are equal)
Hence triangle LMN is isosceles.
So angle LMN = 80^0
Therefore, b = $180^0 - (80^0 + 80^0)$
$= 180^0 - 160^0$
$= 20^0$

Example 3.11

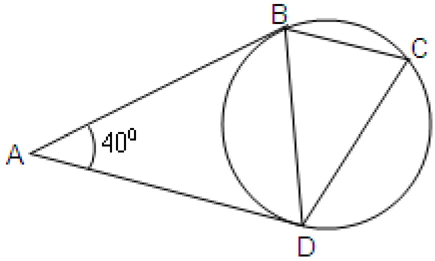
In the following figure, AP is a tangent and AC is a diameter. $A\hat{D}B = x^0$, $C\hat{A}B = y^0$, and $B\hat{A}P = z^0$. Show that x = z.



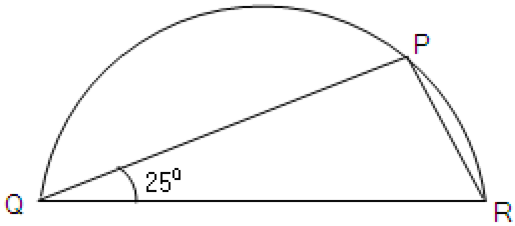
$A\hat{B}C = 90^0$ (angle in a semi-circle)
$A\hat{C}B = x^0$ (angles in the same segment)
Therefore, x + y + 90 = 180 (angle sum of triangle ACB)
x + y = 90 ....(i)
Also z + y = 90 ....(ii)
(tangent is perpendicular to the radius drawn to its point of contact)
Subtracting eqn. (ii) from (i) gives
x - z = 0
Therefore, x = z.

Exercise 3.1

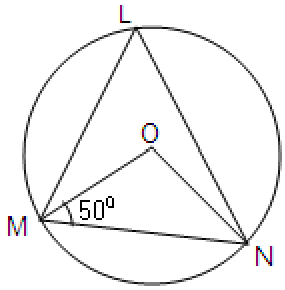
1. In the figure below, AB and AD are tangents to the circle. CD is a diameter and $D\hat{A}B = 40^0$. Find $B\hat{C}D$.



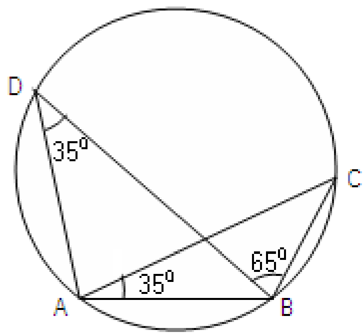
2. The following figure shows a semi-circle with QR as diameter. $P\hat{Q}R = 25^0$. Find angles QPR and QRP.



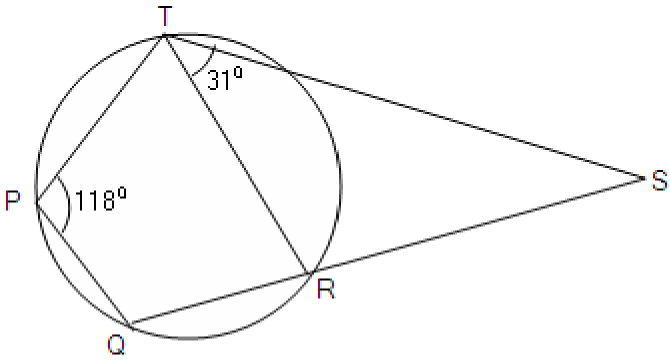
3. Given that O is the centre of the circle and $O\hat{M}N = 50^0$. Find $M\hat{L}N$.



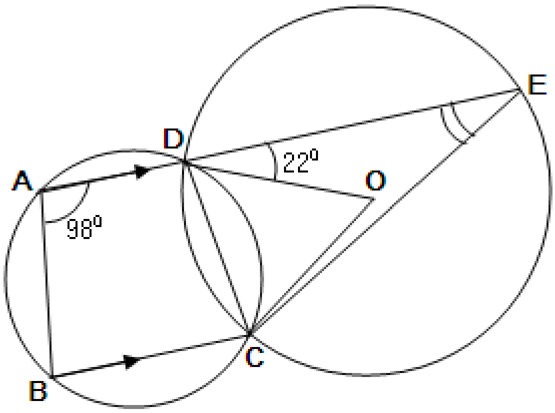
4. In the figure, ABC and ABD are triangles. $A\hat{D}B = B\hat{A}C = 35^0$; $C\hat{B}D = 65^0$. Find $A\hat{B}D$.



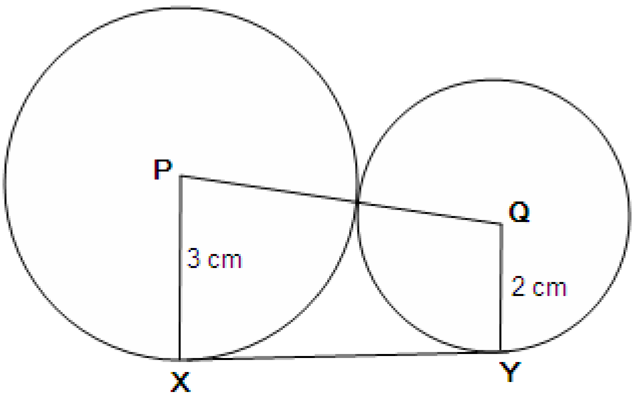
5. In the figure below, QRS is a straight line. $Q\hat{P}T = 118^0$ and $R\hat{T}S = 31^0$. Prove that triangle RST is isosceles.



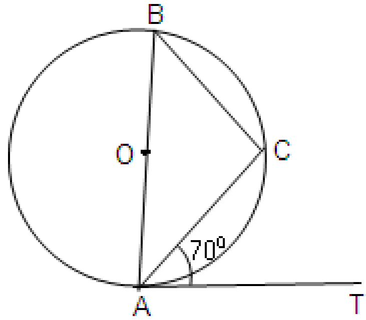
6. In the following diagram, ABCD is a cyclic quadrilateral and O is the centre of the circle through C, D and E. ADE is a straight line and is parallel to BC. $D\hat{A}B = 98^0$ and $O\hat{D}E = 22^0$. Find $C\hat{E}D$.



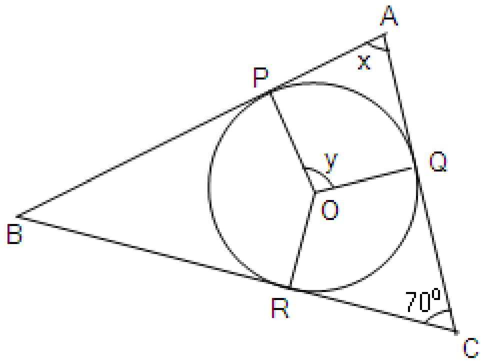
7. P and Q are the centres of two circles which touch externally. XY is a common tangent. PX = 3 cm and QY = 2 cm. Calculate XY



8. TA is a tangent to the circle centre O and angle CAT = 70^0. State the value of:
   (a) angle CBA,
   (b) angle BAC

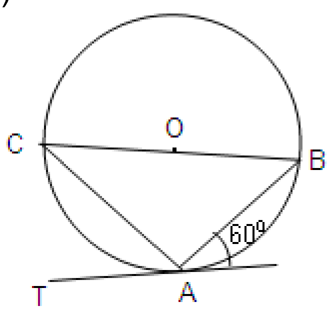


9. In the figure, AB, AC and BC are tangents to the circle centre O, touching the circle at P,Q and R respectively.
   (a) If angle C = 70^0, what is $R\hat{O}Q$?
   (b) If $P\hat{O}R = 120^0$, what is angle B?
   (c) Obtain an equation connecting x and y.

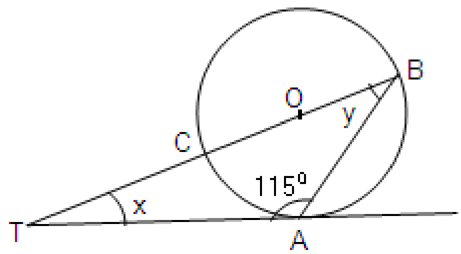


10. Find the value of the angle marked with letters. O is the centre and TA is a tangent.

(a)



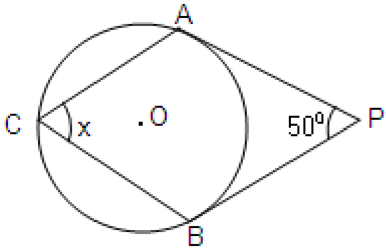
(b)



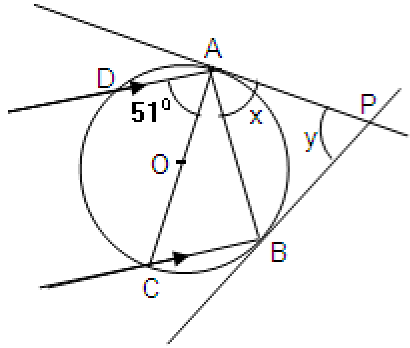
Find x and y given that O is the centre of the circle. Other information is given under each figure.

11.

12.

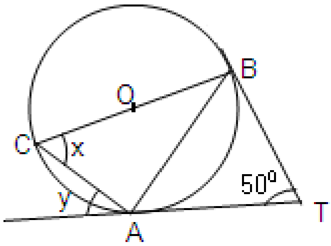


PA and PB are tangents from P.



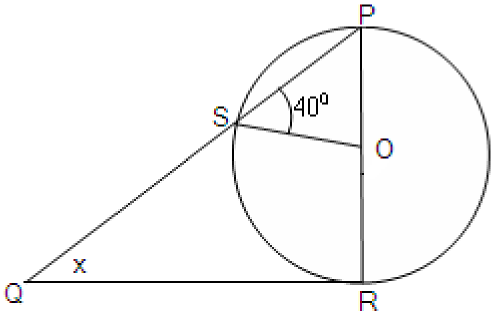
PA and PB are tangents. AD and BC are parallel.

13.



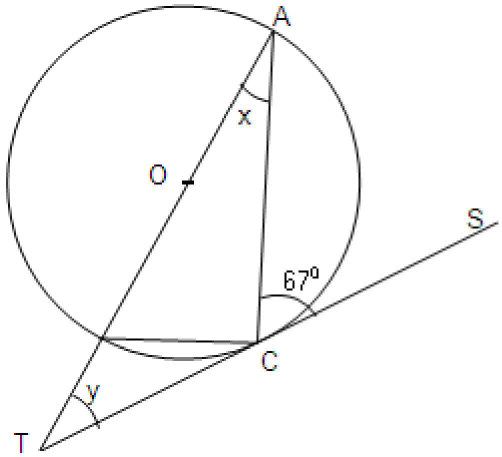
TA, TB are tangents. CB is a diameter

14

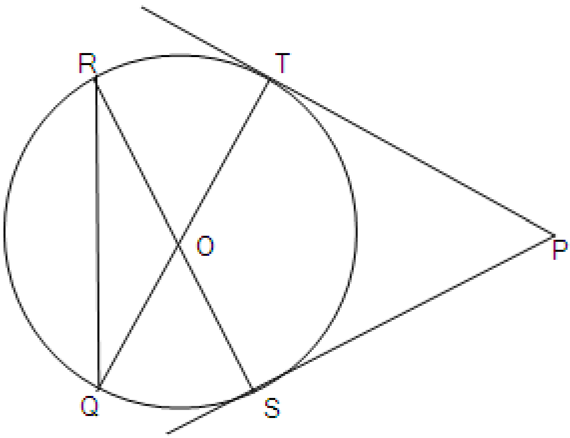


QR is a tangent. PR is a diameter.

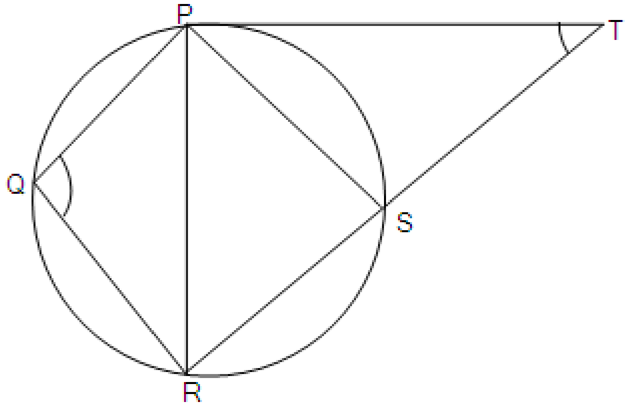
15. SCT is a tangent. ABT is a straight line through O.



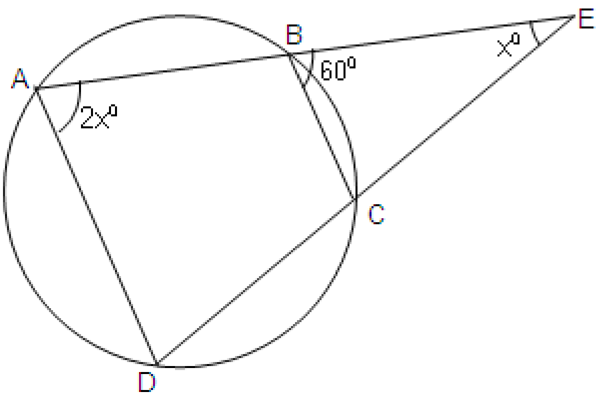
16. In the figure below, ROS and TOQ are diameters of the circle centre O. PT and PS are tangents to the circle and angle TQR = 30^0. Calculate:
(i) angles TQS and POS and
(ii) length of OS and PS if OP = 6 cm.



