## PROBABILITY THEORY REVISION QUESTIONS

1. Events $A$ and $B$ are such that $P(A)=0.4, P(B)=0.45$ and $P(A \cup B)=0.68$. Find;
(a) $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$,
(b) $\mathbf{P}\left(\mathrm{A}^{\prime} \cap \mathrm{B}\right)$
(c) $\mathrm{P}\left(\mathrm{A}^{\prime} \cup \mathrm{B}^{\prime}\right)$.
(d) $\mathrm{P}\left(\mathrm{A} \cup \mathrm{B}^{\prime}\right)$
2. In a competition of several participants, the probability that candidate A wins is 0.1 . Candidate B has a $30 \%$ chance of winning and C has a $2 \%$ chance to win. Find the probability that;
(i) Either A or B or C wins the competition.
(ii) Someone else wins the competition apart from $\mathrm{A}, \mathrm{B}$ and C .
3. If $A$ and $B$ are two events and $P(A)=0.6, P(B)=0.3$ and $P(A \cup B)=0.8$, find;
(i) $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$
(ii) $\mathrm{P}(\overline{\mathrm{A}} \cap \bar{B})$
4. X and Y are independent events where $\mathrm{P}\left(\mathrm{X}^{\prime}\right)=0.6, \mathrm{P}(\mathrm{X} \cup \mathrm{Y})=0.8$. Find;
(i) $\mathrm{P}(\mathrm{Y})$,
(ii) $\mathrm{P}(\mathrm{X} \cap \mathrm{Y})$
(iii) $\mathrm{P}\left(\mathrm{X} \cup \mathrm{Y}^{\prime}\right)$.
5. The probabilities that events $S$ and $T$ occur are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that only one of them occurs is $\frac{5}{12}$. Find the probability that;
(i) Both events occur.
(ii) None of them occurs.
6. Three boys; Paul, Robert and Maurice aim at a target. The probabilities of them hitting the target are $0.8,0.7$ and 0.6 respectively. Find the probability that;
(i) All the three hit the target,
(ii) Exactly two of them hit the target.
7. Events A and B are such that $\mathrm{P}(\mathrm{A})=\frac{4}{7}, \mathrm{P}\left(\mathrm{A} \mathrm{n} \mathrm{B}^{1}\right)=\frac{1}{3}$ and $\mathrm{P}(\mathrm{A} / \mathrm{B})=\frac{4}{5}$. Find
(i) $\mathrm{P}(\mathrm{B})$,
(ii) $P\left(A^{1} n B^{1}\right)$
8. According to the firm's internal survey, of those Employees living more than 2 miles from work, $90 \%$ travel to work by car, of the remaining employees, only $50 \%$ travel to work by car. It is known that $75 \%$ of the Employees live more than 2 miles from work. Determine;
(i) The overall proportion of the Employees who travel to work by car.
(ii) The probability that an employee who travels to work by car lives more than 2 miles from work.
9. The probability that a fisherman catches fish on a clear day is $\frac{1}{5}$ and on cloudy day is $\frac{7}{10}$. If the probability that the day is cloudy is $\frac{3}{5}$, find the probability that the day is cloudy given that the fisherman does not catch fish.
10.Events $A, B$ and $C$ are such that $P(A)=x, P(B)=y$ and $P(C)=x+y$. If $P(A U B)=$ 0.6 and $\mathrm{P}(\mathrm{B} / \mathrm{A})=0.2$,
(i) show that $4 x+5 y=3$,
(ii) Given that B and C are mutually exclusive and that $\mathrm{P}(\mathrm{BUC})=0.9$, determine the values of $x$ and $y$.
11.John wishes to send a message to Mary. The probabilities that he uses e-mail, letter or personal contact are $0.4,0.1$ and 0.5 respectively. He uses only one method; the probabilities of Mary receiving the message if John uses e-mail, letter or personal contact are $0.6,0.8$ and 1 respectively.
(i) Find the probability that Mary receives the message.
(ii) Given that Mary receives the message find the probability that she received it via e-mail.
12.Events $A$ and $B$ are such that $P(A)=0.35, P(B)=0.7$ and $P\left(A^{\prime} n B^{\prime}\right)=0.15$.
Find the;
(i) $\mathrm{P}(\mathrm{A} / \mathrm{B})$
(ii) $\mathrm{P}\left(\mathrm{A} n \mathrm{~B}^{\prime}\right)$
10. In a certain school, the probability that a boy chosen randomly belongs to the football team is 0.6 and that of the chess team is 0.5 . The probability that he belongs to at least one of the teams is 0.9 . Find the probability that he belongs to either the football team or the chess team but not both.
11. Given that $A$ and $B$ are independent events and that $A^{\prime}$ and $B$ ' are respectively their complements.
(i) Prove that A' and B' are also independent.
(ii) If $\mathrm{P}(\mathrm{A})=0.5$ and $\mathrm{P}(\mathrm{B})=0.4$, Find the $\mathrm{P}\left(\mathrm{A}^{\prime} \mathrm{n} \mathrm{B}^{\prime}\right)$
12. Bag A contains 4 red and 6 green sweets. Bag $B$ contains 3 red and 7 green sweets. Bag A is twice as likely to be picked as bag B. A bag is randomly selected and from it a sweet is picked randomly and put into the other bag. A sweet is then picked from the latter bag. Find the probability that the sweet picked from the bag is red.