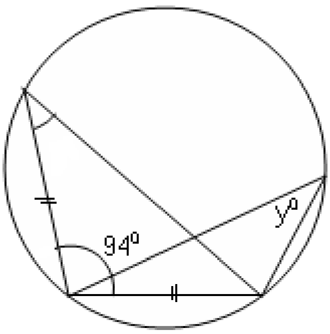
17. PQRS is a cyclic quadrilateral. The tangent at P meets RS produced at T. $P\hat{T}S = P\hat{Q}R$. Prove that:
(a) ΔPRT is isosceles,
(b) ΔPST is isosceles.
Given also that PS = 6 cm and TS = 4 cm, calculate the length of PR.



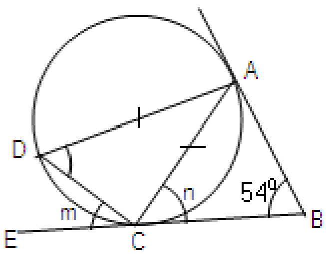
18. In the figure below calculate the value of x giving a reason for each step in your answer.



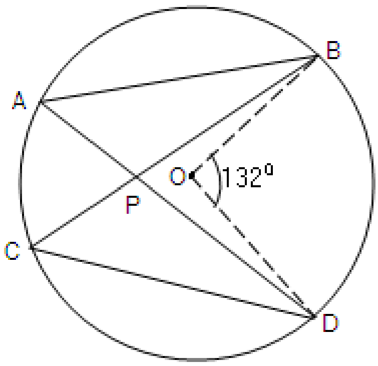
19. Find the angles marked with letters.



20. Calculate the value of the angles marked with letters.



21.

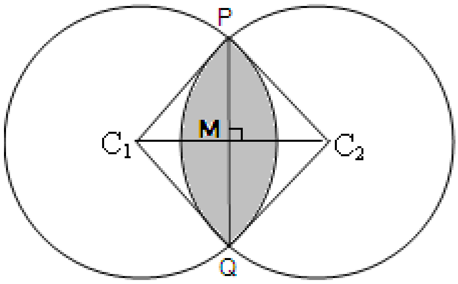


O is the centre of the circle. Angle BOD = 132^0. The chords AD and BC meet at P.

(a) (i) Calculate angles BAD and BCD.
(ii) Explain why triangles ABP and CDP are similar.
(iii) AP = 6 cm, PD = 8 cm, CP = 3 cm and AB = 17.5 cm. Calculate the lengths of PB and CD.
(iv) If the area of triangle ABP is n cm^2, write down, in terms of n, the area of triangle CPD.

(b) (i) The tangents at B and D meet at T. Calculate angle BTD.
(ii) Use OB = 9.5 cm to calculate the diameter of the circle which passes through O, B, T and D, giving your answer to the nearest cm.

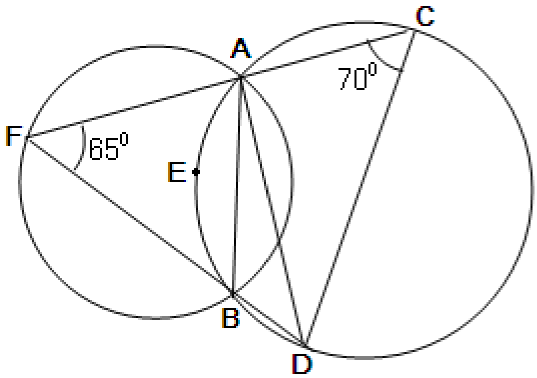
22. The figure below shows intersecting circles whose centres are C_1 and C_2. C_1P = 3 cm. Angle $PC_1M = 30^0$ and angle $PC_2M = 43^0$.



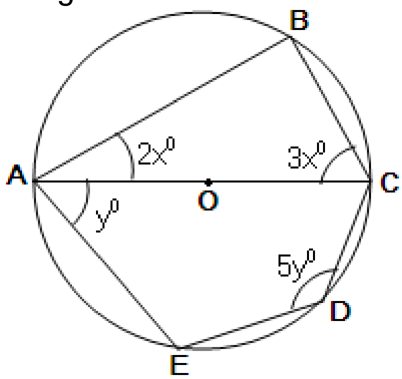
Find the area of the shaded region.

23. The following figure shows two circles that intersect at points A and B. Point E is the centre of the smaller circle and lies on the circumference of the larger one. FAC and FBD are straight lines. $A\hat{F}B = 65^0$ and $F\hat{C}D = 70^0$. Calculate:

(a) the obtuse angle AEB
(b) angle ACB
(c) angle BAD
(d) angle EBF



24. A, B, C, D and E lie on the circumference of a circle, centre O. AOC is a straight line. Find the values of x and y.



Chapter 4

Linear programming

Linear programming is a method used to find the best solution to problems that can be expressed in terms of linear equations or inequalities. Solutions are usually found by drawing graphs of inequalities and looking for optimum values that satisfy the required conditions. This method is widely used in business and industrial contexts and the problems often relate to obtaining maximum profits for given costs and production levels.

Forming linear inequalities

Some of the terms and symbols that imply inequality include:

- less than ($<$).
- Greater than / more than ($>$).
- Less than or equal to ($\leq$)
- Greater than or equal to. ($\geq$)

In forming inequalities, we need to define the variables that represent the various quantities under consideration. For example in a game using a pair of dice, a player must score a total of more than 9 in order to win. Using y for a number on one dice and x for a number on the other die, inequalities can be formed

The phrase used to imply inequalities in this case is **must score a total of more than 9.** Therefore, the inequality for the winner is $x + y > 9$ and the statement for the loser is $x + y \leq 9$.

Example 4.1

A student went shopping with sh. 15,000 to buy exercise books and pens. An exercise book costs sh. 500, while a pen costs sh. 200. He had to buy at least 3 exercise books and not more than 2 pens. Write down inequalities to represent this information.

Solution

We should first define the variables that represent the quantities.
Let x represent the number of exercise books purchased and let y represent the number of pens purchased.
The inequalities that show the number of exercise books and pens purchased are $x \geq 3$ and $y \leq 2$.

Each exercise book costs sh. 500 and a pen costs sh. 200. The total amount must not exceed sh. 15,000. Therefore, $500x + 200y \leq 15,000$. This can be simplified to $5x + 2y \leq 150$

The required inequalities are:

$x \geq 3$; $y \leq 2$; and $5x + 2y \leq 150$.

Exercise 4.1

1. Mary went shopping with sh. 8,500 to buy magazines and exercise books. The cost of a magazine was sh. 1700, while that of an exercise book was sh. 450. She had to buy at least 2 magazines and 6 exercise books. Write down three inequalities to represent this information.

2. John has x five thousand shillings notes and y ten-thousand shillings notes. He has more than sh.30,000 but less than sh. 50,000 in total. The five-thousand shillings notes are more than the ten-thousand shillings notes. Write down inequalities relating x and y.

3. On a farm, sh. 750,000 is available for the preparation of planting beans. Two types of beans, A and B, are to be planted on 4ha. Planting Type A costs sh. 120,000 a hectare, while planting type B costs sh. 150,000 a hectare. Each type is planted on at least 1 ha. Assuming that labour is available for planting, form inequalities to represent this information.

4. A wholesaler for animal feeds wishes to transport 240 bags of feed. She has a lorry that can take 90 bags at a time and a pick-up that can take 20 bags at a time. The cost of each trip is sh. 50,000 by lorry and sh. 35,000 by pick-up. The pick-up should make more trips than the lorry. The cost must not be more than sh. 800,000. Write down inequalities to represent this information.

5. A businessperson had 96 m^3 of space available for storage of boxes. There were two types of boxes, A and B. The volume of type A boxes is 8 m^3 each while the volume of type B boxes is 8 m^3 each. The businessperson had sh. 720,000 to buy the boxes. Type A boxes cost sh. 3,000 each and type B boxes cost sh. 4,200 each. Write down inequalities to represent this information.

6. A pick-up can carry a mass of 2000 kg. It is supposed to carry type A and type B bags of animal feed. Type A bags weigh 40 kg each, while type B bags weigh 70 kg each. Type A bags cost sh. 32,000 a bag while type B bags cost sh. 56,000 a bag. A farmer had sh. 1,600,000 to purchase the feed.
   (a) Write down inequalities to represent this information.
   (b) Show the inequalities on a graph.

Forming and solving inequalities

Example 4.2

A girl went to a book shop with sh. 1200. She bought x exercise books at sh. 200 each and y exercise books at sh. 250 each.

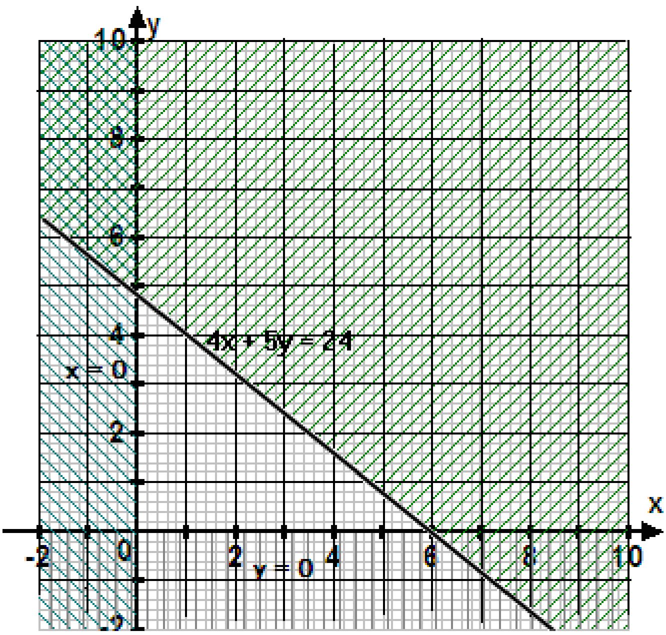
(a) Form inequalities to represent this information.
(b) Draw graphs to represent this information.
(c) Use the graphs to list all the possible solutions.

Solutions

(a) The inequalities are:

- $x \geq 0$ and $y \geq 0$, because x and y cannot be negative.
- $200x + 250y \leq 1200$, because she could not spend more than sh. 1200. This simplifies to $4x + 5y \leq 24$

(b) The graphs of the inequalities are shown in the figure below.



The required region is usually determined by substituting selected values of x and y into the inequalities to check whether they satisfy the inequalities. The coordinates of the origin are often used. The unwanted region is usually shaded.

(c) From the graphs, the values of x and y must be integers. Therefore, the possible solutions are the integral coordinates in the wanted region. That is, points such as (1, 1), (2, 1), (4, 1).
Which other points satisfy the inequalities?

Example 4.3

To transport 165 crates of bread, a pick-up and a van are used. The pick-up can carry 15 crates per trip while the van can carry 33 crates per trip. The pick-up makes x trips and the van makes y trips. The total number of trips must be less than 8.

(a) Write the inequalities that represent this information.

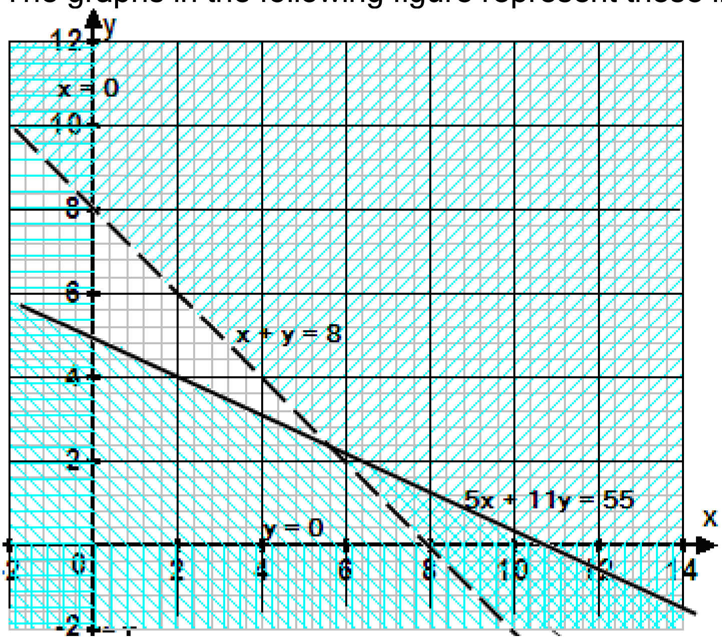
(b) Using graphs, determine the possible number of trips each vehicle should make in order to transport the crates most economically.

Solution

(a) The inequalities representing the information are:

- $x \geq 0$, the number of trips the pick-up makes.
- $y \geq 0$, the number of trips the van makes.
- $15x + 33y \geq 165$, the total number of crates to be transported. This simplifies to $5x + 11y \geq 55$.
- $x + y < 8$, the maximum number of the sum of the trips the pick-up and the van make.

(b) The graphs in the following figure represent these inequalities.



The solutions are in the unshaded region and on the continuous line. The integral coordinates in the unshaded region are possible solutions but the most cost effective ones are close to or on the continuous line. These include (1, 5), (0, 6), (0, 7), (3, 4), e.t.c.

- list other points that give cost effective solutions

Exercise 4.2

1. Ali is 3 years older than Moses. Their total age is less than 30 years but more than 26 years.
   (a) Form inequalities to represent this information.
   (b) Represent this information on graphs.
   (c) List all the possible ages of Ali and Moses.

2. A home library should not have more than 140 books. More than 60 of the books should be fiction and at least 40 non-fictions.
   (a) Write inequalities to represent this information.
   (b) Use graphs to find the possible number of books of each type in the library

3. Nadia had sh. 2000, enough to buy some mandazi and cakes. She had to buy at least 3 pieces of each item. Mandazi cost sh. 200 and cakes cost sh. 300 each.
   (a) Form inequalities to represent this information.
   (b) Represent this information on graphs.
   (c) Use the graphs to find all the possible numbers of each item bought.

4. Transport is to be arranged for 420 students. There are two types, A and B, of passenger vehicles to be used. Type A carries 14 passengers and type B carries 35 passengers. There are to be at least 10 vehicles of type A and not more than 9 vehicles of type B.
   (a) Write down inequalities to represent this information.
   (b) Draw graphs to represent the information.
   (c) List three solutions which satisfy all the inequalities.

5. Some fruit juice is obtained from x oranges and y lemons. For a pleasant taste, 3x + 5y must be at least 30. For an attractive colour, 3x must be greater than y.
   (a) Write inequalities to represent this information.
   (b) Represent the information on graphs.
   (c) List three solutions having integral values that satisfy the inequalities.

6. A transport company has buses that carry 70 passengers each, and minibuses that carry 35 passengers each. There should be at least 10 drivers available and more than 700 passengers should be transported in a day.
   (a) Write down inequalities to represent this information.
   (b) Draw graphs to show this information.
   (c) List solutions having integral coordinates that satisfy these inequalities.

7. A magazine is to be produced with at least 32 pages but not more than 40 pages. The number of advertisement pages must be at lest half the

number of text pages. There must be at least 27 text pages.
(a) Form inequalities to represent this information.
(b) Draw graphs to represent this information.
(c) List three possible coordinates that satisfy these inequalities.

8. A man had sh. 192,000 to buy trousers and shirts. A pair of trousers costs sh, 24,000 and a shirt costs sh. 16,000. He had to buy at least two shirts.
(a) Write down inequalities representing this information.
(b) Draw graphs to represent the information.
(c) List the possible coordinates that satisfy the inequalities.

9. A vegetable seller bought x tones of cabbages and y tones of carrots. Cabbages cost sh. 1000 per kg and carrots cost sh. 2000 per kg. He had sh. 96,000 to spend.
(a) Write down inequalities to represent this information.
(b) Show the inequalities on graphs.
(c) Use your graphs to list the possible solutions that would satisfy the inequalities.

Optimization

Optimization is the process of finding the ordered pairs of variables that give the maximum or the minimum values of given expressions. These ordered pairs of variables are found in the region that satisfies all the inequalities. The process of finding the maximum and minimum values of linear functions under limiting conditions is called **linear programming**.

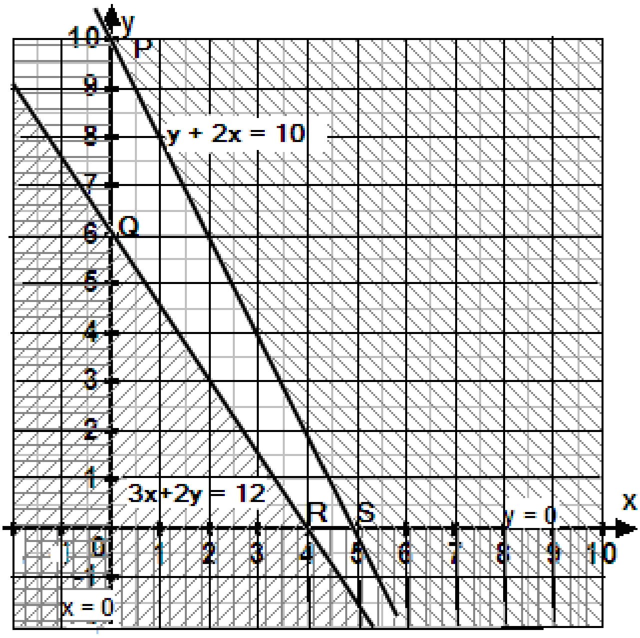
Example 4.4

(a) Draw the graphs of $x \geq 0$, $y \geq 0$, $3x + 2y \geq 12$ and $y + 2x \leq 10$.
(b) Find the maximum and the minimum values of $3x + y$.

Solutions

(a) The graphs of the inequalities and the required region are shown in the following figure.
(b) To find the maximum and minimum values of $3x + y$, we consider the values of the integral coordinates at the vertices of the required region. These are P(0, 10), Q(0, 6), R(4, 0) and S(5, 0).
At P(0, 10); $3x + y = 10$.
At Q(0, 6); $3x + y = 6$.
At R(4, 0); $3x + y = 12$.
At S(5, 0); $3x + y = 15$.

The maximum value of $3x + y$ in the given situation is 15 and the minimum value is 6. All the other coordinates give values between these two.



Example 4.5

A shopkeeper bought two types of animal feed. Type A costs sh. 750 per kilogram and type B costs sh. 600 per kilogram. She had sh. 15,000 available and decided to buy at least 18 kilograms altogether. She also decided to buy at least 6 kg of type A feed. She made a profit of sh.200 per kg of type A and sh. 250 per kg of type B.

(a) Write down inequalities to represent this information.

(b) Show these inequalities on graphs.

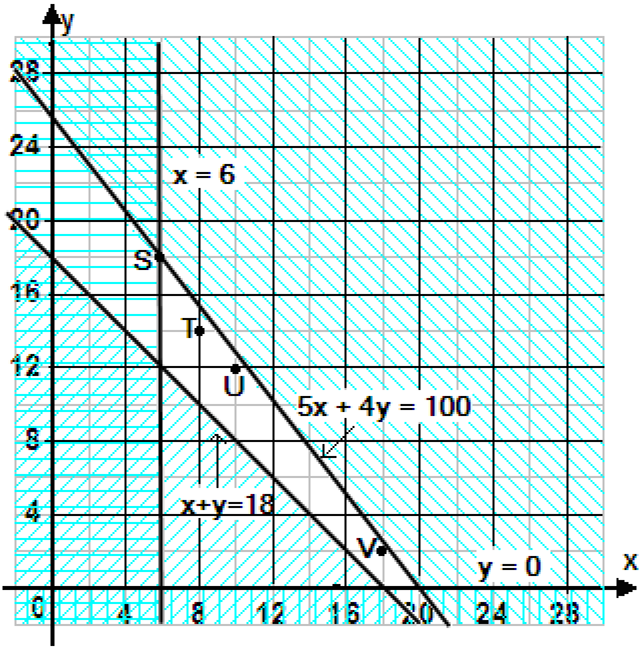
(c) Assuming that she can sell all the feed, find how many kilograms of each type of feed she should buy to maximize her profits and find how much the profit is.

Solution

(a) Let x represent the number of kilograms of type A and y represent the number of kilograms of type B. The inequalities are:

- $x \geq 6$, the number of kg of type A.
- $y \geq 0$, the number of kg of type B.
- $x + y \geq 18$, the total number of kg.
- $750x + 600y \leq 15000$, the amount of money available. This simplifies to $5x + 4y \leq 100$.

(b) The graphs in the following figure represent the above information.



(c) From the information, the profit made is 200x + 250y. To find the maximum profit, we substitute the integral values of x and y at points S, T, U and V in the expression 200x + 250y.
At S(6, 17); (200 × 6) + (250 × 17) = 5,450
At T(8, 14); (200 × 8) + (250 × 14) = 5,100
At U(10, 12); (200 × 10) + (250 × 12) = 5,000
At V(18, 2); (200 × 18) + (250 × 2) = 4,100

She should buy 6 kg of type A and 17 kg of type B to make maximum profit. The maximum profit is sh. 5,450.

When looking for the maximum value, we use the integral values near or on the upper boundary of the region.

The objective function

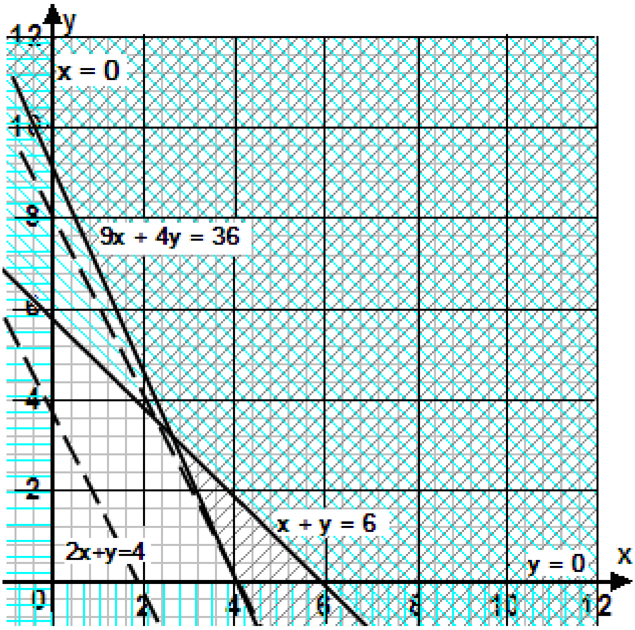
In the above examples, we used trial and error method in order to determine the maximum or the minimum values of a situation. An alternative method is to use the **objective function** or a **search line**. The line is of the form ax + by = k, where a, b, and k are constants.

Example 4.6

(a) Draw the graphs of the following inequalities: x ≥ 0, y ≥ 0, x + y ≤ 6 and 9x +4y ≤ 36.
(b) From the graphs, find the maximum value of 2x + y.

Solutions

(a) The graphs of the inequalities are shown in the following figure.



(b) To find the maximum value of 2x + y, the objective function would be 2x + y = k. The constant k can take any value, but it is appropriate to take a value of k that is a common multiple of the coefficients of x and y. The multiples of 1 and 2 are 2, 4, 6, … In this case, we can take k = 4. The equation becomes 2x + y = 4.
When y = 0, x = 2. when x = 0, y = 4. We can draw the graph of this line using the points (0, 4) and (2, 0). The graph drawn is called a **search line**.
With a ruler and a set square, draw a parallel line to the search line as follows:

- Place the longest side of the set square on the search line.
- With the ruler firmly held onto one of the shorter sides of the set square, slide the set square along the ruler until the last point in the required region nearest to the upper boundary is reached.
- If the last point does not have integral values, take the nearest point within the neighborhood that has integral values. This is the required point.

In this case, the line passes through (2, 4), (4, 0) and (3, 2) all of which give 2x + y = 8. Thus the maximum value of 2x + y is 8.
To find the minimum value of a function you follow the same steps, but this time you look for integral values on the lower boundary of the region.

Exercise 4.3

1. Draw graphs t represent the following inequalities by shading the unwanted regions: x + y ≤ 10, y ≥ 2, and y ≤ 2x.
Use the graphs to find the maximum value of:
(a) 2x + 3y. (b) 3y – x.

2. Show the region represented by the following inequalities: $5x + 4y \leq 40$, $9x + 12y \geq 36$, $y > x$, and $x \geq 1$.
   (a) List all the solutions having integral coordinates.
   (b) From the graphs, find the minimum and maximum values of $3x + 5y$.

3. Show the region represented by the inequalities: $x + y > 10$, $x < 3$, $2x + y < 16$, and $x > 0$.
   Find the points with integral coordinates which satisfy all the inequalities simultaneously. For these points find:
   (a) the maximum value of $2x + y$.
   (b) the minimum value of $2x + y$.
   (c) the maximum value of $x - y$.

4. Graphically show the region represented by the inequalities: $y \geq 0$ $x + y \geq 6$ and $x + 3y \leq 12$.
   (a) Find the values of $3x + 2y$ at each point with integral coordinates in the region.
   (b) Which is the greatest value?
   (c) Find the maximum and minimum value of $4x + 5y$.

5. Caroline had sh. 1,500 to buy oranges and mangoes. Oranges cost sh. 150 each, while mangoes cost sh. 100 each. She bought at least 11 fruits altogether.
   Let x be the number of oranges and let y be the number of mangoes she bought.
   (a) Write down the inequalities representing this information.
   (b) Represent the information in a graph.
   (c) Find the possible combinations of the number of fruits she could buy.

6. Jackline was to buy pens and pencils. She had sh. 9,000 to spend. Each pencil cost sh. 100, while a pen cost sh. 150. She had to buy at most 30 pencils and more than 20 pens. She was supposed to buy not more than 70 pens and pencils altogether.
   (a) Write down inequalities to represent this information.
   (b) Draw graphs to represent the information.
   (c) Use your graphs to find the maximum number of pens and pencils she could buy.