13. $A$ and $B$ are two independent events with $A$ twice as likely to occur as $B$. If $P(A)=$ $1 / 2$, find the; (i) P (A or B but not both) (ii) $\mathrm{P}(\mathrm{A} / \mathrm{B}$ ')
14. One bag of chocolates contains five hard centred and three soft centred chocolates. Another bag of chocolates contains eight hard centred and seven soft centred. A chocolate is chosen at random from either of the bags. Find the probability that a soft centred chocolate came from the first bag.
15. A man travelling from Masaka to Kampala by a private car goes through three police checking points. The probability that he is delayed by checking point A is 0.3 , for checking points B and C it is about 0.5 and 0.7 respectively. Find the probability that;
(i) He is not delayed at all.
(ii) He is delayed at only one checking point.
16. Events $A$ and $B$ are such that $P(A / B)=\frac{1}{3}, P\left(B / A^{1}\right)=\frac{5}{8}$ and $P\left(A^{1} \cap B^{1}\right)=\frac{3}{20}$. Find;
(i) $\mathrm{P}\left(\mathrm{A} \cap \mathrm{B}^{1}\right)$
(ii) $\mathrm{P}(\mathrm{A} / \mathrm{B})$
20.A box contains 6 black, 5 red and 4 green balls. Three balls are picked at random one at a time without replacement. Find the probability that:
(i) All the balls picked are of the same colour
(ii) The third ball picked is the second black ball to be picked
17. If events $X$ and $Y$ are independent.
(a) Show that events X ' and Y are also independent.
(b) Find $\mathrm{P}\left(\mathrm{X} \cap Y^{\prime}\right)$, given that $\mathrm{P}\left(\mathrm{X}^{\prime}\right)=3 / 4$ and $\mathrm{P}(\mathrm{Y})=2 / 5$.
18. Dan's probabilities of passing Biology, Chemistry and Mathematics are $0.6,0.75$ and 0.8 respectively.
(i) Find the probability that he will pass at least two subjects.
(ii) If it is known that he passed at least two subjects, what is the probability that he failed Chemistry?
19. Events $A$ and $B$ are such that $P(A)=\frac{4}{7}, P\left(A \cap B^{1}\right)=\frac{1}{3}$ and $P(A / B)=\frac{5}{14}$. Find; (i) $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$, (ii) $\mathrm{P}(\mathrm{B})$.
20. In a car assembly plant, machines A, B and C produce $37 \%, 42 \%$, and $21 \%$ respectively of the total production. If $0.6 \%$ of the production form A is defective and that from B and C are $0.4 \%$ and $1.2 \%$, find the probability that a car selected at random from the plant;
(i) Is defective
(ii) Came from C given that it is defective.
21. Two events $A$ and $B$ are such that $P(A)=0.75 \mathrm{P}(\mathrm{B}), \mathrm{P}(\mathrm{A} \cup \mathrm{B})=\frac{7}{10}, \mathrm{P}(A \mid B)=\frac{7}{12}$,
calculate the; (i) $\mathrm{P}(\mathrm{B})$
(ii) $\mathrm{P}(\mathrm{A} \cap \mathrm{B})$
(iii) $\mathrm{P}(B \mid A)$
22. $A$ and $B$ are independent events with $P(A)=\frac{3}{8}$ and $P\left(A^{\prime} \cup B\right)=\frac{3}{4}$, Find; (i) $P(B)$, (ii) $P(A \cup B)$
23. A box $P$ contains 1 red bead, 3 green beads and 1 blue bead. Box $Q$ contains 2 red beads, 1 green bead and 2 blue beads. A balance die is thrown and if the throw shows a six, box P is chosen otherwise box Q is chosen. A bead is drawn at random from the chosen box. Given that a green bead is drawn, find probability that it came from P .
24. $A$ and $B$ are mutually exclusive. If $P(A U B)=3 / 4$ and $P\left(A^{1} / B^{1}\right)=2 / 3$; find; (i) $\mathrm{P}(\mathrm{A})$, (ii) $\mathrm{P}(\mathrm{B})$.
25. A bag contains 5 black marbles and 3 white marbles. A second bag contains 3 black marbles and 5 white marbles. A marble is drawn at random from the first bag and placed in the second bag. A marble is then drawn at random from the second bag and placed in the first. Find the probability that each bag now contains;
(i) 4 black and 4 white marbles, (ii) exactly the same number of each colour as it did initially.
30.If $\mathrm{P}(\mathrm{X})=0.4, \mathrm{P}(\mathrm{X} / \mathrm{Y})=0.4$ and $\mathrm{P}(\mathrm{X} \cap \mathrm{Y})=0.25$, find; (i) $\mathrm{P}(\mathrm{XUY})$, (ii) $\mathrm{P}\left[\mathrm{XUY} / \mathrm{Y}^{\mathrm{I}}\right]$.