7. A chef makes cakes of type A and type B. He has 2 kg of flour and 1.2 kg of sugar. Type A cakes use 500g of flour and 100g of sugar. Type B cakes use 300g of flour and 200 g of sugar. He wishes to make more than 4 cakes altogether.
   (a) Write down inequalities to represent this information.
   (b) Represent this information on graphs.
   (c) Find the number of type A and type B cakes that he can make.

8. A painter can spray a van in 3 hours and a car in 2 hours. He sprays for at least 15 hours a week. He must spray at least three times as many cars as vans and not more than 8 cars in a week.
Let x be the number of vans and y be the number of cars he sprays each week.
   (a) Write down inequalities representing this information.
   (b) Draw graphs to represent the information.
   (c) Use the graphs to list the combination of the vehicles he could spray in a week.
   (d) Find the maximum number of vehicles he could spray in a week.

9. The manager of a badminton team has sh. 180,000 to buy new uniforms for the players. She can buy type A uniform for sh. 36,000 each and type B uniform for sh. 27,000 each. She must buy at least two uniforms of type A. The total number of uniforms must be at least 5.
   (a) Write down inequalities to represent this information.
   (b) Draw graphs to represent the information.
   (c) Use your graphs to list down the combinations of the uniforms she could possibly buy.

10. An electronic dealer wishes to stock 20 television sets. She can buy Coloured ones for sh. 150,000 each, and black and white at sh. 90,000 each. She has a total of sh. 2,700,000 to spend and must have at least 6 sets of each type.
   (a) If she buys x sets of black and white and y sets of coloured TVs, write down the inequalities representing this information.
   (b) If she makes a profit of sh. 60,000 on each of the black and white TV set and sh. 80,000 on each colour TV set:
      (i) Write down the equation for the profit.
      (ii) Use a search line to find the maximum profit.

11. Mubarak has 36 acres of land. He declares to prepare the land for planting wheat and maize. The cost of planting maize is sh. 30,000 per acre, while it cost sh. 90,000 to plant an acre of wheat. Maize takes 3 labourers per acre, while wheat takes 6 labourers per acre. He hired 72 labourers and spent sh. 1,500,000 for labour costs. He hopes to make a profit of sh. 200,000 per acre of maize and sh. 450,000 per acre of wheat.
   (a) Write down inequalities representing this information.
   (b) Show the inequalities on graphs.
   (c) Find the maximum profit Mubarak should make.
   (d) What would be the most profitable arrangement?

12. Ten students went to buy snacks from a restaurant. They each bought either a scone or a cake. A scone cost sh. 50, while a cake cost sh. 200. The students had sh. 10,000 altogether. They bought more scones than cakes.
   (a) Form inequalities to represent this information.

(b) Graph these inequalities, showing the wanted region.
(c) List the possible solutions.

13. A transportation firm has 6 lorries which can carry 8 tonnes each The firm also has 4 lorries which can carry 12 tonnes each. The cost of running an 8-tonne lorry is sh. 80,000 per trip, while that of a 12-tonne lorry is sh. 100,000. There are 600 tonnes of rice to be transported and sh. 1,200,000 available to run the lorries.
(a) Write inequalities to represent the information.
(b) Draw the inequalities on graphs.
(c) How should the firm use the lorries in order to incur the lowest costs possible?

14. Some fruit juice is made from x oranges and y lemons. For a strong taste, 3x + y should be at least 24. For an attractive colour, 2x must be greater than y. An orange costs sh. 200, while a lemon costs sh. 150
(a) Form inequalities to represent to represent this information.
(b) Show the inequalities on graphs.
(c) Find the cheapest way of making the fruit juice.

15. Ali has sh. 240,000 to buy shirts and trousers. The cost of a shirt is sh. 32,000, while the cost of a pair of trousers is sh. 50,000. He has to buy at least two shirts and one pair of trousers.
(a) Write down inequalities representing this information.
(b) Draw graphs to represent the information.
(c) List the possible number of shirts and pairs of trousers Ali can buy.
(d) What is the maximum number of shirts and pairs of trousers he can buy?

16. A factory makes two types of items, A and B. To produce item A costs sh. 800 plus a labour cost of sh.1,200. To produce item B costs sh.700 plus a labour cost of sh. 1,600. The cost of producing the items should not exceed sh.140,000 and labour costs should not exceed sh. 25,000.
(a) Write down the inequalities representing this information.
(b) Draw graphs of the inequalities in (a).
(c) Find the maximum number of items A and B that can be produced when the production cost is maximum.

17. A cultural theatre has a capacity of 240 seats. The seats are sold at sh. 8000 or sh. 12000. To cover the costs, the management has to collect at least sh. 144,000.
(a) Write down linear inequalities to represent this information.
(b) Graph the inequalities, showing the wanted region.
(c) List down the possible solutions.
(d) Determine the solution that gives the maximum profit.

18. A firm is planning to build a factory which will occupy a space of 720 m^2. Two types of machines, X and Y, are to be installed. Machine X occupies

a space of 60 m^2, requires 3 labourers and produces 8 units. Machine Y requires a space of 80 m^2, 5 labourers and produces 12 units. There are 75 workers available.
Determine the number of machines of each type that should be installed in order to maximize production and profits.

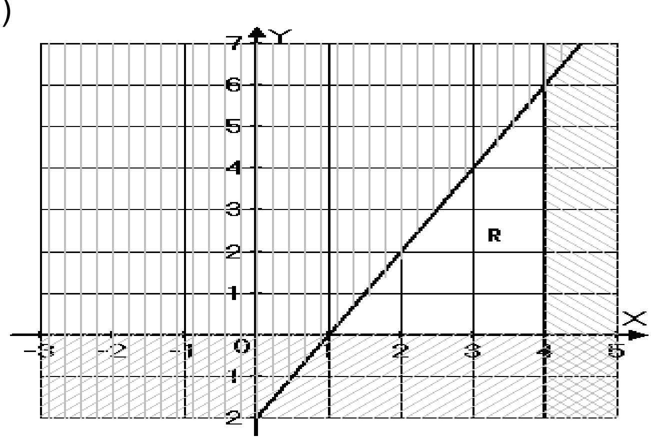
19. A hotel is to buy at most 5 bags of cabbages and at most 7 bags of potatoes. A bag of cabbages costs sh. 45,000, while a bag of potatoes costs sh. 80,000. The total cost must not exceed sh. 600,000.
    (a) Form inequalities to represent this information.
    (b) Draw graphs of the inequalities, showing the wanted region.
    (c) Determine the maximum number of bags of cabbages and potatoes that can be purchased.

20. A depot for famine relief should have at most 20 bags of rice and 35 bags of maize. The mass, volume and number of meal ratios per bag are as shown in the table below.

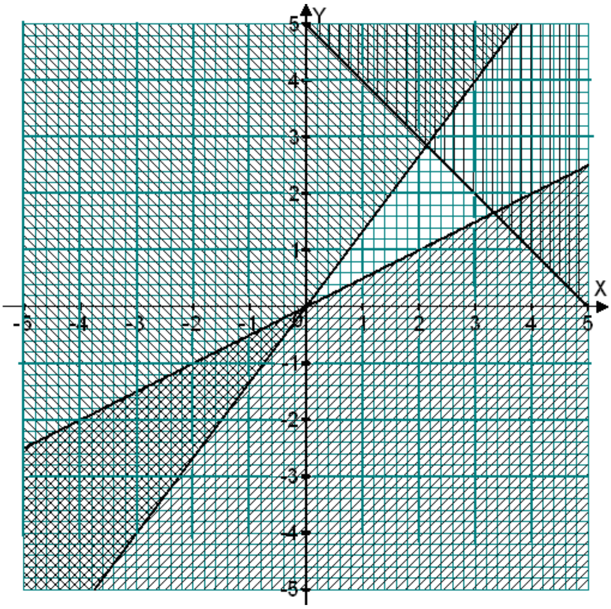
Bag of	Mass (kg)	Volume (m3)	No. of meals
Rice	25	0.05	800
Maize	10	0.05	160

A delivery van is to carry the largest possible number of bags. It can carry a mass of up to 600 kg occupying 2 m^3.
    (a) If a load is made up of x bags of rice and y bags of maize, write down four inequalities (other than $x \geq 0$ and $y \geq 0$) which govern the relationship between x and y.
    (b) Draw a graph to show the inequalities in (a) above.
    (c) Determine the number of bags of rice and maize that should be delivered to make the total number of meal ratios the largest.

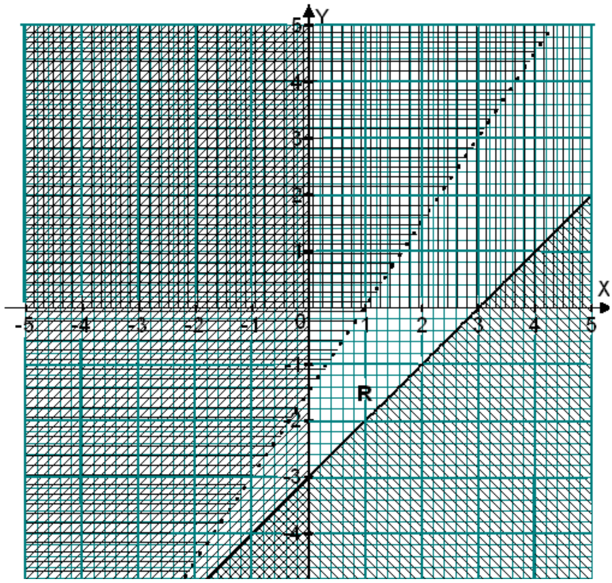
21. Give the inequalities that define the unshaded region R in each of the following.
    (a)



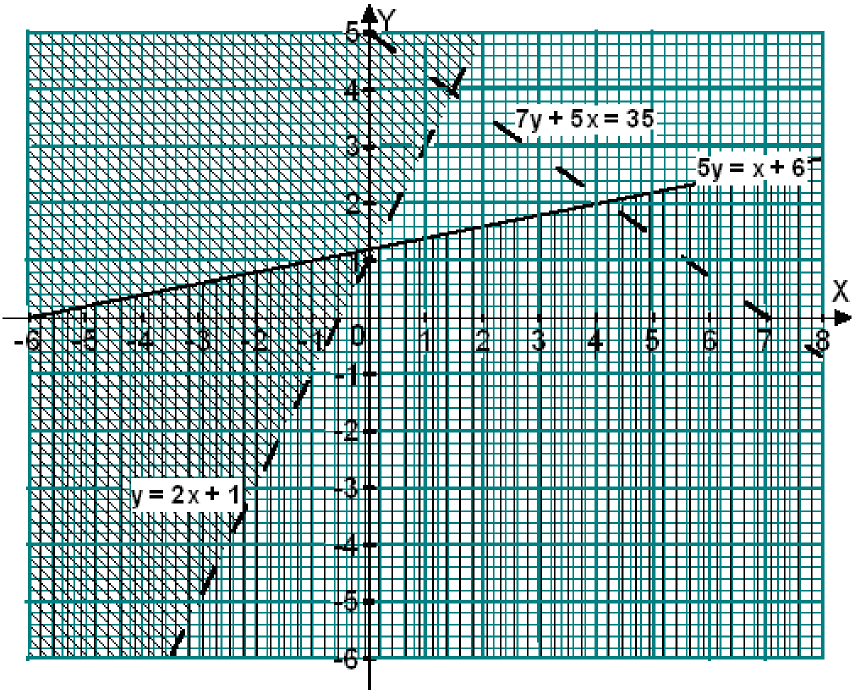
(b)



(c)

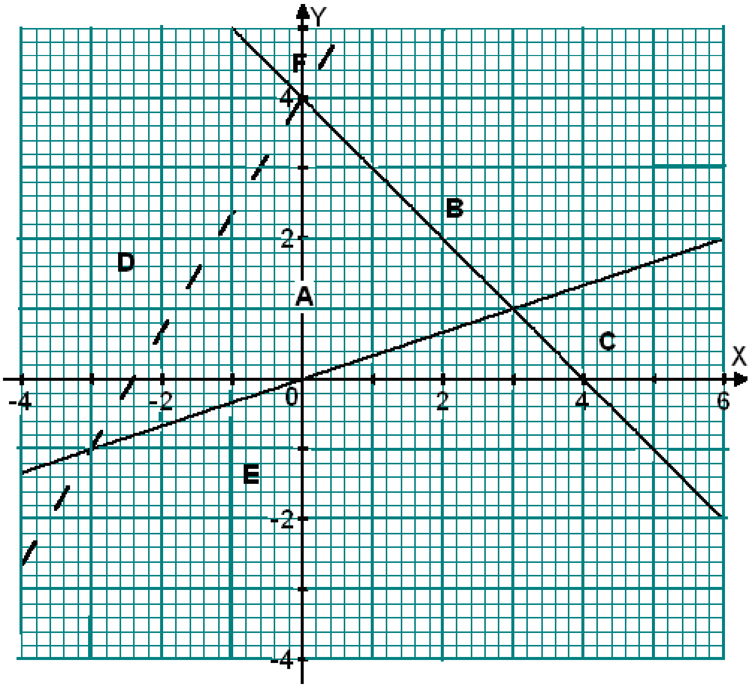


22. Give the points whose coordinates are integers and lie in the unshaded region.



23. Draw a graph and give the integral coordinates of the points that lie in the region defined by the inequalities:
$y > x - 2$, $2y < 3x + 6$, $x + y > -2$ and $x + y \leq 3$

24. The graph below shows regions A to F enclosed by lines x + y = 4, $y = \frac{1}{3}x$ and $y = \frac{5}{3}x + 4$.



Use inequalities to describe region:
(a) A (b) B (c) C (d) D (e) E (f) F

Chapter 5

Constructions and Loci

A geometrical construction is the accurate drawing of a plane figure. For an accurate construction, one must:

(a) use a properly sharpened pencil,
(b) draw fine and clear lines,
(c) avoid repeating already drawn lines.

Perpendicular bisector of a line joining two points

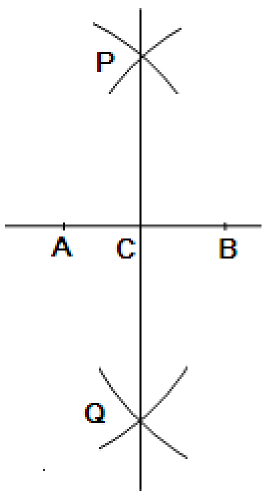
Suppose the length of line AB is 6 cm.

1. adjust a pair of compasses to a radius that is more than half the length of AB
2. Place the sharp end of the pair of compasses at point A and make an arc above and below line AB. Repeat this with the pair of compasses at point B such that the arcs cut (intersect) each other at points P and Q as shown below.
3. Join points P and Q with a straight line and mark the point of intersection of lines AB and PQ as C.

Line PQ divides line AB into two equal parts.

Also $P\hat{C}B = P\hat{C}A = 90^0$

Line PQ is said to be the perpendicular bisector of line AB.

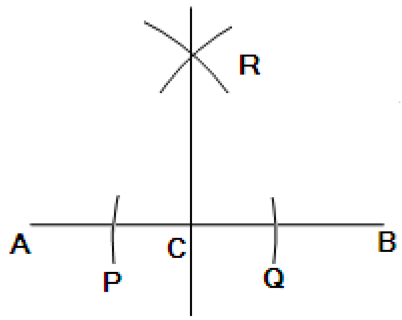


Drawing a perpendicular bisector through a given point on a straight line.

Suppose AB is a straight line and C is a point on line AB.

1. Adjust the pair of compasses to a suitable radius.
2. Place the sharp point of the pair of compasses at point C and draw two arcs that cut line AB at points P and Q. Using P as centre of a circle, draw

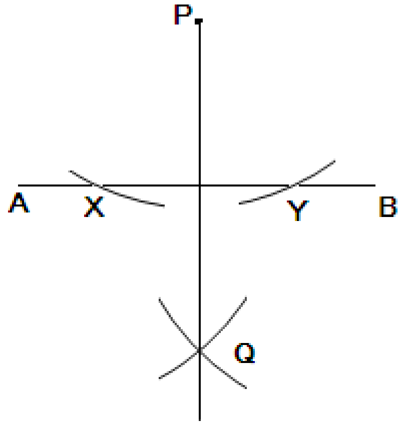
an arc above line AB. Without adjusting the pair of compasses and using Q as the centre, draw another arc so that the two arcs cut each other at point R as shown below.
3. Join points R and C with a straight line and extend the line beyond C.



Drawing a perpendicular bisector from a given point outside the line.

Suppose AB is a straight line and P is a given point not on the line.

1. Adjust the pair of compasses to a suitable radius so that with P as the centre, you can draw arcs that cut line AB at points X and Y as shown in the following figure.
2. Using any radius and point X as the centre, make an arc below line AB. With the same radius and point Y as the centre, make another arc so that it cuts the first arc at point Q. Join points P and Q.

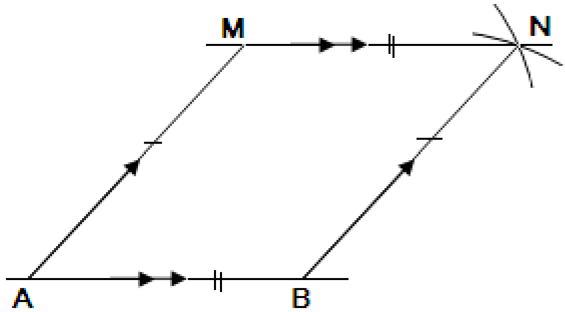


Constructing parallel lines

Constructing a parallel line through a given point

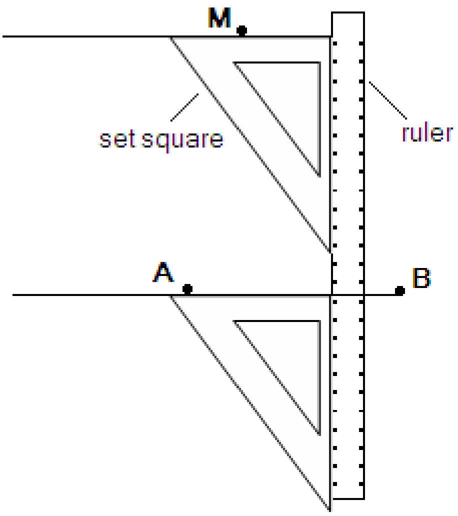
Let AB be a line and M be a point not on line AB.

1. With M as the centre, and using radius AB, draw an arc above point B as shown in the following figure With B as the centre, and using radius AM, draw another arc to intersect the first arc at point N. Join points M and N.



If point A is joined to M and B joined to N, ABNM is a parallelogram in which AB is parallel to MN

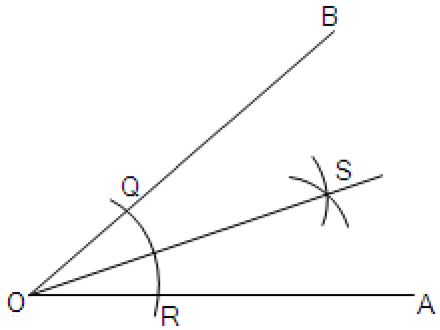
2. Consider the same line AB and point M. Place one of the edges of a set square along line AB so that the edge is exactly on the line.
3. Put a ruler along the second edge of the set square as shown in the following diagram. Hold the ruler firmly on the paper and slide the set square along the ruler until the edge (previously along AB) touches point M.
4. Draw a line along this edge passing through M. The drawn line is parallel to AB.



Bisecting an angle

Let AOB be an angle .

1. With centre O and any radius , draw an arc that cuts OA and OB at P and Q respectively.
2. Using P and Q as centres, in turns, and using any suitable radius, equal in both cases, draw arcs to intersect each other at S.
3. Join O to S. OS is the line bisecting angle AOB.



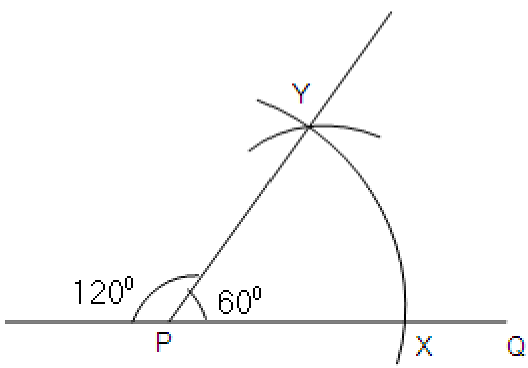
Constructing angles

It is important to note that an angle of 90^0 is formed when a perpendicular bisector to a line is drawn. Thus, the angle on a straight line (180^0) is bisected by a perpendicular line. Many angles are obtained in a similar way by bisecting a given angle. You should now be able to construct an angle of 45^0.
How would you construct an angle of 135^0?

Constructing an angle of 60

1. Draw a line PQ of a suitable length.
2. With centre P and any radius, draw an arc to cut PQ at X and extending further up above line PQ.
3. With centre X and the same radius, draw an arc to cut the first arc at Y.
4. Join Y and P.

   $Y\hat{P}Q = 60^0$



The obtuse angle at P is 120^0. Thus, in constructing an angle of 60^0, we are also constructing an angle of 120^0.

What kind of triangle is PXY?

Note that an angle of 30^0 is obtained by constructing an angle of 60^0 and then bisecting it.
Remember that in all the constructions involving these special angles, we use a pair of compasses and ruler only.

Example 5.1

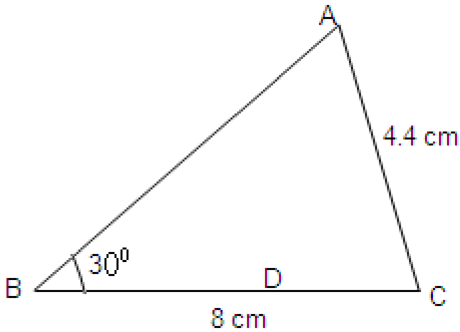
Using a ruler and a pair of compasses only, construct triangle ABC given that $A\hat{B}C = 30^0$, BC = 8 cm and AC = 4.4 cm.

(a) Measure line AB and $A\hat{B}C$.

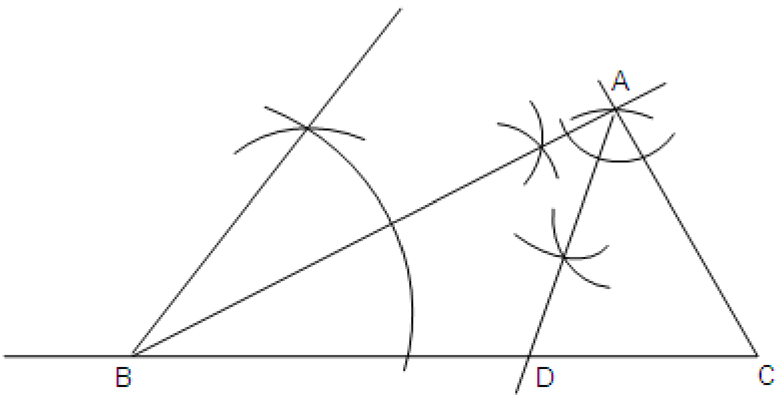
(b) Bisect $B\hat{A}C$ and produce the bisector to meet BC at D. Measure AD and BD.

Solution

Draw a sketch to represent the information in the question.



(a) Draw line BC and measure 8 cm. At point B, construct an angle of 60^0 and bisect it. Draw the bisector. Using a pair of compasses and with C as the centre, mark off 4.4 cm to cut the bisector at A



If done accurately, you will get: AB = 8.6 cm. and $A\hat{C}B = 82^0$

(b) Bisect angle CAB and draw the bisector to meet BC at D.
AD = 4.9 cm and BD = 5.2 cm.

Exercise 5.1

1. Construct the following angles using a ruler and a pair of compasses only.

(a) 135^0 (b) 150^0

(c) 240^0 (d) 202.5^0

2. Using a ruler and a pair of compasses only, construct the following:
   (a) Triangle ABC in which $A\hat{B}C = 60^0$, AB = 4 cm, BC = 5.4 cm. Measure $B\hat{C}A$ and line AC.
   (b) Triangle PQR in which $P\hat{Q}R = 75^0$, $Q\hat{R}P = 60^0$ and QR = 6 cm. Measure PQ and line PR.
   (c) Triangle ABC in which AB = 5.8 cm, BC = 7 cm, CA = 4.6 cm. From point A , draw a perpendicular line to BC to meet BC at D. Measure BD and angles ABC and BCA.
3. Using a ruler and a pair of compasses only, construct:
   (a) Square ABCD given that AC = BD = 7 cm. Measure AB.
   (b) Rectangle PQRS given that PQ = 6 cm, and QR = 3.5 cm. Measure line SQ and $P\hat{Q}S$.
   (c) Rectangle PQRS given that PQ = 8.1 cm, PR = 8.8 cm. Measure PS and $P\hat{R}Q$.

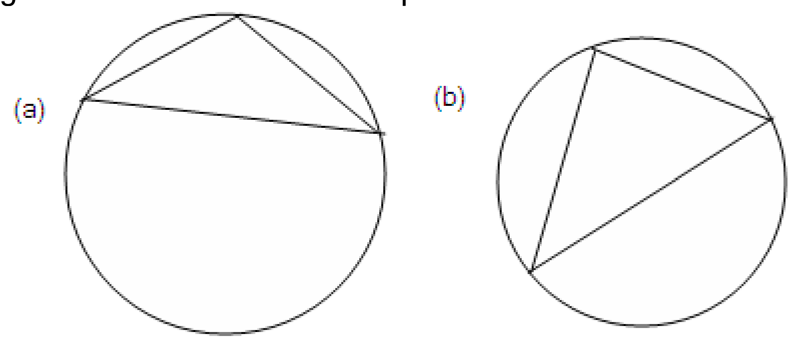
4. Construct an isosceles triangle ABC given that AB = 4.5 cm, AC = BC = 5.5 cm. Bisect $A\hat{C}B$ and draw the bisector to meet AB at D. Measure CD and the angles of the triangle.

5. Construct an equilateral triangle PQR given its sides are 5 cm each. Bisect angles PQR and RPQ such that the bisectors intersect at C and meet PR and PQ at S and T respectively. Measure SC and $Q\hat{C}T$.

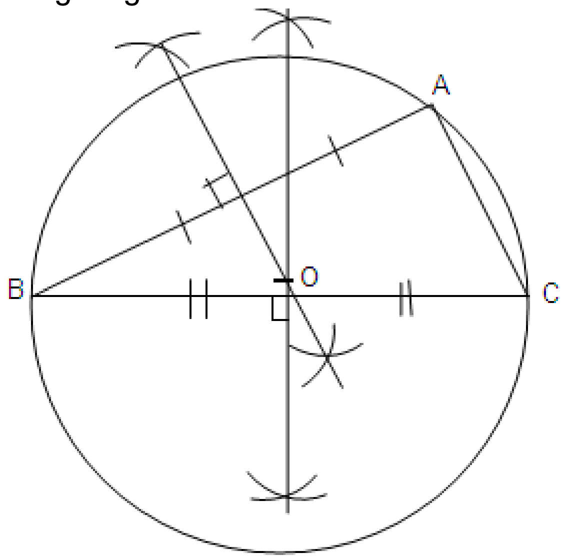
6. Construct triangle ABC given that AC = 8 cm, BC = 7 cm and $A\hat{B}C = 70^0$. Measure AB and $B\hat{A}C$.

Circumscribed circle

A circumscribed circle, or circumcircle, of a triangle is the circle which passes through the vertices of the triangle. A circumcircle can be drawn for any triangle The figure below shows two examples.



To construct a circumscribed circle, bisect any two side of the triangle (ABC) and draw the bisectors to meet at a point (O). Using this point of intersection as centre, adjust the pair of compasses to touch at any vertex of the triangle (A or B or C). Draw the circle with OA or OB or OC as its radius. See the following diagram.



The construction of the circumcircle in the above example, uses the fact that the perpendicular bisector of a chord of a circle passes through its centre.

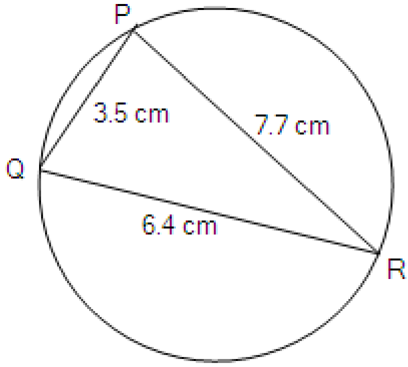
The perpendicular bisectors of the three sides of any triangle are concurrent. The point of concurrency O is called the circumcentre of the triangle. The length OA (or OB or OC) is the circumradius of the triangle.

Example 5.3

Using a ruler and a pair of compasses only, draw a triangle PQR in which PQ = 3.5 cm, QR = 6.4 cm and PR = 7.7 cm. Draw the circumcircle of triangle PQR and measure its radius.

Solution

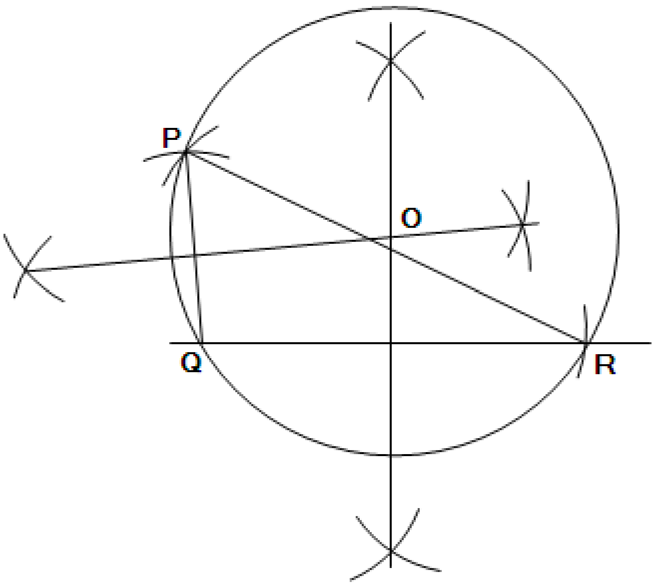
Rough sketch



Accurate drawing

- Draw line QR and measure 6.4 cm.
- Using a pair of compasses and Q as centre, mark off 3.5 cm and draw an arc above point Q.
- Using a pair of compasses and R as centre, mark off 7.7 cm to cut the arc previously drawn above point Q. Label the point of intersection of the two arcs as P.
- Join point P to Q and R to obtain the required triangle PQR.

To draw the circumcircle, bisect any two sides of ΔPQR, say, PQ and QR. As shown. The bisectors will meet at point O. Using O as centre and radius OP (or OQ or PR), draw the circumcircle.



Radius of circumcircle = 3.9 cm.

Inscribed circle

The **inscribed circle**, or **incircle**, of a triangle is the circle drawn inside the triangle such that the circumference touches (internally) the three sides of the triangle.

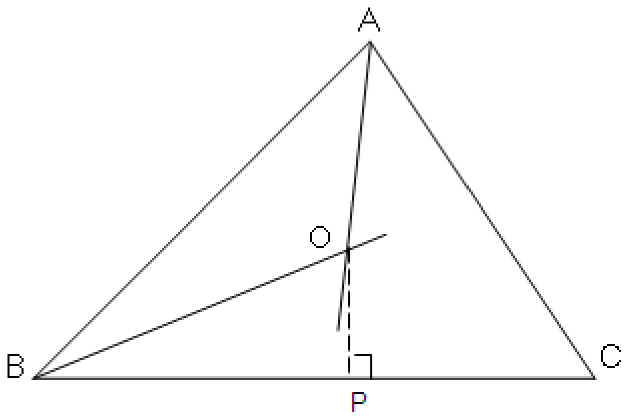
To construct an inscribed circle:

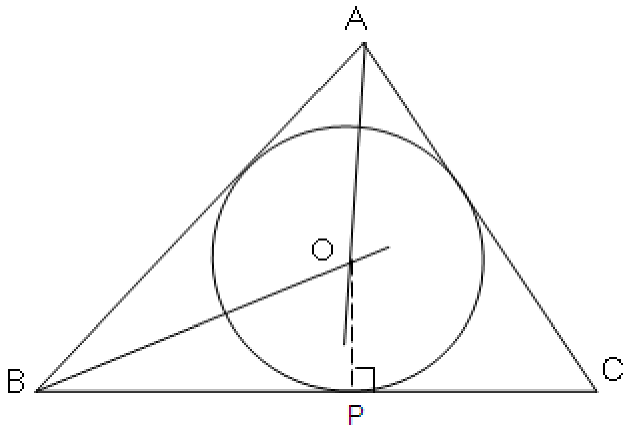
Consider triangle ABC.

(a) Construct the bisectors of $\hat{A}$ and $\hat{B}$. Let them meet at O.

(b) Use a ruler and set square to construct the perpendicular from O to meet BC at P.

(c) With centre O and radius OP, draw the circle which touches AB, BC and CA.





The Locus of a point

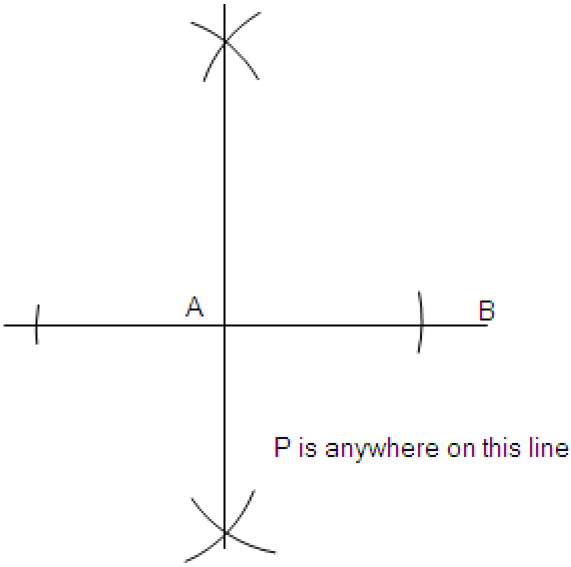
The locus of a point is the path which it describes as it moves.

Example 5.4

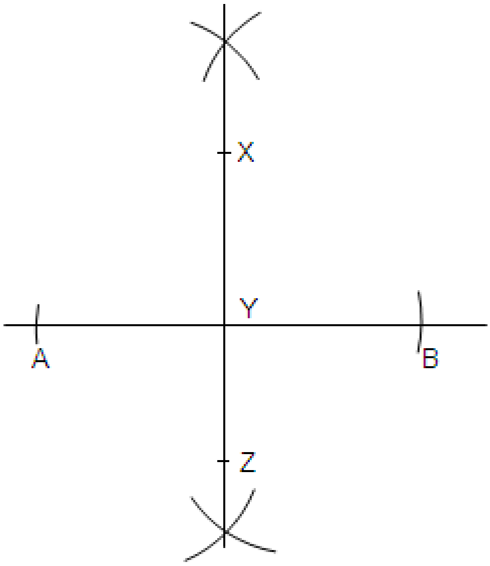
Draw line AB of length 8 cm. Construct the locus of a point which moves so that $B\hat{A}P = 90^0$.

Solution

Construct the perpendicular at A. This line is the locus of P.



The locus of a point equidistant from two fixed points, A and B, is the perpendicular bisector of line AB

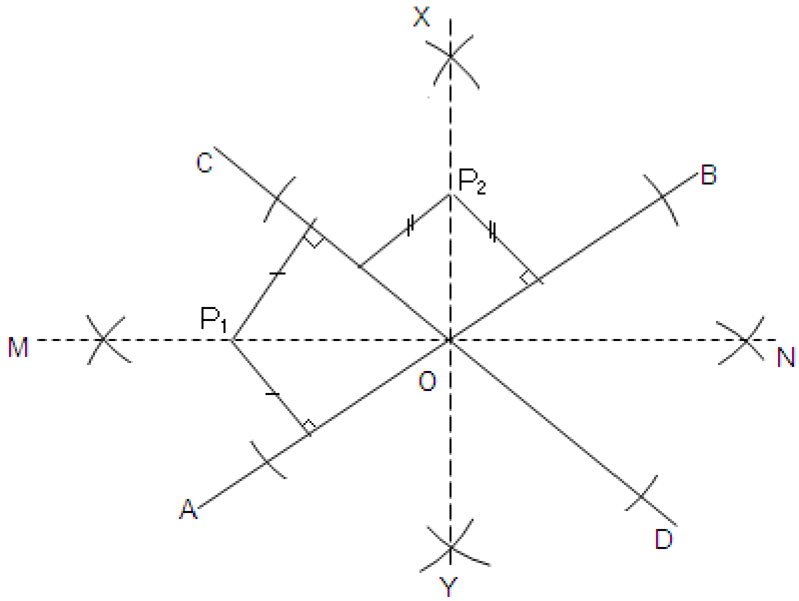


From the diagram above, XA = XB, YA = YB and ZA = ZB.

The locus of a point equidistant from two intersecting lines

In the following figure, AB and CD are straight lines that intersect at O. Lines MN and XY are the angle bisectors of the acute angle and the obtuse angle, respectively, formed by the intersecting lines.

Point P_1 is equidistant from AB and CD. Similarly, point P_2 is equidistant from AB and CD.



The locus of a point equidistant from two intersecting lines is the pair of bisectors of the angles between the intersecting lines. These bisectors are always at right angles to each other.

The shortest distance of a point from a line is its perpendicular distance from the line.

Intersecting loci

Under certain conditions, different loci can have a common point or region.

Example 5.5

Construct triangle ABC in which AB = 9 cm, AC = 8 cm and BC = 6 cm.

(a) Construct the locus of the points equidistant from A and B.

(b) Construct the locus of the points that are equidistant from AB and AC,

(c) Mark the position of point N that is equidistant from A and B and also equidistant from AB and AC.

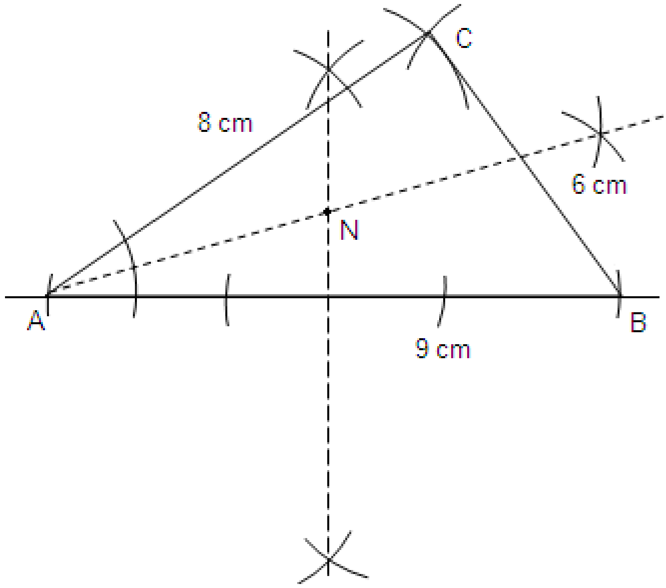
Solutions

(a) The following figure shows the constructions.
The locus of the points equidistant from A and B is the perpendicular bisector of line AB.

(b) The locus of the points that are equidistant from AB and AC is the bisector of angle BAC.

(c) Point N is located at the intersection of the bisectors of line BC and angle BAC as shown on the diagram.

All the construction marks and arcs should be clearly shown. It is important to *use a sharp pencil in order to draw neat diagrams.*



Exercise 5.2

1. Using a ruler and a pair of compasses only, draw a triangle ABC in which AB = 6.2 cm, BC = 5.0 cm and $A\hat{B}C = 60^0$. Construct the line passing through C and perpendicular to AB, to cut AB at X. Measure CX.

2. Only a ruler and a pair of compasses should be used in this question. Draw a parallelogram ABCD in which AB = 4.0 cm, BC = 7.6 cm, and $A\hat{B}C = 45^0$. Also draw the perpendicular line to BC through A. Measure the distance between A and BC. Hence calculate the area of ABCD.

3. Using a ruler and a pair of compasses only, draw a triangle PQR such that PQ = QR = 8.5 cm and $P\hat{Q}R = 120^0$.
Draw the incircle of triangle PQR and measure its radius. Calculate the area of the incircle.

4. ABC is a triangle in which AB = 6.0 cm, BC = 9.0 cm and AC = 4.5 cm. P is a point such that $A\hat{P}B = 90^0$ and PB = PC. Using a ruler and a pair of compasses only, construct ΔABC and the locus of P. How many possible positions are there for P?
Measure the distance between the vertex B and each of the possible positions of P.

5. Using a ruler and compasses only:
   (a) Construct the triangle ABC in which AB = 7 cm, BC = 5 cm and AC = 6 cm.
   (b) Construct the circle which passes through A, B and C and measure the radius of the circle.
6. Answer the whole of this question on a sheet of graph paper.
   Using a scale of 1 cm to represent 1 unit on each axis, draw a pair of axes for $0 \le x \le 18$ and $0 \le y \le 14$.
   (a) On your axes:
      (i) draw the line $y = 2x$,
      (ii) mark the two points A(10, 0) and B(16, 5),
      (iii) construct the locus of points which are equidistant from the points A and B,
      (iv) draw the circle which touches the x-axis at A, and which passes through B.
   (b) Draw the tangent to the circle at B, and write down the coordinates of the point at which it cuts the x-axis.
7. Draw a triangle with sides of 7 cm, 9 cm and 10 cm. Construct its circumscribed and inscribed circles. By measurement, find the difference in length of the radii of these circles.

8. Using a ruler and pair of compasses only, draw triangle ABC such that AB = 6 cm, BC = 5 cm and CA = 4 cm. Find by construction, a point P (other than B) which lies on the circumcircle of triangle ABC and is equidistant from AB and BC. Measure PA.

9. Without using a set square or a protractor,
   (a) Construct triangle ABC in which BC is 6.3 cm, $A\hat{B}C = 60^0$ and $B\hat{A}C = 90^0$.
   (b) Mark point D on line BA produced such that AD = 3.3 cm.
   (c) Construct:
      (i) a circle that touches lines AC and AD.
      (ii) a tangent to this circle parallel to line AD.

10. Points P, Q and R lie on a circle, centre O and radius 3.5 cm. Angle POR = 120^0 and Q lies on a minor arc PR. Arc PQ is 4 times arc QR. Lines OQ and PR meet at point X.
   (a) Draw an accurate sketch to represent this information.
   (b) Calculate:
      (i) $R\hat{P}O$
      (ii) $R\hat{X}Q$
      (iii) $P\hat{Q}R$
      (iv) $R\hat{P}Q$

11. The following diagram shows a sketch of a field.

(a) Using a scale of 1 cm to represent 5 m, construct an accurate plan of the field.

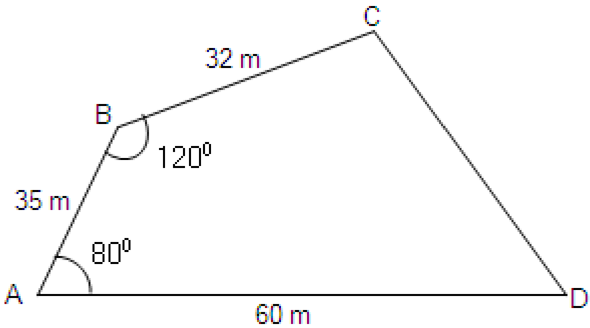
(b) A post P is situated in the field, such that it is equidistant from sides CD and CB, and also equidistant from points A and B. On your diagram, construct, using a ruler and a pair of compasses only:

(i) the locus of points that are equidistant from CD and CB.

(ii) the locus of points that are equidistant from A and B.

(c) Indicate the positions of post P on your diagram.

(d) A goat is tethered to post P by a rope of length 20 m. Shade the part of the field which the goat cannot reach.



Chapter 6

Commercial arithmetic

Currency conversion

Currency is the money system in use in a country, for example, Uganda shillings (Ush.), Kenya shillings (Ksh.), US dollar (US $), Pound sterling, e.t.c.

The currency of another country, known as, **foreign currency**, can be bought and sold at a given **exchange rate**

Example 6.1

(a) Given that 1 US dollar is equivalent to Ush. 1800, what is the value, in US $, of Ush. 54,000?

(b) Convert US $ 50 to Uganda shillings (Ush.) if US$ 1 = Ush. 1750.

Solution

(a) $ 1 is equivalent to Ush. 1,800.

Therefore, Ush. 1 will be equivalent to $\$\frac{1}{1,800}$

Hence, Ush. 54,000 is equivalent to $\$\frac{1}{1,800}\times 54,000 = \$30$.

(b) US$ 50 = Ush. (1750 $\times$ 50)
= Ush. 87,500.

Exercise 6.1

1. A man in Nairobi received Ush, 500,000 which he exchanged for Kenya shillings at the rate of Ksh. 1 to Ush. 22. How many Kenya shillings did he receive?

2. Given that 1 Japanese Yen = Ush. 1,300, convert 2,800 Japanese Yen to Uganda shillings.

3. A tourist had 12,000 US dollars. He exchanged this money for UK sterling pounds and then for Uganda shillings. How many Uganda shillings did he get given that 1 UK pound sterling = 1.4711 US dollars and 1 Sterling pound = Ush. 2,780.

4. Musa visited Kenya for a holiday. He had Ush.1,500,000 The exchange rate was Ush. 23 to Ksh. 1. He spent Ksh, 60,000 during his stay in Kenya and changed the remainder back to Uganda shillings at the rate of Ksh. 1 = Ush. 24.8. How many Uganda shillings did he receive?

5. A firm in France bought coffee worth Ush. 35 000 000 from Uganda. How much did the firm pay in Euros given that:
   1 Euro = 0.9461 US dollars and 1 US dollar = Ush. 1 950.

6. A Ugandan bought a car from Japan for Ush. 6 000 000. How much did he pay for the car in Japanese Yen if 1 Japanese Yen = 1.13 US dollars and 1 US dollar = Ush. 1 720.

7. A Forex Bureau buys one US dollar at Ush.1,900 and sells one Pound Sterling at Ush.3,450. Atim wants to exchange 3,000 US dollars to pound sterling. How many pounds sterling will she get?

Profit and Loss

Cost price is the price at which goods are bought. It is also called the buying price or purchase price. **Selling price** is the price at which goods are sold by the second seller (retailer).

If the selling price is more than the cost price, we say that a **profit** has been made. Thus, profit = selling price – cost price.

Sometimes the cost price is more than the selling price. We call the difference between the two a loss. Thus, loss = cost price – selling price.

Percentage profit and loss

It is common to express the profit or loss as a percentage. Usually the percentage profit or loss is calculated as a percentage of the cost price. Thus,

$$\text{percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\% \text{ and,}$$

$$\text{percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%.$$

Example 6.2

A trader buys goods for sh. 300 000 and sells them for sh. 360 000. Calculate the percentage profit.

Solution

Cost price = sh. 300 000
Selling price = sh. 360 000
Profit = sh. 360 000 – sh. 300 000
= sh. 60 000

$$\text{Percentage profit} = \frac{60000}{300000} \times 100\%$$
$$= 20\%.$$

Example 6.3

A shopkeeper sells a dress for sh. 23 000 thereby making a profit of 15%. Calculate the cost price of the dress.

Solution

Selling price = sh. 23 000
This is 115% of the cost price which is 100%

Therefore, cost price = $\frac{100}{115} \times 23000$ = sh. 20 000.

Alternatively, let the cost price be sh. X
Selling price = sh. 23 000
Percentage profit = 15%

$$= \frac{23000 - x}{x} \times 100\%$$

Therefore, $\frac{23000 - x}{x} \times 100$ = 15

2 300 000 - 100x = 15x
2 300 000 = 115x

$$x = \frac{2300000}{115} = 20\,000$$

Therefore, cost price = sh. 20 000.

Exercise 6.2

1. A man bought a bicycle at sh. 58 000. He sold it at a profit of 10%. What was his selling price?

2. A dealer buys an item for sh. 8 000. He wishes to make a profit of 35%. What should the selling price be?

3. A radio that cost sh. 15 000 was sold at a loss of 18%. What was the selling price?

4. A shopkeeper sells a bag for sh. 32 000, making a loss of 20%. What is the cost price?

5. By selling a table for sh. 31 500, a shopkeeper makes a profit of 15%. Calculate the actual profit.

6. An agent buys 200 items at a total cost of sh. 600 000. She sells 150 of them at a profit of 25% and the remainder at a loss of 8%. Find the

amount of her net profit and express it as a percentage of the initial cost of all the items.

7. A retailer buys 30 boxes of strawberries at sh. 3000 each and sells 27 boxes at 30% profit. How much profit does he make?

8. At a sale, the marked prices were reduced by 10%.
   (a) How much would a buyer have to pay for a shirt marked sh. 25 000?
   (b) If a customer pays sh. 48 000 for a dress. What is the marked price?

Discount

Sometimes a business reduces a small fraction of the selling price if the customer either pays cash or buys a lot of goods. This reduction is called discount. Most of the time, discounts are expressed as a percentage of the original price.

Example 6.4

A television set is selling at sh. 150 000. A customer is offered 4% discount for paying cash. How much does the customer pay for the TV?

Solution

Discount = 4% of sh. 150 000
= sh. 6 000.

The customer pays sh. 150 000 – sh. 6 000 = sh. 144 000

Commission

Sometimes firms use agents who are not actually their employees to sell their goods. The agent is usually paid a commission for selling the goods. Commission is normally given as a percentage of the sales made.

Example 6.5

A saleslady sold goods worth sh. 750 000. She was paid 2% commission on the sales. How much commission did she get?

Value of goods sold = sh. 750 000

Commission paid is = $\frac{2}{100} \times 750000$
= sh. 15 000

Interest

Simple interest

If you borrow money from a bank or other financial institution then you will have to pay **interest** (the charge paid for borrowing) in addition to your repayments. The money borrowed or lent is called the **principal**. When interest is paid at fixed intervals, yearly, half-yearly, quarterly or monthly, the principal is said to be lent (or borrowed) at simple interest.
The interest is calculated on the original principal only. The investor receives interest at regular periods, the principal remains the same. Simple interest is calculated using the following formula

$$\text{Interest} = \frac{P \times R \times T}{100}$$

Where, P = principal, R = rate of interest per annum (%); T = time (in years).
Note that the units for R and T must be consistent, i.e.
If R is per annum, T must be in years,
If R is per month, T must be in months, e.t.c.
When the simple interest for any given time is added to the principal, the sum is called the **amount** at simple interest for that time.

Amount = Principal + Interest, i.e. A = P + I.

Example 6.6

Find the simple interest on sh. 25 000 for 3.5 years at 18% per annum.

Solution

$$I = \frac{P \times R \times T}{100} = \frac{25000 \times 18 \times 3.5}{100}$$
$$= \text{sh. } 15\ 750.$$

Example 6.7

Find the simple interest on sh. 20 000 for $1\frac{3}{4}$ years at $1\frac{1}{2}\%$ per month. Find also, the amount after $1\frac{3}{4}$ years.

Solution

Time = $1\frac{3}{4}$ years × 12 = 21 months; P = sh. 20 000; R = $1\frac{1}{2}\%$ per month

Therefore, $\frac{20000 \times 1\frac{1}{2} \times 21}{100}$ = sh. 6 300.

Amount = Principal + Interest
= 20 000 + 6 300 = sh. 26 300.

Compound interest

In most financial institutions, interest is added to the money borrowed or lent and then the interest is calculated on this total amount for the next period. Adding the interest is known as compounding the interest, or just compound interest.

Compound interest = Final amount – original principal.

Note: Simple interest is the same for each period, compound interest becomes greater for successive periods.

Example 6.8

Calculate the compound interest on sh. 2 000 for 2 years at 8% per annum.

Solution

First year: Principal = 2 000

Interest = 160 calculated as $I = \frac{2000 \times 8 \times 1}{100} = 160$

Amount = 2000 + 160 = 2 160

Second year:

Principal = 2 160

Interest = 172.80, calculated as $I_2 = \frac{2160 \times 8 \times 1}{100} = 172.80$

Amount = 2 160 + 172.80 = 2 332.80

Compound interest = Amount – Principal

= 2 332.80 – 2 000 = sh. 332.80

Alternatively, the compound interest can be calculated using the following formula:

$$A = P\left(1 + \frac{R}{100}\right)^n$$

where, A is the amount after n years; P = principal; R is the rate % p.a. and n is the number of years. If interest is added half yearly, the value of R is half of the given R% p.a. value and n is doubled.

So, when P = 2 000, R = 8 and n = 2,
Amount = $2000(1.08)^2$ = 2 332.80
Interest = Amount – principal
= 2 332.80 – 2000
= sh. 332.80

Appreciation and depreciation

Appreciation is a term used to describe an increase in value of an asset. On the other hand, depreciation is used to mean a decrease in value of an asset. The increase or decrease in value is usually expressed as a percentage of the original value.

Note: The depreciation is calculated as a percentage of the remainder after the first period's depreciation has been subtracted.

For example, if a car depreciates at the rate of 10% per year, car costing Sh. 5 000 000 will depreciate by $\frac{10}{100} \times 5000000$ after 1 year.

Therefore, its value after 1 year = 5 000 000 – 500 000 = sh. 4 500 000.

After 2 years its depreciation will be $\frac{10}{100} \times 4500000 = 450\ 000$

Therefore, its value after 2 years will be 4 5000 000 – 450 000 = sh. 4 050 000.

(The value of the car after 2 years is **not** 5 000 000 – 50 000 – 50 000 = 4 900 000)

Exercise 6.3

1. As a result of a civil war, the population of a town decreased by 4% of its total population of 425 000 at the beginning of the year. Then after, it increased by 2% of its size at the beginning of the year for consecutive five years. Calculate, to the nearest thousand, its population at the end of this period.
   Calculate also, correct to one decimal place, the percentage increase over this period.

2. Find the amount to which sh. 10 000 accumulates in 12 years at 9% per annum compound interest.

3. A savings and credit society granted a short-term loan of sh. 65 000 at 20% per month simple interest. Calculate the interest after 3 months.

4. The amount of an investment at 4% p.a. compound interest after 6 months is sh. 54 200. Calculate the principal.

5. A man loans a friend sh. 500 000 for 2 years at 7% compound interest . How much will he receive when the loan is repaid?

6. A car rental company hires out cars as follows: sh. 23 000 per day and sh. 1 000 per kilometer covered. They offer a discount of 40 km free each day of hire. A man hires a car for 5 days and drives for 350 km. Calculate the total cost.

7. The price of a car when new is sh. 14 000 000. After one year its market value depreciates by 15%. In each subsequent year it depreciates by 10% of its value by the beginning of the year. Find its value to the nearest shillings, at the end of three years.

8. A business man had sh. 1 200 000 and divided it in the ratio 3:2. He used the larger amount to buy a car, and invested the remainder in a bank which paid simple interest at a rate of 8% per annum. After 18 months, he sold the car at 30% less than what he bought it at. He also withdrew his money and interest from the bank. Calculate:
   (a) the amount he invested in the bank.
   (b) the amount for which he sold the car.
   (c) the total amount he withdrew from the bank.
   (d) the percentage of the sh. 1 200 000 he had after selling the car.

9. A car valued at sh. 8 900 000 is supposed to depreciate each year at 10% of its value at the beginning of the year. Find its value after three years.

10. A plot of land bought for sh. 5 000 000 appreciated by 12% in the first year and subsequently for 2 more years at 10%. Find its value after 3 years.

11. A man is offered a choice for his salary:
   (a) Starting salary: Ush. 15 000 000 per year with an annual increase of 10% of the salary at the beginning of the year for three years.
   (b) Starting salary: Ush. 16 500 000 per year with an annual increase of 5% of the salary at the beginning of the year for 3 years.
      (i) what will be his total earnings in the first 3 years in each case?
      (ii) What will be his salary at the beginning of the fourth year in each case?

12. The value of the machinery in a factory depreciates each year by 25% of the value at the beginning of the year. If it was valued at sh. 750 000 after 3 years, find its value when new.

13. For a fixed deposit for 4 years, a bank offers the rates of interest: 10% in the first year, 12% in the second year and 15% in each subsequent year. Find the sum to which a fixed deposit of sh. 800 000 will amount after 4 years.

14. The price of a car was increased by 10% to sh.8,800,000.
   (i) What was the original price of the car?
   (ii) If Tom bought this car at this increased price and sold it a year later at 20% discount. Express Tom's selling price as a percentage of the original price.

15. Find the rate of compound interest levied on sh.8,000 when it has been deposited for 3 years and earned sh.27,000.

Hire Purchase

This is a system of payment where a customer is allowed to buy an item by paying part of the price in cash and then making a fixed payment each month for a number of months.
The first payment is called **deposit** or **down payment**, the monthly fixed payment is called monthly **installment**.
The hire purchase price is usually more than the marked price.

Example 6.9

The marked price of a gas cooker is sh. 450 000. A dealer charges 20% more under hire purchase. If the deposit is sh. 30 000, calculate the amount of monthly installments if there are 12 equal installments.

Solution

Marked price = sh. 450 000.
Hire purchase price = sh. 120% of sh. 450 000

$$= \frac{120}{100} \times 450000$$

= sh. 540 000
Deposit = sh. 30 000

Monthly installments = $\frac{540000 - 30000}{12}$

= sh. 42 500

Example 6.10

A colour TV set is available under hire purchase on payment of a deposit of sh. 20 000 and ten equal monthly installments of sh. 20 000 each. If the cash price is sh. 200 000, calculate what percentage goes the dealer charge extra over the cash price?

Solution

Deposit = 20 000
Installments = 10 × 20 000 = 200 000
Hire purchase price = Deposit + Installments
= 20 000 + 200 000 = 220 000
Cash price = 200 000
Extra payment = 220 000 - 200 000 = sh. 20 000.

Therefore, percentage = $\frac{20000}{200000} \times 100\%$ = 10% of cash price.

Exercise 6.4

1. A sewing machine is sold under hire purchase: a deposit of sh. 25 000 and 12 monthly installments of sh. 16 500 each. If the hire purchase is 18% higher than the cash price, determine the cash price.

2. A dealer marks a price sh. 4 800 of an article. He charges 10% extra under hire purchase with a deposit of sh.800 and four equal monthly installments. Calculate the amount of monthly installments.

3. The cash price of a refrigerator is sh. 290 000 and its hire purchase price is 15% higher under 12 monthly installments of sh. 25 000 each. Determine the amount of deposit.

4. A dealer displays the following label on an article:
   Cash or down payment: sh. 250 000.
   12 equal monthly installments of sh. 500 000.
   If he is charging 25% higher than the cash price, determine the cash price of the article

5. The cash price of a bicycle is sh. 70 000. If the same is bought under HP terms, then there is a deposit of 10% and 12 monthly installments of sh. 7 000. Find the difference between the two prices.

6. A scientific calculator is marked at sh. 45 000. Under hire purchase it is available for a down payment of sh. 20 000 and six monthly installments of sh. 6 000 each. Calculate the
   (a) hire purchase price
   (b) extra amount paid over the cash price.

7. The marked price of a article is sh. 25 000. James bought it by paying a deposit of sh. 5 000 and a number of equal monthly installments of sh. 2 500. If the hire purchase price is 20% higher than the marked price, calculate the number of installments.

8. A cash discount of 10% is allowed on a refrigerator whose marked price is sh. 200 000. Hire Purchase terms are: a deposit of 20%, and 12 monthly installments of sh. 19 500 each. Find the difference between the cash price and the payment required under HP terms

Income Tax

This is a tax levied on peoples' incomes. Income is a payment received by someone who gets involved in a legal gainful activity. Examples include: Profit, salary, wages, interest, commission, fees, rent, overtime pay e.t.c.

Gross income is the total income that an individual receives from wages, salary, leave pay, overtime pay, medical allowance, transport allowance e.t.c.

Taxable income is the income on which income tax is levied. It is arrived at by deducting from the gross income the amount of allowances.
Thus, **taxable income = gross income – allowances**

Example 6.11

In a certain country income tax is levied as follows: A person's monthly gross Income has certain allowances deducted from it before it is subjected to taxation.

Each child below ten years sh. 5 000
Each child above ten years but less than 18 years – sh. 7 000
Married man: sh. 18 000
Transport allowance: sh. 17 000

David earns sh. 450 000 per month, he is married with 3 children of ages between 2 and 10 years, 2 children above twelve but less than 18 years.
Calculate:

(i) the taxable income of David
(ii) the income tax he paid; if the rates are:

Taxable income (sh)	**Rate(%)**
0 – 100 000	10
100 001 – 200 000	20
200 001 – 300 000	30
300 001 – 400 000	45
400 001 - and above	50

Solution

(i) David's allowances

Marriage		18 000
Transport		17 000
3 children	3 x 5 000 =	15 000
2 children	2 x 7 000 =	14 000
Total allowances		64 000

Taxable income = 450 000 – 64 000
= sh. 386 000

(ii) income | | | tax (sh.)

income			tax (sh.)
100 000	$100\ 000 \times \frac{10}{100}$	=	10 000
100 000	$100\ 000 \times \frac{20}{100}$	=	20 000
100 000	$100\ 000 \times \frac{30}{100}$	=	30 000
86 000	$86\ 000 \times \frac{45}{100}$	=	38 700
Total income tax	=		sh. 98 700

Example 6.12

The table below shows the income tax rate of a certain country for government employees. This is applied after the allowances have been already deducted.

Taxable income (sh)	Rate(%)
0 - 100 000	0
100 001 - 200 000	5
200 001 - 300 000	10
300 001 - 450 000	20
450 001 - 550 000	30
550 001 - and above	50

An employee has a gross monthly income of sh. 600 000 and is entitled to the following allowances.
Marriage: sh. 120 000 per annum
Housing and transport: 10% of the gross monthly income.
Medical care: sh. 240 000 per annum.

Calculate:
(i) the amount an employee pays as monthly income tax.
(ii) the net monthly income.

Solution

(i) Allowances:

Marriage $120\ 000 \div 12$ = 10 000

Housing and transport $\frac{10}{100} \times 600000$ = 60 000

Medical care sh. 240 000 ÷ 12 = 20 000

Total allowances = 90 000

Taxable income = monthly gross income - monthly allowances
= 600 000 - 90 000
= sh. 510 000.

Income (sh.):			tax (sh.)
100 000			0
100 000	$\frac{5}{100} \times 100000$	=	5 000
100 000	$\frac{10}{100} \times 100000$	=	10 000
150 000	$\frac{20}{100} \times 150000$	=	30 000
60 000	$\frac{30}{100} \times 60000$	=	18 000
Total monthly income tax		=	63 000.

Therefore, the total monthly income tax is sh. 63 000

(ii) Net monthly income = gross monthly income - income tax.
= 600 000 - 63 000
= sh. 537 000.

Exercise 6.5

1. In a certain country, income tax is computed after deducting the following allowances:

TYPE OF ALLOWANCE	AMOUNT (USH.)
Marriage	10 000
Single	4 000
Each child above 10 but below 20 years	3 000
Each child under 10 years	2 000

Omoja is married with 3 children, two below 10 years of age and the other child 12 years old. Mbili is single but has two dependants aged 11 and 15 years. Each month Omoja and Mbili earn gross incomes of sh. 130 000 and sh. 120 000 respectively. The income tax is calculated as follows:

Ush.	%age tax

1st:	01 - 10 000	20
Next:	10 001 - 50 000	15
Rest:	50 001 - and above	10

(a) Calculate the:
    (i) taxable income for Omoja and Mbili,
    (ii) income tax for Omoja and Mbili,
(b) Express the total income tax for each man as percentage of their respective taxable incomes.

2. In a certain country, tax is levied on income per month as follows:

Income per month (sh)	Tax Rate(%)
0 - 50 000	10
50 001 - 100 000	15
100 001 - 150 000	20
150 001 - 200 000	30
200 001 - 250 000	35
250 001 - 300 000	40
300 001 - 350 000	45
350 001 - and above	55

Tom's gross monthly income is sh. 766 000.

The allowances given to him are:

Housing allowance:	sh. 10 000 pe month
Marriage allowance:	sh. 919 200 per annum.
Medical allowances:	sh. 50 000 per month
Transport allowances:	sh. 120 000 per annum
Insurance premium:	sh. 72 000 per annum

Tom is married with 5 children: 2 above 10 years but below 18 years, 1 is below 10 years while 2 are above 18 years. The rate per child are as below:

Age	rate:
Below 10 years	sh. 3 000
Between 10 and 18 years	sh. 4 000
Above 18 years	sh. 5 000

(a) Calculate:

(i) Tom's taxable income,
(ii) the income tax Tom pays.

(b) Express his income tax as a percentage of his monthly income.

3. (a) Adikini bought a TV set for which the cash price was shs. 599,000. She bought the TV set on hire purchase terms and had to pay an extra sh. 71 000. If she made eight equal monthly instalments, how much did she pay per month?

(b) Mukasa wants to buy a house which is priced at sh. 56,000,000. A deposit of 25% of the value of the house is required. A bank will lend him the rest of the money at a compound interest of 15% per annum and payable after two years.

Calculate the:

(i) deposit Mukasa must make.
(ii) amount of money Mukasa will have to pay the bank after two years.
(iii) total money which Mukasa will spend to buy the house.